MATHEMATICS

Paper 9709/12
Pure Mathematics

Key messages

It should be noted that where answers or conclusions are given, as in **6(i)**, **11(i)** and **11(ii)**, the working out leading to these should be clear and complete. Often the presence of an un-rounded final answer which rounds to the given result can show the candidate has used a correct method.

Where answers are requested in a specific form, as in **4(i)**, **4(ii)** and **7(ii)**, it is advisable and sometimes required to use that form throughout the question.

Candidates are expected to be able to calculate and clearly state the range of a function from its domain.

General comments

Most candidates were well prepared for the examination and demonstrated a broad range of mathematical skills. The questions involving calculus and the binomial expansion were particularly well answered.

Comments on specific questions

Question 1

A very accessible question for most candidates and generally answered very well. The integration and substitution to find the constant were almost always seen and often led to a completely correct answer. Un-simplified answers were acceptable.

Answer: $y = 2x^{\frac{1}{2}} - 3x + 2$

Question 2

(i) Candidates were very familiar with this type of problem with many choosing the appropriate terms to find the correct coefficients rather than using the whole or part of the expansion.

(ii) The majority of candidates who completed part **(i)** were able to use their terms to find the required coefficient. Again a minority found the whole expansion but most were careful to pre-select the required products.

Answers: **(i)** 84 and –280 **(ii)** –140

Question 3

(i) Most candidates were able to identify the arithmetic progression with $d = 1.2$ and apply the nth term formula. Only those who realised that for $a = 40$, $n = 61$ obtained the correct term.

(ii) As in part **(i)**, most candidates were able to identify the geometric progression and find its common ratio correctly. Although most knew the nth term formula, here again the use of $n = 60$ rather than $n = 61$ was often seen.

Answers: **(i)** 112 (cm) **(ii)** 236 (cm)

Question 4

(i) Since the question requested the answer in exact form only those candidates whose methods led to a final exact answer gained credit in this part. Those who were successful used a variety of methods as alternatives to the simple application of $\tan\frac{\pi}{6}$ or $\tan\frac{\pi}{3}$.

(ii) Although the final answer was required in exact form some allowance was made to candidates who used correct methods with their non-exact answers to part **(i)**. Methods for finding the equation of the perpendicular bisector were well understood and applied correctly by the majority of candidates.

Answers: **(i)** $2\sqrt{3}$ **(ii)** $y = x\sqrt{3} - 2$

Question 5

(a) The majority of attempts cleared the fraction successfully to reach a quadratic in tan. Although there were many correct solutions, incorrect rounding or presentation of solutions in degrees were often evident. Some candidates did not consider the solution obtained from $\tan x = -2$.

(b) It was expected that k would be found from substitution of $(0, 2)$ into the curve equation and that α would be found from substituting $(150, 0)$. Some candidates proceeded in this way but others chose to expand $\sin(\theta + \alpha)$ and proceeded from there with varying levels of success.

Answers: **(a)** 0.464, 2.03 **(b)** $k = 4$, $\alpha = 30°$

Question 6

(i) A very well answered question with the cosine rule used slightly more frequently than simple trigonometry in one half of the isosceles triangle. As the answer is given it is important that full detail of working is shown.

(ii) Those candidates who saw that arc QR was part of the circle, centre P, subtended by angle OPQ usually went on to produce a completely correct solution. Some candidates preferred to work in degrees but most used a sufficient level of accuracy to find the perimeter correct to three significant figures.

Answer: **(ii)** 26.2

Question 7

(i) Most candidates were able to interpret the diagrams and made good attempts at answering this question. A lot of correct answers for $\overrightarrow{CE}$ were seen and for most of these the magnitude was calculated directly from the vector. Some candidates chose to return to the diagram and used Pythagoras' theorem to find the magnitude.

(ii) The use of the scalar product for this type of question was seen from many candidates. Those who were consistent in their use of $\overrightarrow{CE}$ and $\overrightarrow{CA}$ or $\overrightarrow{EC}$ and $\overrightarrow{AC}$ usually reached a correct expression. The surd form which followed from correct expressions for the scalar product was not always expressed in the required form.

Answers: **(i)** $-4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$, $\left|\overrightarrow{CE}\right| = 9$ **(ii)** $\cos^{-1}\frac{-1}{\sqrt{18}}$

Question 8

Most candidates made very good attempts at all three parts of this question and many completely correct solutions were seen.

(i) The requirement to differentiate was well understood and successfully applied. Some candidates solved a quadratic in $\sqrt{x}$ and squared their solutions whilst others chose to rearrange their quadratic and square both sides, usually correctly, to obtain *x* directly from the resulting quadratic in *x*.

(ii) Most correct expressions for $\frac{dy}{dx}$ were successfully differentiated to obtain the second derivative correctly.

(iii) Nearly all candidates who attempted this part used their result from part **(ii)** to examine the sign of the second derivative for their values of *x* from part **(i)**. The use of the sign change in $\frac{dy}{dx}$ around the turning points was rarely seen.

Answers: **(i)** 4 and 16 **(ii)** $1-3x^{\frac{-1}{2}}$ **(iii)** $x = 4$ *Maximum*, $x = 16$ *Minimum*

Question 9

(i) Nearly all candidates started this part by equating the expressions for *y* and removing the fraction to obtain a quadratic in *x* with two coefficients involving *c*. Many went on to use the discriminant to find the critical values for *c* and a few appreciated that the critical values were included in the required range.

(ii) It was expected that the critical values of *c* from part **(i)** would be used in the quadratic in *x* from part **(i)**. Some candidates chose this route but a significant number recalculated the critical values of *c* before substituting these into the quadratic. Those who chose to equate the gradient of the curve to *c* should have obtained four values of *x* from which two had to be eliminated for full credit. A better route for those using the gradient of the curve was to substitute this expression for *c* in the quadratic from part **(i)**.

Answers: **(i)** $c \leqslant -9$, $c \geqslant -1$ **(ii)** $x = -1$, $x = \frac{1}{3}$

Question 10

(i) Very few correct answers were seen to any of the three parts. Some otherwise correct answers were spoiled by being expressed as domains in terms of *x*.

(ii) Some correct explanations linking the range of *f* to the domain of *g* were seen but often the range and domain were confused.

(iii) The notation in this part was well understood with many correct expressions for $f'(x)$ and $f^{-1}(x)$. Those candidates who managed the challenge of solving the resulting equation were most successful when they used a substitution of $u = x - 2$ or $u = (x-2)^{-1}$. The few who solved the equation and used the critical values correctly almost always forgot to consider the domain of *f*.

Answers: **(i)(a)** $f(x) > 2$ **(b)** $g(x) > 6$ **(c)** $2 < fg(x) < 4$
(ii) The range of f is (partly) outside the domain of g **(iii)** $2 < x < 6$, $x > 14$

Question 11

(i) Most candidates realised they had to show the gradient of the curve to be equal to the gradient of the line *AB* at point *B*. The differentiation required to find the gradient of the curve was often completed correctly and the substitution of $x = \frac{1}{2}$ made to obtain the required gradient. It was sufficient to state that the gradient of the line was –2 to conclude this part. Those who chose to find the equation of the tangent to the curve at B and note that this was the equation of the line also gained full credit.

(ii) Many completely correct solutions to this part were seen, invariably starting from the premise that the required area was the area under the curve subtracted from the area under the line and demonstrating this with algebraic expressions of the appropriate integrals. Those who chose to evaluate these integrals at this stage had to be very thorough in their explanation to gain the available marks as did those who viewed the situation as the area under the curve taken from the area of triangle *OAB*.

(iii) The evaluation of this type of integral was well understood and many completely correct answers to this part were seen. The final answer was dependent on the sight of a correct integral so a final answer with no working gained no credit.

Answer: **(iii)** $\frac{1}{8}$

MATHEMATICS

Paper 9709/22
Pure Mathematics

Key messages

Candidates are reminded to read the rubric on the front of the examination paper and also ensure that they answer each question fully and to the required level of accuracy.

General comments

With only a very small cohort of candidates, general comments are difficult. The report has been written to reflect the methods that candidates were expected to use and highlight areas where common errors are made.

Comments on specific questions

Question 1

It was expected that candidates would use one of the two following methods:

EITHER

Form two linear equations to obtain the relevant critical values and hence the appropriate inequality.

OR

Form a three term quadratic equation by squaring both $5x+2$ and $4x+3$ and equating in order to obtain the critical values and hence the appropriate inequality.

A correct form of the inequality was needed to obtain the final accuracy mark. An answer of $1 < x < -\frac{5}{9}$ is not acceptable for the final accuracy mark.

Answer: $x < -\frac{5}{9}, x > 1$

Question 2

It was necessary to differentiate the given equation making use of the product rule. A substitution of $x=\pi$ into both the original equation and the derivative obtained gave enough information to be able to form the equation of the tangent required.

Answer: $y = 4x$

Question 3

(i) For four intervals it was necessary to have five *y*-values, meaning the width of each interval was 2. Keeping the *y*-values in terms of logarithms rather than evaluating them straightaway made the calculation using the trapezium rule far easier to do with less scope for error.

(ii) It was necessary to recognise that $x^2+4x+4=(x+2)^2$. An application of the power rule for logarithms meant that the integrand could be simplified to $6\ln(x+2)$. This meant that the answer to part **(i)** could be used, as implied by the word 'Hence' at the start of the question.

Answers: **(i)** 13.5 **(ii)** 81, 81.0 or 81.1

Question 4

(i) It was essential that each term of the expression was shown evaluated once a substitution of $x=-3$ had been made into p(x). Candidates were asked to show that (x+3) is a factor, hence the need to show each stage of the working clearly.

(ii) Either factorisation by observation or by use of algebraic long division in order to obtain a quadratic factor were acceptable. This quadratic factor could then be factorised to linear factors. Candidates must be careful if using 'solutions' to p(x)=0 in order to produce factors, as an answer of $(x+3)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{2}\right)$ is not the factorised form of p(x).

(iii) Candidates were expected to make use of their answer to part **(ii)** having simplified the appropriate terms in the given equation to form a cubic equation in 2^u which could be compared directly with p(x). Only one solution for 2^u was possible and hence just one solution for u, making use of logarithms and/or calculator.

Answers: **(ii)** $(x+3)(2x-5)(2x+1)$ **(iii)** 1.32

Question 5

(i) It was essential to show each part of the working in this part of the question as there is a given answer. Correct integration together with a correct substitution of the given limits were necessary, followed by correct rearrangement to obtain the given answer.

(ii) Requests of this type are often found difficult by candidates. It is expected that candidates form another function making use of $a=\frac{1}{2}\ln\left(12.5+e^{-4a}\right)$, such as $f(a)=a-\frac{1}{2}\ln\left(12.5+e^{-4a}\right)$ or $f(a)=\frac{1}{2}\ln\left(12.5+e^{-4a}\right)-a$. Substitution of both a=1 and a=1.5 into either equation will result in a change of sign between the two values obtained. It is important that candidates state a conclusion with words to the effect that this change of sign indicates a root between the two values.

(iii) It is enough to make use of the equation in part **(i)** by writing it as, or dealing with it as $a_{n+1}=\frac{1}{2}\ln\left(12.5+e^{-4a_n}\right)$. Many candidates do not write down this iterative formula but know how to use it. A starting point of any value between 1.0 and 1.5 is expected and candidates are also expected to make full use of their calculator and the 'ANSWER' function to enable a quick and accurate evaluation of each iteration. It is essential that both the workings and the final answer are given to the required level of accuracy stated in the question.

Answer: **(iii)** 1.263

Question 6

(i) It was expected that candidates simplify each term so that the expression was in terms of both sin2*x* and cos 2*x*. Writing the two terms as one and making use of the double angle formula $\cos 2x = 2\sin^2 x - 1$, or equivalent, gave the required result. Again, it is important that candidates show each stage of their working in full in order to obtain full marks.

(ii) It was essential that the result in part **(i)** was used, by writing $\cot\frac{\pi}{12} = \operatorname{cosec}\frac{\pi}{6} + \cot\frac{\pi}{6}$ and hence evaluating each of the two terms in $\frac{\pi}{6}$ exactly.

(iii) A recognition that part **(i)** could be used again by writing cosec4*x*+cot4*x* was essential in order to progress. The resulting integrand simplified to cos2*x* which could be integrated easily. Candidates are reminded that indefinite integration should involve the use of an arbitrary constant.

Answers: **(ii)** $2+\sqrt{3}$ **(iii)** $\frac{1}{2}\sin 2x + c$

Question 7

(i) Candidates were expected to make use of parametric differentiation in order to obtain the gradient function in terms of *t*. The value of the parameter at the point *P* needed to be found and substituted into the gradient function to give the required answer.

(ii) It was then necessary to find the value of the parameter at the point *M* by equating the gradient function found in part **(i)** to zero and solving. This value could then be used to find the coordinates of *M*.

(iii) Equating the gradient function found in part **(i)** to *m* and subsequent simplification lead to the given result. It was important that each step of working was shown. Use of the discriminant of the given quadratic equation lead to the critical values required in order to find the values of *m* for which there were real roots.

Answers: **(i)** $\frac{9}{10}$ **(iii)** $m \leqslant -9-6\sqrt{2}$, $m \geqslant -9+6\sqrt{2}$ or equivalent.

MATHEMATICS

Paper 9709/32
Pure Mathematics

Key messages

Candidates need to:

(i) know the difference between roots of an equation and its factors,

(ii) accompany the finding of an angle between a line and plane with a clear diagram,

(iii) know how to obtain the asymptotes of exponential graphs and their intersections with the *y*-axis,

(iv) use laws of logarithms in the solution of numerical equations.

General comments

The standard of work on this paper varied considerably, although all questions were accessible to well-prepared candidates. Unfortunately some candidates found certain questions extremely difficult since either they had not studied the complete syllabus or they had omitted to test themselves on the vast number of available past question papers, together with the viewing of their detailed mark schemes. The questions or parts of questions that were generally done well were **Question 2** (binominal expansion), **Question 3(i)** (trigonometrical relations), **Question 5(i)** (parametric equations), **Question 8(i)** (partial fractions), **Question 9(i)(a)** (evaluation of complex numbers) and **Question 10(i)** and **(ii)** (line, plane and angle).

Those that were done less well were **Question 1** (trapezium rule), **Question 3(ii)** (integration of trigonometrical functions), **Question 4(i)** and **(ii)** (solution of numerical equations using the laws of logarithms), **Question 8(ii)** (integration of partial fractions), **Question 9(i)(b)** (roots, real and complex, of cubic equations) and **Question 10(iii)** (finding equation of plane).

In general the presentation of the work was good, although there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black** pen, reasonable sized lettering and symbols, and present their work clearly.

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this is the context of solving a quadratic equation and substituting limits into an integral. There are some extremely important points that candidates need to address: they must use the method asked for in the question (**Question 3(ii)**) and note carefully what final answer is acceptable (**Questions 1**, **4(ii)**, **5(ii)**, **6(ii)**, **9(i)(b)** and **9(ii)**). These points will be discussed in detail below.

Where answers are given after the **Comments on specific questions**, it should be understood that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question was generally not well done. The range of errors was huge. For example, two intervals used instead of three, incorrect interval width, degrees instead of radians, different interval widths when finding ordinates, actual *x*-values used instead of ordinate values, etc. Occasionally the answer was not given to three decimal places. This use of the Trapezium Rule appears to have been one of the topics that many candidates were not expecting.

Answer: 0.525

Question 2

This question was usually well done, apart from the odd sign and/or arithmetical error. However, a few candidates were careless and used +4*x* instead of –4*x* in the terms of their expansion. A few too many candidates were unable to translate their expression successfully into $(1-4x)^{\frac{1}{4}}$, as a consequence their expansion was far from what was required.

Answer: $1-x-\left(\frac{3}{2}\right)x^2-\left(\frac{7}{2}\right)x^3$

Question 3

(i) This part was well done, with most candidates scoring full marks. However, a few candidates made little progress since they opted to apply the double angle formula to cos 4*x* and to cos 2*x*.

(ii) Although the question stated 'hence', many candidates used integration by parts. This approach did nothing to simplify the integral and most candidates terminated this approach and moved to the next question. The candidates who used their expression from **(i)** usually had a correct integration in which to substitute the limits, although there were those candidates who had 2 and 4 in their numerators instead of their denominators. A few candidates carelessly wrote sin *x* instead of sin 4*x* and sin 2*x*. Although limits were usually correctly substituted, many candidates did not show sufficient working to convincingly obtain the given answer.

Question 4

(i) Many candidates believed that some kind of 'word explanation' was being asked for, when really candidates were expected to takes lns of the given equation, hence obtaining $\ln y=\left(\frac{3}{n}\right)\ln x+\left(\frac{1}{n}\right)\ln A$. This then should have been compared with the straight line equation y = *mx* + c, together with relating the appropriate terms. Although many managed the first mark, a complete explanation was not often seen.

(ii) Another numerical question which could either have been done directly or by using the linear equations from **(i)**. Most restarted this section with a repeat of taking lns. Unfortunately instead of ln *A*, many had just *A*, or believed that 3ln *x* should be evaluated with $(\ln x)^3$. Numerous other such basic ln errors were very prevalent with the result that few candidates had even a correct method for *n* and for *A*, with even fewer obtaining the correct answers. Those that did manage to get this far often truncated their answers during their calculations and hence finished with small errors in their answers. In addition, many either failed to round to two decimal places, usually giving only one decimal place accuracy.

Answer: $n = 1.70$ and $A = 2.90$

Question 5

(i) Many candidates obtained full marks here. However, there were the usual errors in differentiating regarding coefficients, and the omission of 2 from the trigonometric functions. Most candidates were able to use the double angle formulae correctly throughout, although a few candidates decided to stop with $\sin 2t$ and $\cos 2t$ still present.

(ii) Few candidates scored both marks, and many candidates who reached $\tan t = \left(-\frac{1}{4}\right)$ were unable to move to $t = \tan t^{-1}\left(-\frac{1}{4}\right)$, the usual error being the omission of the minus sign. The expressions for $\tan t$ were usually +1, –1 or $\left(\frac{1}{4}\right)$. Occasionally the candidates' sound mathematics was spoilt by careless rounding at the end.

Answer: $x = -0.961$

Question 6

(i) Some candidates seemed to believe that this relationship was to be found within the differential equation, hence made no progress. However, many used the chain rule or the product rule very successfully.

(ii) Separation of variables was not the real problem in this part, as many performed this task well and integrated to obtain $\frac{x^2}{2}$. The problem was the failure to realise that the right side integral needed the $2\tan\theta$ and the +1 splitting following division by $\cos^2\theta$. Many who did reach this stage obtained the $\tan\theta$ term correctly but were unable to use the result from **(i)**. Candidates should realise that parts of questions are sometimes very closely linked, as here, and in **Question 3(ii)**. A few candidates cleverly performed the integral with the substitution $z = 2\tan\theta + 1$ and so avoided the need to use the result from **(i)**.

Answer: $x = 2.54$

Question 7

This question was often omitted by candidates.

(i) Few candidates scored any marks in this part for a variety of reasons. It is impossible to tell that there is just one root by restricting the domain to a small region such as $-1 < x < 1$, or even less, since what happens outside this region? A minimum domain should have $-2 < x < 1$ for e^{2x} and $-1 < x < 2$ for $6 + e^{-x}$. Furthermore the graphs should have been sketches not plots, and if intersection points are stated, they should be correct. Many candidates had the intersection points on the y-axis marked as 2 and 6 instead of as 1 and 7. Likewise they had an asymptote at $y = 0$ but the other well below $y = 6$. In some situations this latter asymptote was approaching the x-axis, or somewhere between the x-axis and $y = 6$. Then there is the issue of showing that the given equation has exactly one real root. Obviously this relates to the crossing point of the two graphs, but the solution is for one value of x, not for the y value, and hence an arrow should be displayed from the crossing point down to its corresponding x value on the x-axis.

(ii) Most candidates took a relevant expression and correctly looked at its values at $x = 0.5$ and $x = 1$, followed by a completely correct sign argument. However, the exponential expressions with the negative sign and the factor 2 appeared to cause problems and a considerable number of candidates had at least one value incorrect.

(iii) Another part that candidates found very difficult. They were meant to realise that x_n and x_{n+1} both tended to x and to work this limited expression to the equation in **(i)**. Far too many candidates either simply repeated what they were intending to present for their answer to **(iv)** or they decided to take the logarithm of each value within $(1 + 6e^x)$, resulting in such incorrect statements as $\ln 1 + \ln 6 + \ln e^x$, etc.

(iv) Most candidates knew what was required here and tended to obtain the correct result, although too many failed to show convergence to the required accuracy, or gave the incorrect answer 0.927 instead of 0.928.

Answer: 0.928

Question 8

(i) This question was extremely well done and many candidates obtained the values for all three coefficients. However, one or two candidates started with the incorrect partial fractions, whilst others made arithmetical errors.

(ii) Integrating the partial fractions proved testing, as many had the incorrect coefficient for their $\ln(2x + 1)$ term. Unfortunately, their attempts at the integration of $\frac{x}{(x^2+9)}$ were even further from the correct answer, since there was often a spare x accompanying the $\ln(x^2 + 9)$. If candidates are attempting to apply a substitution such as $u = x^2 + 9$, then this, and $\frac{du}{dx}$, should be clearly displayed prior to any substitution. The substituting of limits mark was only available for candidates who, when integrating, had obtained something close to the correct answer. If candidates wish to introduce a constant of integration then it must appear in both limits and cancel out. Introducing a single +C or ln C at the end of a definite integral is incorrect, unlike in an indefinite integral.

Answers: **(i)** $\frac{3}{(2x+1)} + \frac{x}{(x^2+9)}$ **(ii)** $\ln 45$

Question 9

(i) (a) In this question the use of the calculator is not permitted and all working had to be displayed. Hence $(1 + 2i)^3 = -11 - 2i$ received no credit. However, $(1 + 2i)^3 = (-3 + 4i)(1 + 2i) = (-11 - 2i)$ or expanded by the binomial expansion are both fine.

Despite this question requiring little more than the product of a couple of complex numbers undertaken twice, most candidates had an arithmetical error in their evaluation of k.

(b) The majority of candidates realised that one of the other roots was $1 - 2i$, but were then unclear how to establish the quadratic term, or the correct quadratic term but struggled with the long division. Unfortunately many of the candidates who did perform everything correctly then muddled the definitions of factor and root, so instead of $x = -\frac{3}{2}$ gave the answer as $2x + 3$.

(ii) Candidates were unable to make much progress with this part unless they had a reasonably accurate figure; that is correct centre and with radius unity. The centre was usually correct but sometimes did appear in the other quadrants. However, the radius often distorted the figure by being greater than unity; hence some of the circles appeared in the second quadrant or in the region $y < 1$ and $y > 3$. Few candidates could produce a complete method for the least value of arg z. This was due to candidates not clearly relating the required angle either to arg of the centre of the circle or to the angles between the tangents from the origin of the circle.

Answers: **(i)(a)** $k = 15$ **(b)** $1 - 2i$ and $-\frac{3}{2}$ **(ii)** 0.64

Question 10

In general the work in this question was of a good standard and candidates often scored nine marks or more.

(i) Most candidates expressed the general point in component form but, instead of substituting this point into the equation of the plane, many equated their x coordinate with the x coefficient of 2 from the plane.

(ii) Whilst often candidates incorrectly took the point (4,3,–1) for the direction of the line, they usually used the correct processes for both scalar product, moduli and inverse sine or cosine of their result. Unfortunately far too many candidates who started with the correct vectors then made a basic arithmetical error in their scalar product evaluation and finished with $2 - 6 + 2 = -6$ instead of –2. As all candidates opted not to show a diagram it is hardly surprising that incorrect angles were calculated, for example 79.7° or 110.3°, instead of the correct value of 10.3°.

(iii) Candidates needed to find the vector product of the direction vector of the line l and the normal to plane p, hence establishing the normal to the required plane. Unfortunately too often one of these vectors was either a point on the line l, for example (4,3,–1) or the point on the plane q, for example (0,4,–1).

Answers: **(i)** $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ **(ii)** 10.3° **(iii)** $8x + 3y + 7z = 5$

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates are also reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers.
- When dealing with problems involving calculus, candidates are reminded of the need to show working particularly if using a calculator for problems involving definite integration.

General Comments

The paper was very well done by many candidates. Only a small number of candidates appeared to be ill prepared for the paper. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3sf as requested. Marks were also lost due to prematurely approximating within their calculations leading to the final answer. This was particularly noticeable in **Questions 2** and **4**.

Candidates should also be reminded that if an answer is required to 3sf, as is the rubric on this paper for non-exact answers, then their working should be performed to at least 4sf.

In questions where an answer is given, candidates should be particularly careful to show all of their working in order to justify their proof of a given answer

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases, such as in **Question 1**, it is impossible to achieve the correct given answer unless this value is used.

Comments on Specific Questions

1 Most candidates found this to be a very straightforward question and it was well done by almost all candidates. The best method of approach is to write down Newton's second law for each particle in terms of the acceleration a and the tension T and then to solve these simultaneous equations for both a and T. When an answer is given in the question as it is here, candidates must be particularly careful to show all working. Some candidates tried to remember the system equation for the acceleration, a, and a separate system equation for the tension, T, but in some cases these were quoted incorrectly. It is always safer to work from first principles and use Newton's law for each particle.

Answers: $a = 6\text{ ms}^{-2}$ (answer given) and $T = 3.2$ N

2 Again most candidates performed well on this question. The two main approaches used were to resolve forces horizontally and vertically or to use Lami's theorem.

Those who resolved forces should have equated the vertical components of the P and $2P$ forces and as the P cancels this gave an equation in θ which can be solved directly. The three term horizontal equation leads to an equation in P which can be solved by using the value already found for θ.

Those who used Lami's theorem needed to look first at the angles between the forces which are in this case 120°, $(60 + \theta)$° and $(180 - \theta)$°. Once these angles have been found the two Lami equations give the results in a similar manner to the resolving forces method. Some candidates lost marks either due to premature approximation or by only giving their answers to 2 significant figures.

Answers: $\theta = 25.7°$ and $P = 4.34$

3 (i) A significant number of candidates used an incorrect method to solve this question. Since the motion is not in a straight line, the acceleration cannot be assumed to be constant. However, many candidates wrongly chose to use the constant acceleration formulae. The only method which can be used in problems such as this is the work-energy equation. In this case there is no friction and so the potential energy which the girl possesses at the top of the slide is transferred to kinetic energy when she reaches the bottom. By taking the level from which the potential energy is measured as the level of the bottom of the slide, then the equation PE loss = KE gain will give the required result.

Answer: The speed of the girl at *B* is 12 ms^{-1}

(ii) In this part there is now a frictional effect. It is necessary to continue to use the work-energy equation. In the first case when the girl starts from rest, the PE lost is equated to the work done against friction plus the KE gain. In the second case, where she has an initial speed *V*, the initial PE plus the initial KE is equated to the final KE plus the work done against friction. If both of these equations are stated correctly then the work done against friction can be eliminated between the two equations and the value for *V* can be found directly. However, most candidates correctly found that the work done against friction is 880 J and then successfully determined the value of *V*. Again some candidates wrongly chose to use constant acceleration formulae.

Answer: $V = \sqrt{21} = 4.58$

4 This question proved to be the most difficult one on the paper for many candidates. In fact a large number of candidates misinterpreted the given information. It is stated in the question that the force *P* is applied horizontally. However, a significant number of candidates wrongly thought that this meant that the force acted directly up the plane. The method adopted by most candidates was to resolve forces along and perpendicular to the plane. Due to the fact that *P* acted horizontally, this meant that the normal reaction, *R*, consisted of two terms. Those who took *P* to act along the plane wrongly found *R* to be $12g \cos 25$. Once the two correct equations were found and $F = \mu R$ was used, the value of *P* is determined by substituting for *R* and solving for *P*.

Answer: $P = 242$

5 (i) In this question one of the answers is given. It is particularly important to realise that in such cases, it is vital to show all of your working. Most candidates made very good attempts at this question. There are various different ways of solving the problem. Two possible approaches are shown here. By using the equation, $s = (u + v)/2 \times t$ with $s = 200$, $u = 0$ and $t = 10$, the value of the speed *v* of the rocket after 10s can be found. Once *v* is found the acceleration, *a*, can be determined by using the equation $v = u + at$. Alternatively, the constant acceleration equation $s = ut + \frac{1}{2}at^2$ can be used with the values $s = 200$, $u = 0$ and $t = 10$ and this will determine the value of *a*. Once *a* is found the value of *v* can be determined by using the equation $v^2 = u^2 + 2as$ with the values $u = 0$, $s = 200$ and $a = 4$ (found earlier) used.

Answer: Speed of rocket after 10 seconds is 40 ms^{-1} (answer given) $a = 4$ ms^{-2}

(ii) In this part many candidates wrongly thought either that the acceleration of the rocket was still 4 ms^{-2} or that the time until the rocket reached its highest point was 10 seconds. In fact the rocket now moves under the action of gravity and so the acceleration is $-g$ and this can be used to determine the extra height which the rocket reaches beyond 200 m. Some candidates found this extra height but did not complete the question since it asked for the maximum height above ground level which involved adding the 200 m to the value found for the extra height.

Answer: The maximum height above the ground is 80 m + 200 m = 280 m

(iii) In this part of the question there are again several different approaches which can be taken. The most direct method is to determine the time taken to reach the maximum height once the fuel runs out. Use of the equation $v = u + at$ with $v = 0$, $u = 40$ and $a = –g = –10$ will determine the time between the fuel running out and reaching maximum height. The result from **part (ii)** can now be used to find the time to fall back to ground level by using $s = ut + ½at^2$ with $s = 280$ (answer to **part (ii)**), $u = 0$ and $a = g = 10$. Finally the three times for the journey must be added. Some candidates correctly found that the total time to reach maximum height is 14 seconds but then merely doubled this value. This cannot be done since the acceleration on the upward part of the motion is different to that for the downward part.

Answer: The total time is 10 + 4 + 7.5 = 21.5 seconds

6 (i) This question was well done by most candidates. It must be recognised that the driving force required for the speed to be constant is equal to the resistance force. In this case the resistance force is given to be 35 v, where v is the speed of the car, and so with a speed of 60ms^{-1} the resistive force must be 35 × 60 = 2100 N. The majority of candidates found this correctly and then used $P = Fv$ to find the greatest power of the car. Some wrongly thought that the driving force required must balance the weight of the car.

Answer: The greatest power of the car is 126 000 W

(ii) When acceleration is required, as in this case, it is generally best to apply Newton's second law to the problem. If we are to find the greatest possible acceleration then the maximum power will be used. For a speed of 30ms^{-1}, use of $F = P/v$ shows that the driving force is 126 000/30. The resistance force in this case is 35 × 30. These two forces combine to produce the acceleration and application of Newton's second law will give the required acceleration. Again this is a question where the answer is given and so particular care must be taken to show all workings. Several candidates could see from the given answer that the required force was 3150 N but unless the correct full working was seen the marks could not be scored.

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Answer: The greatest possible acceleration is 2.625 ms^{-2} (answer given)

(iii) In this part many candidates continued to use the acceleration found in **part (ii)** but in fact the question refers to the constant speed up the hill and so the acceleration is zero. For this to happen there has to be a balance between three forces. The greatest driving force 126 000/v must balance the sum of the friction force 35 v and the weight component down the hill, 1200 g (7/48). When the equation stating this balance is used, the resulting equation is a quadratic. The resulting solution has two cases, one of which is impossible.

Answer: The greatest speed up the hill is 40 ms^{-1}

7 (i) This question was well done by most candidates. The velocity for time $0 \leqslant t \leqslant 10$ is $v = 4 + 0.2t$. To find the required acceleration either differentiate the expression for v or just notice that the expression is of the form $v = u + at$ and so the value of a can be written down immediately.

Answer: The acceleration is 0.2 ms^{-2}

(ii) The velocity in the time period $10 \leqslant t \leqslant 20$ is given as a function of t and is clearly not going to lead to constant acceleration. In this case it is necessary to differentiate the expression for velocity in order to find an expression for acceleration and then to substitute the value $t = 20$ in to the expression found. Most candidates attempted this part of the question using differentiation but some did not differentiate correctly.

Answer: The acceleration of P at $t = 20$ is $–0.2\text{ ms}^{-2}$

(iii) In this part the two different expressions for velocity must be represented in a velocity-time graph. For the region $0 \leqslant t \leqslant 10$ the velocity is linear and so is represented in terms of the (t,v) coordinates by a straight line from (0,4) to (10,6). The line representing the time $10 \leqslant t \leqslant 20$ involves a curve which is concave upwards joining the points (10,6) to (20,0). Many candidates thought that the curve representing $10 \leqslant t \leqslant 20$ was also a straight line. Also the line representing the first ten seconds was often shown starting at the origin. However, many correct solutions were seen. The axes should also be correctly annotated with at least $v = 4$ and $v = 6$ shown on the v-axis and $t = 10$ and $t = 20$ shown on the t-axis.

Answer: Straight line from (0,4) to (10,6). Curve, concave upwards from (10,6) to (20,0)
The points $v = 4$, $v = 6$, $t = 10$, $t = 20$ all shown correctly on the axes

(iv) The distance travelled in the first ten seconds can be found by either using the area under the straight line segment which is a trapezium or by integration of the expression for v in that region. The distance travelled from $t = 10$ to $t = 20$ must be found by integration. Most candidates who attempted this part performed the integration correctly. Once this had been done, careful correct use of the limits must be shown. Finally the two parts must be added to give the final result. Some made numerical errors in the evaluations. Because a number of candidates thought that the velocity in the time zone from $t = 10$ to $t = 20$ was represented by a straight line, they wrongly used the area of a trapezium or a triangle to find the distance travelled. Overall the question was well attempted. Some candidates performed the definite integration on a calculator, showing very little working. If any errors occur and no working is seen then the candidate would not be able to score many marks.

Answer: The distance travelled is 50 m + 20 m = 70 m

MATHEMATICS

Paper 9709/52
Mechanics

Key messages

Candidates should be reminded that if an answer is required to three significant figures then their working should be performed to at least four significant figures.

When an angle is given in the form of a trigonometric equation, for example $\tan\theta = \frac{3}{4}$, it is a good idea to recognise that $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$. By doing this an exact answer can usually be found.

General comments

This paper was of a similar standard to the one set in March last year.

Most candidates work was neat and well presented.

g=10 is now being used by most candidates as instructed on the front cover of the examination paper.

A formula booklet is provided. Candidates should always check the booklet when using a formula.

The easier questions proved to be **2**, **3(i)** and **4(i)**.

The harder questions proved to be **1**, **6(iii)** and **7(i)**.

Comments on specific questions

Question 1

This question proved to be very difficult. Most candidates could not visualise the position of the block on the inclined plane. It was necessary to use the fact that *AC* was up a line of greatest slope of the plane with *A* below *C*. The block would topple about the point *A*, so the weight would pass through *A*. this would result in

$$\tan 30 = \frac{\left(\frac{AC}{2}\right)}{\left(\frac{h}{2}\right)}.$$

Answer: 0.98(0)

Question 2

This question proved to be one of the easier questions on the paper. Candidates needed to find the horizontal and vertical velocities after 2 seconds. After that the resultant velocity and the required angle could be calculated.

Answer: 16.8 ms^{-1}, 42.8° below the horizontal

Question 3

(i) Most candidates were able to solve this part of the question.

(ii) Candidates needed to realise that the greatest velocity occurred at the equilibrium position. After that a 4 term energy equation had to be set up.

Answers: **(i)** 0.9 kg **(ii)** 2 ms^{-1}

Question 4

(i) This part of the question was generally well done. Candidates needed to compare the given equation with the trajectory equation given in the formula booklet.

(ii) Most candidates realised that the horizontal and vertical distances were the same. By putting $y = x$ the required result was found.

(iii) This part of the question proved to be rather difficult for a lot of candidates. The quickest and easiest solution was to find $\frac{dy}{dx}$ and equate it to zero. This gave the value of x at the highest point and by using this value in the given equation, the maximum height could be calculated. The required time was found by considering horizontal motion.

Answers: **(i)** 71.6°, 31.6 ms^{-1} **(ii)** (40,40) **(iii)** 45 m, 3.01 s

Question 5

(i) This part of the question was reasonably well done. Initially the candidates needed to realise that the tension would be zero. Next the normal reaction could be found by resolving vertically. Finally, by using Newton's Second Law horizontally, the smallest angular velocity was found. A quicker and neater approach was to equate along the tangent at the point of contact. The resulting equation had only the angular velocity as the unknown.

(ii) This part of the question was quite well done. A few candidates found the angular velocity instead of the velocity. This part required candidates to resolve vertically to find the normal reaction and then to use Newton's Second Law horizontally.

Answers: **(i)** 7.07 $rads^{-1}$ **(ii)** 4 ms^{-1}

Question 6

(i) A reasonable attempt was made by many candidates in this part of the question.

(ii) This part of the question was generally well done with candidates using Newton's Second Law.

(iii) Many candidates found this part of the question rather difficult. Firstly it was necessary to integrate the equation from part **(ii)** in order to find the velocity when $t = 8$. From this point the candidates needed to use the constant acceleration equations. Too many candidates thought that it was necessary to integrate again to find the distance.

Answers: **(i)** 5 s **(ii)** $\frac{dv}{dt} = t - 5$ **(iii)** 2.025 m

Question 7

(i) Candidates found this part of the question one of the hardest on the paper. The centre of mass from *AD*, $\bar{x}$, had to be found. This was done by using $\tan 48 = \frac{\bar{x}}{0.3}$. To complete the solution it was necessary to take moment about either *AD* or *BC*.

(ii) This part of the question was quite well done. The weight of the lamina could be found by taking moments about the point *C*. The final step required was to use the fact that the ratio of the weights was equal to the ratio of the areas.

Answers: **(i)** 0.214 m **(ii)** 25(.0) N

MATHEMATICS

Paper 9709/62
Probability and Statistics

Key messages

Candidates are reminded that to achieve non-exact answers correct to three significant figures, all calculations should be carried out using at least four significant figures.

Candidates are advised to ensure that workings are submitted within their solutions to communicate their thinking, as there are occasions when a correct answer value may not be sufficient to justify full credit. The workings also enable marks to be awarded where there are errors in the process.

General comments

Many candidates seemed well prepared for the examination, and there were many praiseworthy solutions seen to each of the syllabus areas.

Candidates who used diagrams to interpret the information stated in the questions were often more successful, especially when considering the normal distribution. Many candidates were able to gain credit in **Question 7(ii)** because their diagram indicated the approach that they were seeking to achieve, even if there was confusion over the details of the data provided.

Candidates who had prepared well appeared to have sufficient time to attempt all questions. However, a significant number of candidates failed to follow the instructions stated within questions, or to appreciate that the skills developed in Pure Mathematics 1 may be used in this component.

Comments on specific questions

Question 1

The majority of candidates attempted to draw a cumulative frequency graph. However, a significant number of histograms were drawn with accuracy but would gain no credit. A common error was to fail to label the axes fully, and candidates should be aware that both the description and units of the variable are expected. In most cases, good linear scales were used for both axes, although some scales for the cumulative frequency prevented candidates from plotting their values accurately.

The majority of candidates who had an increasing graph were able to obtain an appropriate median value, although weaker candidates' readings were less accurate than expected at this level. It is recommended that candidates indicate on the graph where they are obtaining their value.

Answer: 4.8

Question 2

(i) Good solutions recognised that as one L had to be included, the problem simplified to finding the selections of two letters from COIDER. The majority of solutions interpreted selections correctly as requiring the use of combinations. A common error here was to multiply by 2 because of the repeated L, which assumed that the Ls were identifiable.

The most common misconception was to simply calculate the total number of selections of three letters possible.

(ii) The best solutions recognised that this was an extension of part **(i)**, and calculated in a similar manner the number of selections with both 0 Ls and two Ls and then summing to obtain the total. A number of candidates who did not score in **(i)** recalculated all values correctly in this part.

An alternative approach that was used successfully by a number of candidates was to consider the total number of selections of three letters from COLIDER and adding the number of selections which contains two Ls.

Answers: **(i)** 15 **(ii)** 41

Question 3

Almost all candidates were able to access this question well. Failure to read the question carefully appeared to be a major cause of errors. No penalty was applied where fractional solutions were converted to decimals and rounded inappropriately to two significant figures. It was disappointing to see how frequently decimal conversions were inaccurate.

(i) Most candidates identified the appropriate value within the table and stated the required probability. Common errors were to calculate the probability of an estate car, or the probability of the car being silver if an estate was chosen.

(ii) Almost all candidates correctly calculated the total number of hatchbacks and then stated the required probability.

(iii) The most efficient solutions recognised that the conditional probability could be calculated directed from the data table. However, the majority of candidates used the standard conditional probability formula $\frac{\text{P}\,(\textit{red and hatchback})}{\text{P}\,(\textit{hatchback})}$. Many of these solutions inappropriately calculated the conditional probability for the numerator which resulted in little credit being given.

(iv) The best solutions recognised that 'justifying your answer' was a prompt that clear numerical comparison of $\text{P}(R)\times\text{P}(H)=\text{P}(H\cap R)$ was required. A considerable number of solutions stated the calculations required but did not evaluate to show that the conditions were not met. Many candidates interpreted $P(H\cap R)$ as the conditional probability calculated in **(iii)**, and used this value throughout. Better incorrect solutions used the numerator from **(iii)** at this stage to gain partial credit. A surprising error identified in this question was to compare the values at two decimal places, and justify independence. Very few candidates used the alternative approach of using the conditional requirements for independence P(R|H) = P(R).

Answers: **(i)** $\frac{1}{16}$ **(ii)** $\frac{9}{16}$ **(iii)** $\frac{4}{9}$ **(iv)** 'Not independent' with justification

Question 4

Many candidates were able to access this question well. The standard of algebra was general good, with clear presentation of the processes involved. Candidates are reminded that the skills of Pure Mathematics 1 are expected prior knowledge for this component.

(i) Good solutions clearly stated the two equations that can be generated from the data given. The best solutions showed a clear and accurate solution of the simultaneous equations produced, with elimination being the most common approach. A few candidates simply stated values for *p* and *q* which could gain little credit.

Weaker solutions often only considered the equation for the expected value, and then provided a circular argument for substituting for *q* in the expression to calculate a value for *p*.

(ii) Almost all solutions attempted to use the table information correctly to calculate Σpx^2. Good solutions recalled that μ^2 needed to be subtracted and that $\mu = E(X)$. A few candidates used their values from **(i)** to recalculate μ.

Answers: **(i)** $p = 0.2$, $q = 0.4$ **(ii)** 5.41

Question 5

This question of coded data was attempted more successfully than in previous sessions, with the majority of candidates scoring well in **(i)**. Candidates should be reminded that after they have finished their solution that it is good practice to read the question again to ensure that they have actually reached the required answer. In **(ii)** there appeared to be evidence that some candidates believed that they had reached the end of the question when they had calculated the variance.

(i) Almost all candidates successfully produced an equation to link the coded data with the mean. Good solutions then showed clearly the algebraic processes required to solve for n. Weak solutions often contained simple algebraic or arithmetical errors.

(ii) Most solutions attempted to use the variance formula appropriately. Good solutions recognised that the variance would be the same for the coded and uncoded data. Many identified the required information to apply the variance formula using the coded data, and successfully calculated either the variance or standard deviation.

A large number of solutions failed to state the variance formula accurately for the uncoded data, or equated to an inaccurate variance value. Candidates should be reminded that to achieve the required degree of accuracy at least four significant figures need to be used throughout, or to use their calculator efficiently to use their earlier more accurate value.

Good solutions again presented the algebra within the work clearly and accurately, which possibly assists in avoiding the careless slips seen in some solutions.

Answers: **(i)** 32 **(ii)** 21 128

Question 6

Most solutions recognised that this was a permutation style question, as indicated by the key word 'arranged'. Solutions which presented their working in a clear manner were often more successful.

(i) The best solutions considered separately how many arrangements of the three odd digits and the four even digits could make before attempting to answer the main question. Weaker solutions either failed to divide by 3! (Which removed the effect of the repeated sixes) or multiply by 2 (for odds and evens swapping positions). A considerable number of solutions did not separate the odds and evens but calculated a total number of ways of arranging the digits and then attempting to remove the unwanted solutions.

(ii) Although this part was attempted less successfully, many good solutions were seen. The most common, and successful, approach was considering separately the number of arrangements which ended in 8, ended in 6 and then summing the answers. A common misconception was the need to multiply the number of arrangements ending in a 6 by 3, because of the repeated nature of the digit. Candidates should be encouraged to consider whether an item can be identified uniquely when answering permutation questions.

An efficient alternative approach was to consider the arrangements of the first six digits and then the ways that the final digit could be chosen. This required understanding that the effect of the repeated sixes did need to be removed from the final answer.

Answers: **(i)** 48 **(ii)** 480

Question 7

Many candidates found this normal distribution question challenging. This may be because raw data needed to be used initially in **(i)** and the principles of the normal distributions in **(ii)** to be successful.

(i) The best solutions calculated the number of packets of biscuits that weighed less than 410 grams so that the *z* value for the appropriate probability was available to use with the normal distribution formula. A simple sketch of the normal distribution was seen in a few solutions, but these candidates usually were able to clarify their method successfully to reach the correct answer. A common error was to equate the normal formula with the calculated probability, which could gain little credit. Again, a surprising number of candidates made simple arithmetical or algebraic errors in their solution.

(ii) There was evidence that many candidates did not appreciate what weights of packets were required by the question. Where simple sketches of the normal distribution were seen, solutions were often more successful. A large number of candidates multiplied their standard deviation from **(i)** by 1.5 to calculate the range of weights required. The normal formula was then applied and led back to the anticipated $P(Z < -1.5)$ or $P(Z > 1.5)$. Unfortunately, many candidates did not work with sufficient accuracy resulting in incorrect values at this stage. Almost all solutions were completed to calculate the number of expected packets, although weaker students did not use a probability of sufficient accuracy. The majority of solutions only considered $P(Z > 1.5)$ tail.

Answers: **(i)** 5.62 **(ii)** 66 or 67

Question 8

(i) Most recognised that this was a binomial distribution. Candidates need to be encouraged to understand the requirements of ‘at least’, and seek the most efficient process to achieve the required probability. The best solutions recognised that P(4) + P(5) was required, although good candidates often attempted 1 – (P(0) + P(1) + P(2) + P(3)). Candidates who used their calculators efficiently stated an exact answer. There was evidence of premature approximation in some solutions.

(ii) The anticipated approach to this question was to consider that the probability of ‘at least one’ was equivalent to 1 – P(0), which then produced an exponential equation for the probability. The majority of candidates who attempted this question did find a solution to their equation, although where the initial interpretation of the information was incorrect, many rejected the extremely small value (3.62×10^{-3}) which the correct approach would produce, and calculated the reciprocal. Candidates should be aware that a decimal final answer will not be acceptable in this context.

(iii) The best solutions clearly identified that a normal approximation was appropriate, justified the appropriateness of the decision, and identified clearly the calculations for mean and variance before using the normal distribution formula. Most candidates recognised that as the data was discrete, a continuity correct was appropriate. A number of solutions used the upper bound and it was unclear whether this was due to confusion applying the correction, or failing to interpret the question correctly. An unexpected error was the number of solutions seen which used the calculated value from **(i)** in this part.

Answers: **(i)** $\frac{1}{64}$ **(ii)** 19 **(iii)** 0.959

MATHEMATICS

Paper 9709/72
Probability and Statistics

Key messages

It is important that candidates are aware of the conditions needed to apply any approximating distributions. When a question asks for an explanation ‘in the context of the question’ text book definitions will not be accepted, the answer must relate to the situation given in the question.

Candidates must make sure that they read the question carefully.

General comments

In general, candidates scored well on **Questions 1**, **2** and **4(i)** whilst **Questions 3**, **5** and **7** proved more demanding for some candidates. In particular, candidates found **Question 7**, on calculation and interpretation of a confidence interval, more demanding than has often been the case on this type of question in the past. Candidates were generally able to demonstrate and apply their knowledge in the situations presented, though explanations showing a statistical understanding of the situation were not always well answered. There was a complete range of scripts from very good ones to poor ones.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy. This was particularly seen on **Question 7(v)**. Lack of sufficient accuracy also caused a loss of marks in **7(iii)** (see comments below).

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some very good and complete answers.

Comments specific questions

Question 1

This was a well-attempted opening question. Many candidates used the correct distribution for the sample means $N\left(4.9, \frac{2.21^2}{75}\right)$, and were able to standardise correctly; errors made included standard deviation/variance mixes and misinterpretation of the area (probability) required.

Answer: 0.348

Question 2

This question required the approximation of the Poisson distribution Po(98.4) to the normal distribution N(98.4, 98.4). Many candidates used the correct distribution and went on to standardise and find the required probability. Errors included incorrect or no continuity correction and calculation of the wrong tail probability.

Answer: 0.787

Question 3

This question was not well attempted; candidates found part **(ii)** particularly demanding. Both parts required a mean and a variance to be calculated, and in both parts the mean was better attempted than the variance. In part **(i)**, 5×0.03^2 should have been used to find the variance; many candidates incorrectly calculated $5^2 \times 0.03^2$. Similar errors were made in part **(ii)** when calculating the variance; the expression required was their previous answer to var $H_A + 4 \times 5 \times 0.02^2$; a variety of errors were seen including a minus sign instead of +, 2 used instead of 4 (i.e. 2^2), and 5^2 instead of 5.

Answers: **(i)** 6, 0.0045 **(ii)** 0, 0.0125

Question 4

Part **(i)** of this question was well attempted, but candidates found part **(ii)** more demanding. In part **(i)** Po(2.4) was required; this was used by many candidates, though some tried unsuccessfully to work with Po(0.9) and Po(1.5) and whilst this could lead to the correct answer the full calculation using this method was seldom seen. Misinterpretation of 'less than 4' was a common mistake with candidates incorrectly including the P(4) term in their Poisson expression.

In part **(ii)** the null and alternative hypotheses should have been stated, but this was omitted by some candidates. The calculation required was P($\leqslant$1); many candidates merely calculated P(=1). The subsequent comparison with 0.1 should then have been clearly shown in order to justify the conclusion. Errors here included failure to show a clear comparison and some used z values for their comparison. Candidates should be aware that the conclusion should be written in context and should be non-definite (as below).

Answers: **(i)** 0.779 **(ii)** No evidence that fewer than usual sold

Question 5

Many candidates were unable to set up correct hypotheses, and in part **(ii)** many candidates did not answer in context and merely gave a text book definition of a Type I error. Calculating the probability of a Type I error was reasonably well attempted. The use of B(30, 0.17) was expected in **(iii)**; some candidates attempted to use normal or Poisson distributions. Part **(iv)** was not well attempted and many candidates calculated P(>3) or P(<3) rather than P($\geqslant$3).

Answers: **(i)** H_0:P(orange)=0.17 H_1:P(orange)<0.17
(ii) Wrongly concluding that the percentage is less than 17 per cent **(iii)** 0.0949 **(iv)** 0.188

Question 6

Candidates mainly knew how to find a probability by integrating the probability density function. However, on numerous occasions, candidates left their answer as 0.568, which was the probability that X lies between 0.3 and 0.7. The question wanted the probability that X did not lie between 0.3 and 0.7 so 1–0.568 was required. It is important that candidates carefully check the wording of the question.

The sketch in part **(ii)** was not particularly well attempted. Candidates drew non-symmetrical curves, or curves that were bell-shaped, and some continued their curve outside the range x=0 to x=1. The value of E(X) was often found by calculation as opposed to just using the sketch.

In part **(iii)** a large proportion of candidates attempted to integrate x^2f(x). Errors included poor integration, incorrect limits and many candidates left this as their answer to var(X) rather than subtracting their mean^2.

Answers: **(i)** 0.432 **(ii)** 0.5 **(iii)** 0.05

Question 7

This question was not well attempted. Candidates found the explanations and statistical understanding that were required in parts **(i)**, **(ii)** and **(iv)** particularly challenging.

Part **(i)** was reasonably well attempted; errors made included standard deviation/variance errors, use of an incorrect z value, and some candidates attempted to find an estimate of the population variance from the sample; this was not required as the population variance (0.01) was given.

In part **(ii)** it was important that the explanation given clearly identified that it was the population that was normally distributed, and hence the Central Limit theorem was not required. It was not sufficient for candidates to write ‘it’ was normally distributed.

In part **(iii)** candidates were required to check to see if 11.7 was within their confidence interval found in **(i)**. Candidates whose answers to **(i)** were only shown to three significant figures were unable to fully justify in **(iii)** that the claim was not supported.

Many candidates did not appreciate in part **(iv)** that a 95 per cent confidence interval would be narrower than a 99 per cent confidence interval.

Part **(v)** was well attempted, though marks were lost by candidates not showing three significant figures accuracy and giving their answer as 0.020.

Answers: **(i)** 11.7(5) to 11.9(1) **(ii)** No because population is normal **(iii)** 11.7 not within CI
(iv) No because 95 per cent CI is narrower than 99 per cent **(v)** 0.0201