MATHEMATICS

Paper 9709/11
Paper 11

Key messages

Where answers are given as in **Question 5(i)** and **7(ii)** it is important that all steps to the solution are shown clearly. When a question is unstructured, such as **Question 9**, candidates should be particularly careful in explaining their methods.

When the sign change of the gradient is used to identify a turning point, the values of x that have been used should be clearly identified, as should the values of the gradient given by them.

In questions where a numerical answer is required from use of calculus such as **Question 7(iii)** it is essential that working is shown to justify correct solutions.

General comments

Again it needs to be noted that although no GCSE topics are directly examined basic algebraic, geometrical and trigonometric techniques are required to answer some A-level questions.

Any study of the quadratic function should look closely at the conditions for the inverse function to exist.

Comments on specific questions

Question 1

The best answers to this question were produced by solving a quadratic in $\sqrt{x}$ and squaring the resulting roots. Those who chose to isolate the $\sqrt{x}$ term and square the resulting equation could find the solutions directly but there was more scope for making an algebraic error. Confusing the values of x and $\sqrt{x}$ still gained candidates the first mark.

Answers: $\frac{9}{16}, 4$

Question 2

Most candidates saw the need to form a single quadratic equation with a coefficient of x expressed in terms of b. Some realised that the equal roots of this equation must be either $x = -2$ or $x = 2$ and used the sum of the roots to find the possible critical values of b. The most frequently used method was to set the discriminant equal to zero. The best answers went on to express the required ranges of b correctly.

Answer: $b \leqslant -3$, $b \geqslant 5$

Question 3

Part (i) This question was well answered. Although it was acceptable for a to appear in the numerator and denominator of the gradient for the first mark, it was expected that it should be cancelled and not appear in the final equation.

Part (ii) Correct use of the distance formula was the route to most of the correct answers to this part. A few candidates showed the points on a diagram and formed a right angle triangle with sides $3a$ and $4a$ before using Pythagoras' theorem to find a.

Answers: **(i)** $y = -\frac{3x}{4}$, **(ii)** $\frac{2}{3}$

Question 4

Part (i) Many completely correct solutions to this part were seen from candidates who knew and understood how to use either version of the S_n formula for an arithmetic progression.

Part (ii) The formula for the sum to infinity of a geometric progression was used well with most attempts using either the correct value of r or one in the required range for a sum to infinity to exist.

Answers: **(i)** –12160, **(ii)** 9

Question 5

Part (i) Candidates showed confidence in either expressing the left hand side of the identity as a single fraction or multiplying through the identity by the two denominators. Most who reached this stage used the trigonometric identity correctly to reach the given result. Many answers reflected an awareness that when a result is given the working leading up to it must be shown clearly.

Part (ii) The given result from part **(i)** was used to form a quadratic equation in $\cos\theta$ by every candidate who attempted this part. Most realised that the quadratic formula or completing the square were required to find the one acceptable value of $\cos\theta$. The best answers came from the candidates who realised valid solutions existed in the first and fourth quadrants.

Answers: **(ii)** 78.6° and 218.4°

Question 6

Part (i) Setting the gradient equal to zero at $x = 3$ was seen from many candidates, most of whom went on to find the non-zero value of a correctly.

Part (ii) This part was well answered by those candidates who realised reaching the solution involved a routine integration and substitution to find the constant of integration.

Part (iii) Although the simplest route to identifying the turning point was to find the sign of the second derivative at $x = 3$, successful candidates also clearly showed the sign change of the gradient at either side of $x = 3$.

Answers: **(i)** –3 **(ii)** $y = -x^3 + \frac{9x^2}{2} - 4$ **(iii)** maximum

Question 7

Part (i) This was a straightforward question which was generally answered correctly.

Part (ii) The given result was most easily demonstrated by substitution of $x = 5$ into the equation of the curve. Some chose the more complicated method of solving the equations of the line and the curve simultaneously and showing that one of the results was $x = 5$.

Part (iii) The most popular approach used by candidates was to find the area under the curve between the required limits and subtract the area of the triangle, although the subtraction was not always seen. Some candidates found the area by forming and using a single integral.

Answers: **(i)** $\frac{1}{2}$ **(iii)** $\frac{8}{3}$

Question 8

Part (i) Correct interpretation of the diagram generally resulted in an acceptably presented correct vector.

Part (ii) The method for finding the magnitude of a vector was often seen. The best answers used the vector magnitude correctly to form the required unit vector.

Part (iii) The candidates who had found the correct vectors in the previous two parts were usually able to apply the scalar product formula correctly to find the required angle. It was acceptable to use an obtuse angle to find the correct acute angle.

Answers: **(i)** $\left(-6i + 2k\right)$ **(ii)** $\frac{1}{7}\left(-6i - 3j + 2k\right)$ **(iii)** $25.4°$

Question 9

The most popular and straightforward route to the solution involved finding the areas of the two sectors and subtracting the sum of these from the area of the triangle OAB. The required trigonometrical skills were evident in the answers of many candidates as was the use of the sector area formula. Whist some candidates preferred to work in degrees, most used radians as the given angle in the question suggested. The best answers were set out clearly with intermediate answers given to more than the three significant figures required in the final answer.

Answer: 1.56 – 1.57

Question 10

Part (i)(a) The candidates who realised the gradient of the curve was required were usually able to carry out the necessary differentiation. Many of them went on to find the gradient of the normal correctly and use it with the coordinates of point A to find the normal equation.

Part (i)(b) This part depended on the answer from part **(i)** and most of the candidates who produced an answer there were able to solve their equation with the equation of the curve to attempt to form a quadratic and find the required coordinate of the point of intersection.

Part (ii) The correct detailed use of the chain rule was a feature of the better candidates' answers. Some of these realised that the rate of change of x was a decrease and were able to reach the correct answer.

Answers: **(i)(a)** $y = \frac{x}{2}$ **(i)(b)** $\frac{-1}{4}$ **(ii)** 0.6

Question 11

Part (a)(i) The provision of the function in completed square form enabled candidates who appreciated the significance of f being a one to one function to state the required value of a without any calculation.

Part (a)(ii) Careful thought was required to deduce the range of f, ensuring $f(x) = -1$ was not included. Many candidates were able to change the subject of the given function to find an expression for the inverse, but few realised the range of the inverse demanded the use of the negative square root they had in their expressions.

Part (b)(i) The candidates who kept the expression for $g(2x)$ in factorised form usually went on to find $gg(2x)$ in the required form.

Part (b)(ii) Nearly all completely correct attempts came from the expansion and simplification of the first stage or final result in part **(b)(i)**. Re-checking the products of negative and positive terms would have helped eliminate some errors.

Answers: **(a)(i)** 3 **(ii)** $y > -1$, $f^{-1}(x) = 3 - \sqrt{1+x}$ **(b)(i)** $(2x-3)^4 - 6(2x-3)^2 + 9$
(ii) $16x^4 - 96x^3 + 192x^2 - 144x + 36$

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

It is very important that candidates show all working clearly. For example, correctly labelling angles using 3 capital letters in the proof in **Question 8** and not relying upon Examiners looking at the diagram given in the question. Similarly, when a scalar product is calculated, the individual components should be shown multiplied together, and when definite integrals are evaluated it is important that both of the limits can be clearly seen to have been substituted into the integral. For example $\left(\frac{6^3}{12} - 6(6) - \frac{36}{6}\right) - \left(\frac{2^3}{12} - 6(2) - \frac{36}{2}\right)$.

This is partly so that Examiners can clearly see that the integral function, available on some calculators, has not been used.

General comments

The paper seemed to be generally very well received by the candidates and many good and excellent scripts were seen. There were, however, a number of candidates who missed out a significant number of questions and scored very poorly overall. The paper contained a number of reasonably straightforward questions, giving all candidates the opportunity to show what they had learned and understood, but also contained questions which provided more of a challenge, even for the strongest candidates. The vast majority of candidates appeared to have sufficient time to complete the paper. The standard of presentation was generally good, with candidates setting their work out in a clear readable fashion. Writing answers in pencil and then over writing them in pen later should be discouraged as it makes the answers very difficult to read. The question candidates found easiest was **Question 3**, while **Question 2** proved to be the easiest question for candidates.

Comments on specific questions

Question 1

This question proved to be a very accessible start to the paper, with a great many candidates demonstrating a good knowledge of the binomial expansion. They were very often able to write down the relevant term and evaluate it correctly. Weaker candidates were sometimes unsure which term would have a coefficient of $\frac{1}{x^2}$ and a good number forgot to raise 3 to the power 4. Those who bracketed the term $(3x)^4$ were usually more successful.

Answer: 840

Question 2

Most candidates demonstrated a very good understanding of indices and definite integration and full marks were very commonly scored in this question. Generally sufficient working out was shown but it is important that both of the limits can be clearly seen to have been substituted into the integral. A number of candidates were unsure about the required indices and small number forgot to actually integrate the function.

Answer: $\frac{26}{3}$

Question 3

This question was slightly different to normal and many candidates struggled with it. In part **(i)** many tried to use Pythagoras' Theorem and failed to understand that the length *PQ* was vertical and that the difference in the *x* co-ordinates was therefore 0. Almost 40% of candidates missed out part **(ii)** probably due to the complex answers they had reached for part **(i)**. Those candidates with the correct length *PQ* were generally able to differentiate, set their value to 0 and solve, although many of these stopped when they had found the value of *t* and did not find the maximum length of *PQ*.

Answer: **(i)** $4t - t^3$, **(ii)** $\frac{16\sqrt{3}}{9}$

Question 4

In part **(i)** the vast majority of candidates were able to form the function fg(x) and go on to solve the given equation correctly although many answers were given in degrees rather than the required radians. Part **(ii)** proved to be more difficult, with many candidates failing to draw a curve which clearly turned at both ends. Those who attempted to transform the graph of $y = \cos x$ to the given function were generally much more successful than those who plotted points and joined them up.

Answer: **(i)** $x = 2.46$

Question 5

Many fully correct solutions to this series question were seen, but in part **(i)** weaker candidates did not form correct equations from the information given in the question. Those who were successful in part **(i)** could generally identify the fourth term in part **(ii)**, but those who were unsuccessful in **(i)** could not make much meaningful progress in **(ii)** and almost a third of candidates missed this part out completely.

Answers: **(i)** $x = 8$, $y = 12$ **(ii)** 16, 27

Question 6

The vast majority of candidates attempted part **(i)** of this question and they generally made some progress, usually finding that *BD* was equal to $20\sin\theta$ by using triangle *BDC.* Many realised then that, in triangle *ABD,* angle *DBA* would also equal θ and concluded that *BD* would be equal to $\frac{9}{\tan\theta}$. The two expressions were then equated and the given result shown. Weaker candidates were often unable to make the second connection correctly. Some did not obtain full marks because they did not find *BD* in terms of θ, even though they showed the final given result. Although part **(ii)** was independent of part **(i)**, nearly 20% of candidates omitted it. Those who did attempt it were almost always successful in forming a quadratic in $\cos\theta$ and correctly solving it.

Answer: **(ii)** 36.9

Question 7

Candidates coped better with the information presented in this vector question than in some previous years. The majority were able to correctly identify the required vectors and use the scalar product, as instructed, to correctly find the required angle. A significant number, though, incorrectly rounded their final answer, with 31 and 40 quite common.

Answer: 31.0

Question 8

There were a good number of fully correct solutions seen to this question. Strong candidates clearly identified which angles they were finding in part **(i)** with isosceles triangle *ABC* often being split into two right-angled triangles and angles *CAB* or *ABC* being correctly identified. Weaker candidates often did not clearly identify which angles were being found. In part **(ii)** many strong candidates correctly found the required perimeter of the shaded region but weaker candidates often did not identify which angles and arc lengths were required, with the shaded area sometimes being found.

Answer: **(ii)** 24.2

Question 9

Many fully correct solutions were seen for this question. The vast majority of candidates understood the need to complete the square in part **(i)**, although some did not know what was required for parts **(ii)** and **(iii).** In part **(iv)** many realised the need to use the completed square form and made some progress, but only the strongest were able to correctly identify fully the inverse function.

Answer: **(i)** $2(x-3)^2-11$, **(ii)** $\mathrm{f}(x) \geqslant -11$, **(iii)** 3, **(iv)** $3-\sqrt{\dfrac{x+11}{2}}$

Question 10

This question was quite straightforward for strong candidates and many completely correct solutions were seen. Part **(i)** proved to be more difficult for weaker candidates, who often knew what was required, equating and using the discriminant, but were unable to correctly carry out the algebraic manipulation needed. Some made the discriminant >0 and others could not go from $k^2<144$ to the correct final answer. Part **(ii)** was more accessible for weaker candidates as less algebraic and more numerical manipulation was needed. Those who were successful in **(ii)** were generally able to complete part **(iii)** but some forgot to use the perpendicular gradient or did not use the mid-point.

Answers: **(i)** $-12 < k < 12$, **(ii)** (1, 14) and (4, 11) **(iii)** $y = x + 10$

Question 11

Parts **(i)** and **(ii)** of this final question were very well done, but many candidates struggled to find the required area in part **(iii)**. The standard rules for the differentiation and integration of this type of function were well known, but weaker candidates sometimes forgot to multiply or divide by 4. In part **(ii)**, again many candidates knew to equate their differential to 0 and were able to solve the subsequent equation correctly, although some forgot to find the *y* as well as the *x* co-ordinate of the stationary point. In part **(iii)** candidates generally knew to use the integral from part **(i)** but often had the wrong limits, some using both of the co-ordinates found in **(ii)**. The line was even more problematic with some not including it at all and others using the tangent at *A*. Those integrating the line and those using the area of a trapezium were equally likely to be successful as were those who combined the line and curve together into one integral.

Answers: **(i)** $6(4x+1)^{-\frac{1}{2}}-2$, $\dfrac{(4x+1)^{\frac{3}{2}}}{2}-x^2+c$, **(ii)** (2, 5) **(iii)** 1

MATHEMATICS

Paper 9709/13
Paper 13

Key message

In general, the presentation of work by candidates was satisfactory. However, **Question 3(ii)** required candidates to find certain lengths and certain areas of regions in order to find the area of the required region. Examiners saw numerous different methods and, unless candidates explicitly made it clear at each stage what they were finding (e.g. *BD* or ΔBDC) it was sometimes difficult to see what each calculation referred to and what method was being attempted. Candidates should lay out steps in their working clearly, showing what they are finding at each stage

One of the messages contained in last June's report was that an answer unsupported by correct working will not gain credit. Whilst there were still isolated instances of essential working not being shown, it is pleasing to be able to report an improvement in this aspect of presentation.

General comments

Many very good scripts were seen and candidates all seemed to have sufficient time to finish the paper. One issue, that of accuracy of numerical answers, remains a source of many lost marks. Unless specified otherwise, non-exact numerical answers should be given correct to 3 significant figures, or 1 decimal place in the case of angles in degrees. In order to achieve this it is usually necessary to use *more* than 3 significant figures in the working. This was particularly necessary in **Question 3**.

Comments on specific questions

Question 1

This question was done very well, with many candidates scoring full marks. The most efficient approach was to realise that in order to achieve a final term in $\frac{1}{x^3}$, terms in x^2 and $\left(\frac{-2}{x}\right)^5$ were needed, together with a binomial coefficient of ${}^7C_5$. A common approach, however, was to write out all, or part, of the expansion and a number of candidates stopped before reaching the required term, which was the sixth. A common mistake was to lose the minus sign.

Answer: –672

Question 2

Most candidates started this question correctly by differentiating the given function and many went on to set this to zero in order to find stationary points. After this, many candidates were influenced by the fact that at $x = -2$, for example, the gradient was zero which they thought automatically meant that the function was neither increasing nor decreasing. Although it was the right answer, it was for the wrong reason. Candidates trying to follow this approach needed to show that at $x = \frac{2}{3}$, the second derivative was positive, thereby establishing there was a minimum at this point. Therefore gradients to the left of this point were negative and gradients to the right were positive, so it was neither an increasing function nor a decreasing function. Some candidates found the second derivative but did not appreciate what it showed them. An alternative approach was to find the critical values of –2 and $\frac{2}{3}$, and evaluate the gradient of any point between them. Finding this to be negative, evaluating the gradient of any point to the right of $x = \frac{2}{3}$, and finding it to be positive, was sufficient to determine that f was neither an increasing function nor a decreasing function. A less frequently seen approach was to find f(x) for three or four values of x in the domain of the function and spanning $x = \frac{2}{3}$ in order to show at first a decreasing pattern, and then an increasing pattern, of values. It was not at all uncommon to see scripts which started off in the right way, scoring the first one or two marks, but then reached an incorrect conclusion. Not many used enough specific values of x substituted into f(x) or into f′(x) to be able to make a proper decision and hence often stated 'increasing' or 'decreasing' after only one valid substitution. The wording used by some candidates in their conclusion also requires comment. A function cannot be an increasing *function* for part of the domain and a decreasing *function* for another part of the domain. The word *function* needs to be omitted and the conclusion is then that it is neither an increasing function nor a decreasing function. Examiners were looking for the word 'Neither' being used, or words to that effect.

Answer: Neither

Question 3

Part **(i)** was well answered with almost all candidates scoring the one mark available for this part. Part **(ii)** was notable for the number of different routes candidates found to the end result. The most straightforward route was to find the area of the trapezium and to subtract the area of the sector. As reported in previous reports, some candidates showed a preference for working in degrees even though calculations in radians are less complicated and less subject to rounding errors. In addition to this, as reported in **General comments** above, many candidates obtained an incorrect final answer because they did not work with more than 3 significant figures in their calculations. However, many of these candidates employed a correct method and were able to score four of the five available marks.

Answers: **(i)** 0.8 **(ii)** 1.69

Question 4

In part **(i),** the vast majority of candidates were able to find the gradient of the line *AB* and most were then able to find the equation of *BC*. However, a significant proportion of candidates misinterpreted the question and found the equation of the perpendicular line through *A* or through the mid-point of *AB*. When finding the position of point *C* some candidates substituted $x = 0$ instead of $y = 0$. In part **(ii)**, some candidates did not spot that an answer correct to 4 significant figures was required. This question should have provided a good source of marks to most candidates but, in practice, marks were lost through minor mistakes.

Answers: **(i)** $y = \frac{-4x}{3} + 8$ $x = 6$; **(ii)** 7.071

Question 5

This question was quite well done, with a few candidates dropping the final mark as they did not appreciate that n had to be an integer. Candidates had to eliminate either a or n (most choosing a) leading to a quadratic equation and this was a stage that had to be shown. A number of candidates, after stating the initial two equations, went straight to the solution with no working shown. These candidates scored only the first two marks.

Answer: $n = 40$ $a = –23$

Question 6

The first step required in this unstructured question was to decide which two vectors to combine in a scalar product in order to find the required angle. Most candidates correctly chose **BO** and **BF** or **OB** and **FB**. If one of these vectors was reversed it gave rise to a negative solution which lost the final mark. A few candidates used **OF**, which resulted in finding a different angle and it was usually only possible to award the first mark. A significant number of candidates found **BF** incorrectly. These candidates thought that *CG* was a vertical edge and therefore set **CG** equal to 7 **k**. This did not give a final answer which could be expressed as a fraction in which the denominator was an integer. There was usually no indication that the candidate had gone back to check whether an error had been made.

Answer: $\cos^{-1}\left(\frac{28}{45}\right)$

Question 7

In part **(i)**, candidates were generally able to express the left hand side as a single fraction, despite brackets often being omitted. Candidates were then able in the vast majority of cases to reach the required result. Many candidates had more trouble with part **(ii)**. The first difficulty for candidates was in realising they had to set the numerator to zero. Some of the candidates who successfully negotiated the first obstacle and reached a quadratic equation in $\sin\theta$ were not then able to deal with the quadratic equation.

Answer: **(ii)** 38.2°

Question 8

Part **(i)** was very well done, with and the vast majority of candidates scoring full marks. Part **(ii)** proved to be more difficult. Some candidates made mistakes in deriving the two initial equations, e.g. setting $\frac{dy}{dx} = 3$ instead of 0 when $x = 2$. Mistakes were often made in the simplification, or solving, of the equations.

Answers: **(i)** $y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x + 11$ **(ii)** $a = 3$ $b = -4$

Question 9

In part **(i)**, most candidates knew to find the discriminant. However, mistakes were fairly common. Some candidates made mistakes when collecting like term together to express the 3–term quadratic equation. Brackets were also not always used and this usually led to errors. Finally, $b^2 - 4ac$ was not always applied correctly. Careful candidates usually found their way to the correct quadratic expression, but many candidates did not know how to progress from that point. For example, a significant number of candidates introduced an inequality, arriving perhaps at $9k^2 + 6k + 1 \geqslant 0$ from which they then stated their answer, $k \geqslant \frac{1}{3}$. This was not what was required. Having obtained the correct quadratic expression some candidates simply stated that it was greater than or equal to zero, which was not sufficient. Candidates needed to express the quadratic expression as a perfect square e.g. $(3k + 1)^2$, state that this was greater than or equal to zero and conclude that, for all values of *k*, the line and curve meet. Many candidates scored the first three marks if their arithmetic and algebraic manipulation were correct, but relatively few candidates scored the final mark. In part **(ii)** it was permissible to refer to the work done in the previous part, but many candidates repeated the work in order to derive the value of *k* required. If this value of *k* was correct, candidates were often able to continue and score full marks by following one of several possible routes to the right answer.

Answer: **(ii)** $k = -\frac{1}{3}$, $\left(\frac{2}{3}, -\frac{1}{9}\right)$

Question 10

While the method for finding volume of revolution was familiar to most candidates, the challenge was in finding $\left((3x-1)^{-\frac{1}{3}}\right)^2$. Candidates did not seem confident finding the resultant power and powers such as $\frac{2}{3}$, $\frac{5}{3}$ and $\frac{1}{9}$ were seen frequently. Integrating first and then trying to square the result was a route some took, whilst others integrated y or, having found y^2, forgot to integrate. The actual process of integrating was safer ground and was mostly done effectively. Some final answers were given without showing the working for substituting the limits (for which marks were lost) and 1 was seen occasionally as the lower limit. In contrast to part **(i)**, candidates seemed more comfortable with the process of differentiation in part **(ii)** and many fully correct solutions were seen. Using the gradient of the tangent instead of the normal or assuming the y coordinate of A was zero were errors that were occasionally seen.

Answers: **(i)** 4π **(ii)** $y=\left(\frac{1}{2}\right)x+\frac{5}{3}$

Question 11

The familiar request in part **(i)** was done very well, with most candidates scoring full marks. Part **(ii)** required a little more thought and a variety of wrong attempts were seen. The question asks for the value of k, so answers left in terms of x did not score. In part **(iii)**, most candidates demonstrated that they knew the process for finding the inverse function, but many lost a mark by not choosing the negative square root and some candidates got the domain wrong, often by using the wrong inequality. In part **(iv)**, it tended to be the stronger candidates only who recognised the need to use $x + 3 \leqslant 1$ and went on to score full marks. Most candidates were successful in finding $fg(x)$, although most employed a more laboured method in order to find $f(x + 3)$ rather than using the form found for $f(x)$ in part **(i)**.

Answers: **(i)** $2(x-3)^2-7$ **(ii)** Largest value of k is 3 **(iii)** $f^{-1}(x)=3-\sqrt{\frac{1}{2}(x+7)}$ $x\geqslant-7$
(iv) largest p is -2, $fg(x) = 2x^2 - 7$

MATHEMATICS

Paper 9709/21
Paper 21

Key messages

It is essential that candidates give their answers to the required level of accuracy of 3 significant figures, which is stated in the rubric on the front of the examination paper. If a different level of accuracy is required, it will be stated within the question itself. This means that working must be done to a greater accuracy. When an exact answer is required, a calculator must not be used in the simplification and evaluation involved in the solution.

General comments

The cohort taking this examination was relatively small, so where it is difficult to make generalisations, the intended method of solution will be indicated.

Some candidates did not attempt a large number of questions, or gave working which demonstrated a complete lack of understanding of what was required in some of the questions. Basic mathematical errors were common.

Some candidates did demonstrate a good knowledge of the syllabus and were able to apply the techniques they had used both appropriately and correctly. It was clear from these candidates that there were no issues with timing.

Comments on specific questions

Question 1

(i) Most candidates chose to square each side of the equation and attempt to solve the resulting 2 term quadratic equation. Some candidates showed poor algebraic skills when attempting to square and simplify, with many candidates failing to identify the solution $x = 0$. Better success was had by those candidates who chose to produce 2 separate linear equations.

(ii) It was intended that candidates make use of their solutions to the first part of the question. This was the reason the word 'Hence' was used. Candidates should recognise the use of this word. Very few candidates did so and as a result, correct solutions were few. It was intended that x in part **(i)** was equivalent to 3^y in part **(ii)** and so a solution to the equations $3^y = \frac{2}{3}$, making use of logarithms, was required. It was essential that the answer was given to the correct level of accuracy as some candidates gave an answer of -0.37.

Answer: **(i)** $0, \frac{2}{3}$ **(ii)** -0.369

Question 2

In spite of the hint given in the question that the integral involved a logarithm, some candidates did not integrate correctly to obtain an integral of the form $k\ln(2x+1)$. It was pleasing to see that many candidates did obtain this form and applied the limits correctly to their integral. However, many did not go on to use the power law and subtraction law for logarithms, choosing instead to resort to the use of a calculator. Checking that $\ln 125 = 4.83$ or similar, does not 'show' that the integral is equal to $\ln 125$. It is essential that each step of working is given in a 'Show that..' question and the use of calculators is not usually required in a question of this type. Candidates must also take care when using the subtraction law as $\ln 15 - \ln 3 \neq \frac{\ln 15}{\ln 3}$.

Question 3

There were few correct responses to this question as many candidates were unsure of which trigonometric identities to use. There were 2 different initial approaches that could be used. The first one was to write $\sec^2\theta$ as $\frac{1}{\cos^2\theta}$ and then make use of $\cos^2\theta = 1 - \sin^2\theta$ which then led to a quadratic equation in $\sin\theta$. The second, but longer approach was to $\sec^2\theta = 1 + \tan^2\theta$ and then make use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and the identity $\cos^2\theta = 1 - \sin^2\theta$ to obtain the same quadratic equation in $\sin\theta$. However, many candidates made the incorrect assumption that if $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ then $\operatorname{cosec}\theta = 1 + \cot\theta$. The resulting quadratic equation in $\sin\theta$ does not factorise so use of the quadratic formula or equivalent was necessary.

Answer: 57.9°, 122.1°

Question 4

(i) It was intended that $x = -2$ be substituted into the given expression. Many candidates did so, but immediately stated that the expression was equal to zero. As the candidates were required to show that $x + 2$ is a factor, it is essential that each term is evaluated and shown. Many candidates did not do this and lost the accuracy mark as a result. It is also good practice to conclude questions of the type with words to the effect that ' the remainder is zero, hence $x + 2$ is a factor'. Algebraic long division was acceptable provided a remainder of zero was reached and a conclusion made, as was synthetic division. Forming an identity was also acceptable, but a remainder was needed and this had to be shown to be equal to zero and a conclusion made for both marks. Candidates should recognise that a question of this type is straightforward and little work is needed to get full marks. Algebraic long division, synthetic division and forming an identity are time consuming and also prone to errors.

(ii) It was intended that candidates make use of algebraic long division and obtain a cubic quotient which could be equated to zero and then re-arranged. Those candidates that had used this method in part **(i)** were then able to make use of their quotient from part **(i)** immediately. It was evident that some candidates were unable to produce the steps required. A good approach to questions with several parts is to check if a previous part can be used, if a candidate is having a problem in how to proceed.

(iii) Many candidates demonstrated high levels of competence with the iterative process, making good use of their calculators and the answer function to cut down time consuming calculations. It was necessary that each iteration be shown to the required level of accuracy and that there were enough iterations to deduce the final answer, which also had to be given to the required level of accuracy.

Answer: 2.256

Question 5

(i) Many candidates were able to obtain a correct expression for $\frac{dy}{dx}$ but seldom used it to obtain the equation of the tangent as required. It is essential that candidates check that they have met the demands of the question. Common errors included writing $\frac{dx}{dt} = \frac{1}{t+1}$ rather than the correct $\frac{dx}{dt} = 1 + \frac{1}{t+1}$ and also failing to recognise that $y = 3te^{2t}$ needed to be differentiated as a product. Most candidates did not attempt to find the value of t at the origin.

(ii) Most candidates attempted to equate their $\frac{dy}{dx}$ to zero, but needed to make a valid attempt to solve for t before being able to obtain a method mark. Very few correct solutions were seen.

Answer: **(i)** $y = \frac{3x}{2}$ **(ii)** $x = -1.19$, $y = -0.55$

Question 6

(i) The use of the trapezium rule continues to cause problems for candidates. Common errors included having too many x values, which implies that the interval width is incorrect, calculators being in the incorrect mode, incorrect evaluations and just using the x values rather than the y values. As a result there were few correct solutions.

(ii) Many candidates were able to obtain the correct integrand for the required volume. However many did not recognise the need to make use of an appropriate double angle formula before attempting to integrate. It should also be noted that an exact answer was required and even though candidates were requested not to use a calculator, many did.

Answer: **(i)** 4.84 **(ii)** $\frac{5\pi^2}{2}$

Question 7

(i) Most candidates were able to gain at least one mark in this question, with many completely correct responses seen. A common error was to obtain $\cos 2x - 3\sin 2x$.

(ii) Many candidates knew the steps that they had to take in the solution of this part. However, errors meant that full marks were rarely obtained. Some candidates obtained an incorrect value for α as they had written $\tan\alpha = \frac{1}{3}$ rather than $\tan\alpha = 3$. Some worked in degrees and did not convert back to radians. Questions involving calculus need to have any work done in terms of radians unless otherwise specified. Many candidates obtained the first solution in the range, but were unable to find the second solution. Of those candidates whose expression for $\frac{dy}{dx}$ was incorrect, many were able to gain method marks for correct steps being used.

Answer: **(i)** $2\cos 2x - 6\sin 2x$ **(ii)** 1.979, 3.055

MATHEMATICS

Paper 9709/22
Paper 22

Key messages

It is essential that candidates give their answers to the required level of accuracy of 3 significant figures, which is stated in the rubric on the front of the examination paper. If a different level of accuracy is required, it will be stated within the question itself. This means that working must be done to a greater accuracy. When an exact answer is required, a calculator must not be used in the simplification and evaluation involved in the solution.

General comments

Some candidates did not attempt a large number of questions, or gave working which demonstrated a complete lack of understanding of what was required in some of the questions. Basic mathematical errors were common.

Some candidates did demonstrate a good knowledge of the syllabus and were able to apply the techniques they had used both appropriately and correctly. It was clear from these candidates that there were no issues with timing.

Comments on specific questions

Question 1

It was essential that candidates obtained the critical values using either a quadratic equation or a pair of linear equations or inequalities. By far the most popular method of obtaining these critical values was to form a quadratic equation by squaring each side of the given inequality and equating. However, many candidates squared the left hand side of the inequality and forgot about squaring the coefficient of x^2 on the right hand side which resulted in the incorrect quadratic equation $7x^2 - 15x + 25 = 0$ rather than the correct quadratic equation $5x^2 - 15x + 25 = 0$.

Candidates making use of either 2 linear inequalities or 2 linear equations to find the critical values were usually more successful.

Answer: $1 < x < 5$

Question 2

Very few candidates were able to make much progress with this question. It was intended that a quadratic equation in 3^x be formed, the solutions of which could then be used to solve for x. Quite a few candidates recognised that $9^x = 3^{2x}$, but were unable to progress from there is a correct manner.

Most candidates attempted to take logarithms of each term in the given equation. This was a completely incorrect approach which could not be awarded any marks. It is essential that candidates read the question carefully. In this case, the first demand was to find value of 3^x and **then** use logarithms to find the value of x.

Answer: 15, 2.465

Question 3

Many candidates were able to attempt the differentiation required to find the x – coordinate of the stationary point. Most were able to obtain the form $p\cos 2x - q\sec^2 2x$, with many obtaining the correct values of p and q. Some candidates forgot to take into account the double angle when they were differentiating and gave an incorrect statement of $\frac{dy}{dx} = 5\cos 2x - 3\sec^2 2x$. Rearranging the trigonometric result when $\frac{dy}{dx} = 0$ to obtain $\cos^3 2x = \frac{3}{5}$ was problematic for some who either tried to use the identity $\tan^2 2x + 1 = \sec^2 2x$ (this would then involve a lot more work) or mistakenly thought that $\sec 2x = \frac{1}{\sin 2x}$.
There were however, candidates who completed this question with well thought out and well-presented correct solutions.

Answer: 0.284

Question 4

Most candidates realised that implicit differentiation was needed and differentiated each term of the given equation, with varying degrees of success. Some candidates did not recognise the need to differentiate $3ye^{2x}$ as a product and others did not differentiate the right hand side of the equation, resulting in the derivatives of the left hand side of the equation being equated to 10 rather than zero. It is far easier to substitute values for x and y into an equation which has not been re-arranged and simplified, than to re-arrange the equation and them make substitutions of values. In this case, most candidates attempted to find an expression for $\frac{dy}{dx}$ and then substitute in $x = 0$ and $y = 2$. However, mistakes in simplification prior to substitution often led to incorrect answers. For those candidates who did not realise that implicit differentiation was needed, there was no opportunity for gaining any marks.

Answer: $-\frac{16}{7}$

Question 5

(i) Candidates with algebraic skills gave good answers to this part of the questions It is essential that each step of a candidate's working is shown in a question of this type as the final result is given.

(ii) There were 2 different approaches that could be taken. The first approach, which was attempted by the majority of those who attempted this part, was to make use of the equation obtained in part **(i)**. Some candidates substituted both values into both sides of the equation but made no explanation as to what this was showing. The more successful candidates chose to use the equation in part **(i)** and form a new function of the type $f(x) = x - \frac{1}{2}\ln(1.6x^2 + 4)$ or equivalent and use substitution of both given values to show a sign change.

The second approach was to make use of the original equation $y = 5e^{2x} - 8x^2 - 20$ and substitute the given values to again obtain a sign change. Whichever approach had been used, it was expected that the candidate would make an appropriate conclusion after their calculations.

(iii) It was expected that a value between 0.75 and 0.85 be used as a starting point for the iterative process. It was evident that many candidates were not making the correct use of their calculator as evidenced by the incorrect results obtained after substitution into a correct formula. Some candidates did not give enough iterations. In most cases at least 9 or 10 were needed and it was essential that each of these was shown to the correct level of accuracy. Many gave their final answer as either 0.8095 or 0.8096, not the level of accuracy required in the question. Some candidates chose incorrectly to 'round down' to 0.80955.

(iv) Of those candidates who attempted this part, most were able to gain marks for a correct differentiation and attempt to substitute in their value from part **(iii)** to obtain a value for the gradient of the curve at the given point.

Answer: **(iii)** 0.80956 **(iv)** 37.5

Question 6

(a) Most candidates recognised that the integral involved a logarithm, the clue to this being in the given answer. As the answer was given, it was important that each step of the solution be shown and the actual integral was expected. There were some errors involving the numerical coefficient, with 12 being a common error. The correct use of the square bracket notation was needed and then a clear use of both the subtraction law involving logarithms and the power law involving logarithms. These could be used in either order. Many candidates did not obtain the mark for stating that $4\ln 4 = \ln 4^4$.

(b) As an exact result was asked for, it was expected that no use of calculators be made. It was necessary to make use of the double angle formulae $2\sin^2 x = 1 - \cos 2x$ and $\tan^2 2x = \sec^2 2x - 1$. If either of these was not used, candidates were unable to make much progress. There were occasional slips in coefficients and signs. Few correct exact answers were seen as many candidates were unable to obtain the correct integral at the start.

Answer: **(b)** $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

Question 7

(i) It was expected that candidates substitute $x = -\frac{3}{2}$ into the given expression and obtain a result of zero. The question demanded a result to be shown and those candidates that wrote $8\left(-\frac{3}{2}\right)^3 + 4\left(-\frac{3}{2}\right)^2 - 10\left(-\frac{3}{2}\right) + 3 = 0$ with no evaluation of each of the separate terms were unable to gain the accuracy mark. It is important to show each step in a solution. Algebraic long division was also acceptable provided each step in the process was shown clearly and correctly. An appropriate conclusion was also expected for the accuracy mark, whichever method was being used.

(ii) Many candidates were able to make use of the correct double angle formula and appropriate simplification to obtain the required result. There were occasional slips in manipulation and simplification.

(iii) It was necessary for candidates to make the connection between this part of the question and parts **(i)** and **(ii)**. Those candidates who had completed algebraic long division in part **(i)** were at an advantage over those that had not as they were more readily able to obtain a factorised form of the equation in part **(ii)**. There was only one solution for $\cos\theta$. For those candidates who did not make the connection, often no marks were awarded. Some of the stronger candidates were able to gain full marks for this part by making the correct connections between each of the parts. Candidates should be encouraged to look for 'similar' expressions or equations in longer questions of this type.

Answer: **(iii)** $60°$, $300°$

MATHEMATICS

Paper 9709/23
Paper 23

Key messages

It is essential that candidates give their answers to the required level of accuracy of 3 significant figures, which is stated in the rubric on the front of the examination paper. If a different level of accuracy is required, it will be stated within the question itself. This means that working must be done to a greater accuracy. When an exact answer is required, a calculator must not be used in the simplification and evaluation involved in the solution.

General comments

The cohort taking this examination was relatively small, so where it is difficult to make generalisations, the intended method of solution will be indicated.

Some candidates did not attempt a large number of questions, or gave working which demonstrated a complete lack of understanding of what was required in some of the questions. Basic mathematical errors were common.

Some candidates did demonstrate a good knowledge of the syllabus and were able to apply the techniques they had used both appropriately and correctly. It was clear from these candidates that there were no issues with timing.

Comments on specific questions

Question 1

(i) Most candidates chose to square each side of the equation and attempt to solve the resulting 2 term quadratic equation. Some candidates showed poor algebraic skills when attempting to square and simplify, with many candidates failing to identify the solution $x = 0$. Better success was had by those candidates who chose to produce 2 separate linear equations.

(ii) It was intended that candidates make use of their solutions to the first part of the question. This was the reason the word 'Hence' was used. Candidates should recognise the use of this word. Very few candidates did so and as a result, correct solutions were few. It was intended that x in part **(i)** was equivalent to 3^y in part **(ii)** and so a solution to the equations $3^y = \frac{2}{3}$, making use of logarithms, was required. It was essential that the answer was given to the correct level of accuracy as some candidates gave an answer of -0.37.

Answer: **(i)** $0, \frac{2}{3}$ **(ii)** -0.369

Question 2

In spite of the hint given in the question that the integral involved a logarithm, some candidates did not integrate correctly to obtain an integral of the form $k\ln(2x+1)$. It was pleasing to see that many candidates did obtain this form and applied the limits correctly to their integral. However, many did not go on to use the power law and subtraction law for logarithms, choosing instead to resort to the use of a calculator. Checking that $\ln 125 = 4.83$ or similar, does not 'show' that the integral is equal to $\ln 125$. It is essential that each step of working is given in a 'Show that..' question and the use of calculators is not usually required in a question of this type. Candidates must also take care when using the subtraction law as $\ln 15 - \ln 3 \neq \dfrac{\ln 15}{\ln 3}$.

Question 3

There were few correct responses to this question as many candidates were unsure of which trigonometric identities to use. There were 2 different initial approaches that could be used. The first one was to write $\sec^2\theta$ as $\dfrac{1}{\cos^2\theta}$ and then make use of $\cos^2\theta = 1 - \sin^2\theta$ which then led to a quadratic equation in $\sin\theta$. The second, but longer approach was to $\sec^2\theta = 1 + \tan^2\theta$ and then make use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and the identity $\cos^2\theta = 1 - \sin^2\theta$ to obtain the same quadratic equation in $\sin\theta$. However, many candidates made the incorrect assumption that if $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ then $\operatorname{cosec}\theta = 1 + \cot\theta$. The resulting quadratic equation in $\sin\theta$ does not factorise so use of the quadratic formula or equivalent was necessary.

Answer: $57.9°$, $122.1°$

Question 4

(i) It was intended that $x = -2$ be substituted into the given expression. Many candidates did so, but immediately stated that the expression was equal to zero. As the candidates were required to show that $x + 2$ is a factor, it is essential that each term is evaluated and shown. Many candidates did not do this and lost the accuracy mark as a result. It is also good practice to conclude questions of the type with words to the effect that ' the remainder is zero, hence $x + 2$ is a factor'. Algebraic long division was acceptable provided a remainder of zero was reached and a conclusion made, as was synthetic division. Forming an identity was also acceptable, but a remainder was needed and this had to be shown to be equal to zero and a conclusion made for both marks. Candidates should recognise that a question of this type is straightforward and little work is needed to get full marks. Algebraic long division, synthetic division and forming an identity are time consuming and also prone to errors.

(ii) It was intended that candidates make use of algebraic long division and obtain a cubic quotient which could be equated to zero and then re-arranged. Those candidates that had used this method in part **(i)** were then able to make use of their quotient from part **(i)** immediately. It was evident that some candidates were unable to produce the steps required. A good approach to questions with several parts is to check if a previous part can be used, if a candidate is having a problem in how to proceed.

(iii) Many candidates demonstrated high levels of competence with the iterative process, making good use of their calculators and the answer function to cut down time consuming calculations. It was necessary that each iteration be shown to the required level of accuracy and that there were enough iterations to deduce the final answer, which also had to be given to the required level of accuracy.

Answer: 2.256

Question 5

(i) Many candidates were able to obtain a correct expression for $\frac{dy}{dx}$ but seldom used it to obtain the equation of the tangent as required. It is essential that candidates check that they have met the demands of the question. Common errors included writing $\frac{dx}{dt} = \frac{1}{t+1}$ rather than the correct $\frac{dx}{dt} = 1 + \frac{1}{t+1}$ and also failing to recognise that $y = 3te^{2t}$ needed to be differentiated as a product. Most candidates did not attempt to find the value of t at the origin.

(ii) Most candidates attempted to equate their $\frac{dy}{dx}$ to zero, but needed to make a valid attempt to solve for t before being able to obtain a method mark. Very few correct solutions were seen.

Answer: **(i)** $y = \frac{3x}{2}$ **(ii)** $x = -1.19$, $y = -0.55$

Question 6

(i) The use of the trapezium rule continues to cause problems for candidates. Common errors included having too many x values, which implies that the interval width is incorrect, calculators being in the incorrect mode, incorrect evaluations and just using the x values rather than the y values. As a result there were few correct solutions.

(ii) Many candidates were able to obtain the correct integrand for the required volume. However many did not recognise the need to make use of an appropriate double angle formula before attempting to integrate. It should also be noted that an exact answer was required and even though candidates were requested not to use a calculator, many did.

Answer: **(i)** 4.84 **(ii)** $\frac{5\pi^2}{2}$

Question 7

(i) Most candidates were able to gain at least one mark in this question, with many completely correct responses seen. A common error was to obtain $\cos 2x - 3\sin 2x$.

(ii) Many candidates knew the steps that they had to take in the solution of this part. However, errors meant that full marks were rarely obtained. Some candidates obtained an incorrect value for α as they had written $\tan\alpha = \frac{1}{3}$ rather than $\tan\alpha = 3$. Some worked in degrees and did not convert back to radians. Questions involving calculus need to have any work done in terms of radians unless otherwise specified. Many candidates obtained the first solution in the range, but were unable to find the second solution. Of those candidates whose expression for $\frac{dy}{dx}$ was incorrect, many were able to gain method marks for correct steps being used.

Answer: **(i)** $2\cos 2x - 6\sin 2x$ **(ii)** 1.979, 3.055

MATHEMATICS

Paper 9709/31
Paper 31

Key messages

Candidates need to know:
- that they cannot take logs of negative numbers
- exactly what is meant by a tangent parallel to the x-axis or parallel to the y-axis
- how to sketch a cubic graph
- how to use the chain rule for differentiation of a trig function such as $\sin^3(4x)$
- that a 3-term partial fraction with two linear terms and one linear term squared for the denominators should have only three unknown coefficients. Introducing $Dx + E$ in the numerator above the squared linear term is over-specifying the system.

General comments

The standard of work on this paper varied considerably, although all questions were accessible to the stronger candidates. A significant number of candidates found certain questions extremely difficult. Candidates are reminded that working carefully through past papers and mark schemes will give them a good idea of what to expect in the examination.

Questions or parts of questions that were generally done well were **Question 2** (solution of exponential equation), **Question 3(ii)** (iterative convergence), **(iii)** (iterative formula to find root) **Question 4(i)** (parametric equations), **Question 5** (solution of differential equation), **Question 6(ii)** (solution of trig equation), **Question 9(i)** (partial fractions) and **Question 10(ii)** (angle between planes). Questions that were done less well were **Question 3(i)** (sketching graphs), **Question 4(ii)** (finding where tangent to curve is parallel to y-axis), **Question 6(ii)** (solving trig equation), **Question 7(i)**, **(ii)** (establishing stationary point on curve given by a trig function and determining the exact area under this curve), **Question 9(ii)** (integration of partial fractions) and **Question 10(iii)** (finding position of point on a line a given distance from a plane).

In general the presentation of the work was good, though there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black pen**, reasonable sized lettering and symbols, and present their work clearly. Candidates should avoid using ink that is absorbed into the paper and then appears on the reverse side as this can make it difficult to read the pages when scanned.

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. In recent years additional rubric has been added to individual questions, for example ‘show all your working.’ This is to ensure that candidates can demonstrate understanding of the methods even when their calculator is capable of finding the answer. For example, modulus of a vector, scalar product of two vectors, product or division of two complex numbers, modulus or argument of complex number. No credit is given to answers written directly from the calculator, without working. This will be even more important when the modified syllabus commences, since it has a new rubric ‘no marks will be given for unsupported answers from a calculator’. Very strong candidates who can do some of these tasks mentally should also be encouraged to show the steps in their solution in order to gain credit.

Where answers are given after the comments on individual questions, it should be understood that the form given is not necessarily the only ‘correct answer’.

Comments on specific questions

Question 1

This question proved to be challenging for candidates. The omission of the parameter *a* was usually the source of the problem. Many candidates failed to square the 2 outside the modulus sign or poor algebra prevented them from arriving at the correct equation. Although many candidates took the quadratic equation approach, this question is more easily solved by considering the pair of linear equations or linear inequalities. Many candidates who successfully obtained a correct quadratic equation ran into problems because they did not have the correct power of *a* where required. Factorising was generally more successful. Some candidates decided to set *a* to unity, i.e. to solve for $\frac{x}{a}$, but they did not reintroduce *a* after solving their equation. Those who did obtain $x = -\frac{a}{5}$ and $\frac{5a}{3}$ usually made the correct choice of region at the very end.

Answer: $-\frac{a}{5} < x < \frac{5a}{3}$

Question 2

This question was usually well done, however there were still many candidates who did not reach an equation of the form $ae^{2x} = b$, $ae^{x} = be^{-x}$ or $ae^{x} = \frac{b}{e^{x}}$ with the correct values of both *a* and *b*. Many arithmetic and algebraic errors were seen, as well as multiplication of indices to give an exponential term to the power x^2. Most candidates knew they then needed to take lns but some did not show this step despite the additional instruction to show all necessary working. Too many candidates had a correct equation, but with *a* and *b* negative. Then, before moving to an equation with positive terms, they attempted to take lns. Candidates should be aware that they cannot take lns of negative numbers

Answer: 0.46

Question 3

(i) This part proved challenging more many candidate, for various reasons. The graph of $y = x^3$ should be sketched for at least $-2 \leqslant x \leqslant 2$ to show its shape. Stopping at $x = -1$ does not highlight how the graph behaves for negative *x*. Similarly the graph of $y = 3 - x$ should be sketched for at least $-1 \leqslant x \leqslant 4$. However, leaving the graphs in this form is still insufficient, and adding the comment 'there is exactly one real root' is not enough. The question required a clear indication of the single point of intersection with a dot or some other mark, together with a comment that there is only one point of intersection so only one root. This final statement should be clearly made without it being left to the examiner to interpret.

(ii) Convergence, starting with either form, was usually well done. As it is a proof there should be no jumps in the working; it is better to show too many lines than too few.

(iii) Most candidates found the root to the required accuracy but occasionally candidates left their answer to 2 d.p. or 4 d.p.

Answer: **(iii)** 1.213

Question 4

(i) Most candidates knew what was required and were successful with either $\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$. However many did not apply the chain rule correctly to either $\sin 2\theta$ or $\cos 2\theta$. Although the question didn't ask for the expression in a particular form, most candidates tried to simplify it into an expression in just $\sin\theta$ and $\cos\theta$. This often introduced errors and if candidates continued to use this form into **(ii)** little progress was made in that section. Fortunately most candidates returned to their initial expression that was hopefully free of errors.

(ii) This is where candidates really ran into trouble. Most candidates believed that this question must be about finding where $\frac{dy}{dx} = 0$, 1 or ∞, with about half using 0, 10% using 1 and the remainder using ∞. If candidates cannot reason this mentally they should draw a small sketch to help themselves, as a wrong guess meant zero out of four immediately. Most of the candidates making the correct choice scored three or four marks, since some stopped after finding the value of θ. Some candidates factorised their quadratic incorrectly or had a sign error in their double angle formula.

Answers: **(i)** $-\frac{(2\sin\theta + 2\sin 2\theta)}{(2\cos\theta + 2\cos 2\theta)}$ **(ii)** $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

Question 5

This proved a high scoring question as most candidates could separate variables correctly and obtain the ln *y* term. The integral in *x* proved more difficult, since instead of considering $\frac{2}{x}$ and *x*, some candidates resorted to integrating by parts and increased the possibility of sign errors. A considerable number of candidates left the expression as ln *y* instead of obtaining an expression for *y* as requested in the question. Candidates who left the term exp(2ln *x*) in their final answer were penalised in their final mark.

Answer: $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$

Question 6

(i) Many candidates found it difficult to obtain the given expression, taking far more steps than were required. Fortunately most did achieve it, although several had sign and coefficient errors in their cos *x* and sin *x* terms. Done correctly, *R* cos α and *R* sin α should have been $\sqrt{3}$ and 1 respectively, and the working for *R* and α should not have had any negative signs present. However, a large number of candidates simply quoted the expressions $R = \sqrt{(a^2 + b^2)}$ and $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$, with $(-1)^2$ seen in the expression for *R*. Whilst examiners allowed this error in the evaluation of $R = 2$ they were less generous if the sign error was present in finding α. There is a similar issue arising in **Question 9(i)** when determining the partial fractions. In the present case the use of the formula for *R,* which is perfectly valid, has covered up the incorrect mathematics. A few candidates made the error of writing $\cos\alpha = \sqrt{3}$ and sin $\alpha = 1$ (omitting *R*), leading incorrectly to the correct expression $\tan\alpha = \frac{1}{\sqrt{3}}$ and $\alpha = 30°$.

(ii) A few candidates did not spot the link between **(i)** and **(ii)** and as a result were unable to make any progress. However, most candidates readily found the values of $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$ and hence both required solutions. Some incorrectly believed that there was only one angle arising from $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$ and that the other angle should come from 180° minus their correctly found angle.

Answer: $x = 75°$ and 165°

Question 7

(i) This was a question where candidates usually scored either just a single method mark, for using the product rule, or full marks. Correct solutions required candidates to apply the chain rule accurately twice. It was still possible to proceed to a solution if the errors in the chain rule were restricted to the coefficients. Other errors such as incorrect powers of trig functions meant that it was impossible to reduce the equation to one that could be solved.

(ii) Nearly all candidates struggled with this question despite being given the substitution required. This was evidenced by the amount of working most produced, with little progress made. Common errors were poor substitution of $du = \cos x\, dx$, inability to express $\cos^2 x$ correctly in terms of u, substituting limits 0 and $\frac{\pi}{2}$ for u or substituting limits 0 and 1 for x. In fact very few candidates reached the correct integrand in terms of u, together with the correct limits.

Answers: **(i)** $x = 0.685$ **(ii)** $\frac{2}{3}$

Question 8

(a) The rubric demanded that candidates show detailed working in order to gain marks. Most candidates divided out the complex numbers then converted to polar form, as opposed to converting to polar form and then dividing out. When multiplying the numerator and the denominator by the conjugate of the denominator, candidates often had errors either in the denominator, e.g. $1 + (2i)^2$ instead of $1 - (2i)^2$, or arithmetic errors when collecting the real or imaginary parts in the numerator. Basic errors such as these meant that none of the accuracy marks could be gained. However, many candidates who performed the division correctly then left their modulus in the exact form instead of correct to 3 significant figures. Few candidates managed to obtain the correct argument, the usual answer being $\theta = -1.05$, something that candidates arrived at from their calculators without any reference to the position of the complex number $\frac{-4}{5} + \frac{7}{5}$ i in the Argand diagram. A quick sketch would have helped them.

(b) Many candidates scored full marks, although some restricted their solution to the Argand diagram part only, with no attempt at the least value of $|z|$. Several candidates finished with their circles in the wrong quadrant, whilst others did not indicate that the radius of the circle was unity.

Answers: **(a)** $1.61e^{i2.09}$ **(b)** $\sqrt{13} - 1$

Question 9

(i) This question was extremely well done by most candidates. However, a few candidates introduced a constant term into their partial fractions, something only necessary when the powers of the numerator and denominator are the same. A few others incorrectly used a linear term as opposed to a constant term for the numerator of the term with the denominator $(3 + 2x)^2$ in the 3-term partial fractions. This form will see two of the constants determined uniquely but only a combination of the other two as unique. Candidates need to check very carefully when they remove the denominator in order to find their constants, since slips are possible, e.g. multiplication by a factor from the denominator that is of the incorrect power. Any error at this point leads to errors in the three equations obtained by equating the coefficients of the powers of x and incorrect constants. However, if the candidate opts instead to construct their equations by using the roots of the factors of the denominator then in some cases the incorrect terms will vanish and the correct values of the constants will be obtained. Unfortunately, these cannot be given credit since the equations were in error and it is only by the incorrect terms dropping out that correct answers have been obtained. This feature is like that arising in **Question 6(i)** where the correct solution only resulted from the squaring of an incorrect negative sign.

(ii) Integrating the partial fractions proved testing, as many candidates had the incorrect coefficients for both their ln terms. Hence it was possible to see some candidates who had the correct forms for all three integrals gaining no marks due to mistakes in determining the appropriate coefficients. In obtaining the given answer, it is necessary for candidates to ensure that there are no gaps in their working; again too much detail is better than not enough.

Answers: **(i)** $\frac{1}{(2-x)}-\frac{1}{(3+2x)}+\frac{3}{(3+2x)^2}$ or $\frac{1}{(2-x)}-\frac{2x}{(3+2x)^2}$

Question 10

(i) All details of the working using the scalar product were required. Most candidates were able to show the scalar product was zero, but omitted to verify that one point of *l* does not lie in the plane. If, instead, the candidate substituted coordinates of a general point of *l* in the equation of the plane, it was necessary to establish and state there is no point that satisfies this equation. Again few candidates made the concluding statement.

(ii) Candidates usually showed the full details for their scalar product and moduli hence even with the presence of errors it was possible for them to gain both method marks.

(iii) Those candidates who knew the formula for the distance of a point from a plane usually quickly scored full marks, whilst most others struggled. There were a few candidates who didn't appear to know this formula yet made a reasonable attempt by finding where the normal vector from a point on the line met the plane and then setting the distance between these points equal to 2. Others appeared to think they had solved the problem when they found where the line met the plane at (5, 3, 3) and stated this as their answer. In fact they still had to find the distance between this point and a point on the line and set this equal to the length $\frac{2}{(\cos\alpha)}$, where α is the acute angle between the direction of the line and the normal vector to the plane.

Answers: **(ii)** 74.5° or 1.30 radians **(iii)** $7\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$

MATHEMATICS

Paper 9709/32
Paper 32

General comments

The response to this paper was very varied. There were some very strong candidates who produced clear, accurate solutions, and at the other extreme there were some scripts that were very difficult to follow, sometimes because one solution had been overwritten with another, and sometimes because there was no discernible method being followed.

Most candidates offered solutions to all ten questions, and very few candidates needed to use additional answer booklets. In general the candidates were familiar with the methods examined, but there were many examples of major errors in cancelling fractions, errors in basic arithmetic and the use of incorrect rules such as $\ln(a+b) = \ln a + \ln b$.

Candidates scored well on **Question 1** (modular inequality), **Question 2** (solving equations using trigonometry formulae), **Question 5** (root of equation and iterative process) and **Question 8** (partial fractions and binomial expansion). Parts of some questions were challenging to candidates, most notably **Question 3**ii (evaluate definite integral) **Question 4** (recognition of the quadratic equation in e^x), **Question 6** (variable separable differential equation), **Question 7ii** (calculus and trigonometry), **Question 9b** (finding the argument of a complex number) and **Question 10i and iii** (vectors).

Key messages for candidates

- Read the questions very carefully.
- If the question asks for full and exact working then the method must be clear at each stage, and calculator approximations will not be accepted.
- If the answer is given in the question then full working is expected.
- Never underestimate the need to practise basic arithmetic and algebraic processes – many candidates lose marks through elementary errors.
- Use brackets correctly when evaluating complex calculations on the calculator, and when writing down formulae.
- Never write one solution on top of another (even if you think that you have erased the original) – when the work is scanned it is often illegible.

Where numerical and other answers are given after the comments on individual questions that follow it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question gave many candidates a confident start to the paper. A few drew diagrams or rewrote the modular inequality as two linear inequalities, but the majority started by squaring both sides. A large number of candidates reached the correct critical values. Some candidates gave a decimal approximation to the negative root, which resulted in an inaccurate final answer. The most common errors were in squaring the brackets and in simplifying to obtain a quadratic equation/inequality. Some candidates did not square the 3. The incorrect answer $-\frac{1}{7} > x > \frac{7}{5}$ was a common error. Similarly, some candidates thought that it was possible for " $x < -\frac{1}{7}$ and $x > \frac{7}{5}$ ".

Answer: $x < -\frac{1}{7},\ x > \frac{7}{5}$

Question 2

The candidates recognised that the first step required was to expand $\sin(\theta - 30°)$. This was often done correctly, although several candidates made a sign error, or used the expansion of $\cos(\theta - 30°)$. Some candidates scored no marks at all because they started with $\sin(\theta - 30°) = \sin\theta - \sin 30°$.

In simplifying the equation, some candidates overlooked the "$+\cos\theta$", but the majority went on to form an equation in $\tan\theta$. Some candidates were unsure of the properties of $\tan\theta$ and gave an additional incorrect answer of $156.2°$.

The alternative method of solving $\cos\theta + (\sqrt{3} - 4)\sin\theta = 0$ by first expressing the left hand side in the form $R\cos(\theta + \alpha)$, or equivalent, was seen from a small number of candidates, and was usually completed correctly.

Answer: $\theta = 23.8°$

Question 3

(i) Most candidates did attempt to use integration by parts, and the majority obtained an answer with terms of the correct form, although several had difficulty with the coefficients, making sign errors and placing the 2 in the numerator. Some candidates made errors in integrating x^{-3}, obtaining answers involving x^4 or x^{-4}.

(ii) This was the first stage in the paper where a significant number of candidates offered no response. The first mark was available to any candidate who had an answer of the correct form in part **(i)**. As the answer is given in the question, the candidates were expected to explain how they moved from an expression involving $\ln 2$ to one involving $\ln 4$.

Answer: **(i)** $-\frac{\ln x}{2x^2} - \frac{1}{4x^2}$

Question 4

Those candidates who recognised that the given equation simplified to a quadratic equation in e^x usually went on to obtain a value for e^x. Some candidates worked on an incorrect quadratic equation, usually because they had $4(e^x + 1) = 4e^x + 1$.

The question asks candidates to show "all necessary working", so some indication of the method for obtaining the final answer was expected. Some candidates gave their final answer to 3 significant figures rather than 3 decimal places, and some attempted to obtain a solution for x from a negative value of e^x.

A significant minority of candidates made no progress at all because they attempted to apply incorrect rules of logarithms to the original fraction, or to $e^x + e^{-x} = 4e^x + 4$, in order to remove the exponentials.

Answer: $x = -1.536$

Question 5

(i) Most candidates made a correct start to this question by attempting to use the product rule for differentiation. The error $\frac{d}{dx}\ln(8-x) = \frac{1}{8-x}$ was common, and very few candidates traced their error when they could not obtain the given answer. Processing errors meant that several candidates with the correct derivative did not reach the given answer.

(ii) There were many correct solutions to this question, but also evidence of many candidates not really understanding the difference between looking for a root of $f(x) = 0$ and looking for a root of $f(x) = x$. There were several instances of candidates using $f(x) = 8 - \frac{8}{\ln(8-x)}$, obtaining $f(2.9) = 3.09$ and $f(3.1) = 2.97$ and concluding that there was no root in the interval because there was no sign change.

(iii) The majority of candidates applied the process for using the iterative formula correctly. Some candidates did not work to the accuracy required by the question, but many correct answers were seen. Some solutions were longer than necessary; given that the root is known to lie in the interval $(2.9, 3.1)$ candidates should have been working from a starting point in that interval, but this was not always the case.

Answer: **(iii)** 3.02

Question 6

This should have been a straight forward question about a variable separable differential equation. The first key step was to express the statement of proportionality as an equation. Some candidates did this, but the constant was often implied to be equal to 1, or an incorrect value was used. The incorrect constant usually came from using the gradient of the chord joining the two given points.

Most candidates achieved the next step, separation of the variables, correctly.

The majority of candidates completed $\int \frac{1}{x}dx$ correctly, but $\int \frac{1}{y^2}dy$ proved to be more difficult. Many candidates gave the answer as a multiple of $\ln y^2$.

Those candidates who completed the integration correctly usually reached the equivalent of $\frac{1}{y} = 1 - \frac{1}{2}\ln x$.

Some candidates stopped at that point, but most went on to rearrange the equation to express *y* in terms of *x*. This process generated several algebraic errors.

Answer: $y = \frac{2}{2 - \ln x}$

Question 7

(i) The candidates recognised the need to start by differentiating the equation of the curve. Most candidates used the quotient rule, although some preferred the product rule. Several candidates made errors in quoting the rule they chose to use, and some used only the numerator of the quotient rule without giving any justification for this. There were many errors in moving from the derivative to an equation in $\sin x$. Several candidates used “invisible brackets”, writing down terms like $2 + \sin x. - 3\sin x$ that were accepted if they were then multiplied out correctly at the next stage. However, the majority confirmed that they had not intended to use brackets at all. Some candidates multiplied out the terms in the numerator and the denominator and then "cancelled" the terms in $\sin x$ and $\sin^2 x$. Many of the candidates who did reach $6\sin x + 3 = 0$ did not obtain a negative value for $\sin x$, and those who did have a negative answer often ignored the minus sign. Although the question asks for the exact values of the coordinates, many candidates gave only decimal approximations.

(ii) Many candidates did recognise the form of the integral correctly. The common errors were to attempt to use integration by parts, or to integrate the numerator and denominator separately. Candidates with correct integration often reached a correct final answer. Candidates with errors in their working who found that $|\sin a| > 1$ did not seem to realise that this indicated an error in their working. A common error in the final stages of the solution was the incorrect statement $\ln(2 + \sin x) = \ln 2 + \ln(\sin x)$.

Answer: **(i)** $x = -\frac{\pi}{6}, \quad y = \sqrt{3}$ **(ii)** $a = 0.913$

Question 8

(i) This question provided a welcome source of marks for many candidates, the great majority of whom selected a correct form for their partial fractions. Most did attempt to split the original into three parts, although several opted for the two term alternative. Most errors were caused by slips in the arithmetic rather than by the use of incorrect methods.

(ii) The expansion of $(1-2x)^{-1}$ was often correct, although there were errors in simplifying $(-2x)^2$. The expansion of $(2-x)^{-1}$ and $(2-x)^{-2}$ was more challenging, with several attempts to expand $2\left(1-\frac{x}{2}\right)^{-1}$ or even $2(1-x)^{-1}$. Correct expansions were not always combined with the correct coefficients, and the correct final answer was not common.

Answers: **(i)** $\frac{1}{1-2x} + \frac{3}{2-x} - \frac{2}{(2-x)^2}$, **(ii)** $2 + \frac{9}{4}x + 4x^2$

Question 9

(a) (i) Most candidates understood exactly what they needed to do to divide by a complex number, and many obtained the correct answer.

(ii) The majority of candidates understood how to obtain the values of r and θ, with many reaching a correct value for r. Only a minority of candidates obtained the correct value for θ; despite the frequent use of sketch diagrams, the most common answer was $\theta = -\frac{\pi}{4}$.

(b) The majority of candidates recognised $|z - 3\text{i}| = 1$ as a circle, and most drew a circle of the correct size in the correct place. Although the question asked for a "sketch" of an Argand diagram, candidates were expected to use even scales on their axes. The scales on the two axes should be identical, but unequal scales were acceptable so long as the shape drawn looked like an ellipse.

Many candidates did not attempt to find the greatest value of $\arg z$. Those who did offer a solution usually overlooked the fact that it is the angle between the tangent and the radius that is a right angle; many attempts involved $\tan^{-1}\frac{1}{3}$ in place of $\sin^{-1}\frac{1}{3}$.

Answers: **(i)** $-2 + 2\text{i}$, **(ii)** $r = 2\sqrt{2}, \quad \theta = \frac{3}{4}\pi$, **(iii)** 1.91

Question 10

(i) Although there have been questions in the past using the form $\mathbf{r.n} = \mathbf{a.n}$, many candidates did not recognise this use of $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$. Many simply ignored the $-\mathbf{i} - 2\mathbf{j}$, and some worked with a vector $r\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

(ii) The majority of candidates were familiar with the process for finding the angle between two vectors, although several did not work with the direction vector of the line and the normal to the plane. Many of the candidates who were working with the correct vectors found the angle between the line and the normal to the plane and did not then go on to find the angle between the line and the plane.

(iii) Several candidates offered no attempt at this part of the question. Those candidates who identified that they needed to find a vector perpendicular to both the direction of the line and to the normal vector of the plane usually had a correct method to achieve this. Provided they had an answer to part **(i)** they could complete the task. The final mark was only given for answers expressed in the correct form.

Answer: **(i)** $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, **(ii)** $14.3°$, **(iii)** $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$

MATHEMATICS

Paper 9709/33
Paper 33

Key messages

Candidates need to know:

- that they cannot take logs of negative numbers
- exactly what is meant by a tangent parallel to the x-axis or parallel to the y-axis
- how to sketch a cubic graph
- how to use the chain rule for differentiation of a trig function such as $\sin^3(4x)$
- that a 3-term partial fraction with two linear terms and one linear term squared for the denominators should have only three unknown coefficients. Introducing $Dx + E$ in the numerator above the squared linear term is over-specifying the system.

General comments

The standard of work on this paper varied considerably, although all questions were accessible to the stronger candidates. A significant number of candidates found certain questions extremely difficult. Candidates are reminded that working carefully through past papers and mark schemes will give them a good idea of what to expect in the examination.

Questions or parts of questions that were generally done well were **Question 2** (solution of exponential equation), **Question 3(ii)** (iterative convergence), **(iii)** (iterative formula to find root) **Question 4(i)** (parametric equations), **Question 5** (solution of differential equation), **Question 6(ii)** (solution of trig equation), **Question 9(i)** (partial fractions) and **Question 10(ii)** (angle between planes). Questions that were done less well were **Question 3(i)** (sketching graphs), **Question 4(ii)** (finding where tangent to curve is parallel to y-axis), **Question 6(ii)** (solving trig equation), **Question 7(i)**, **(ii)** (establishing stationary point on curve given by a trig function and determining the exact area under this curve), **Question 9(ii)** (integration of partial fractions) and **Question 10(iii)** (finding position of point on a line a given distance from a plane).

In general the presentation of the work was good, though there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black pen**, reasonable sized lettering and symbols, and present their work clearly. Candidates should avoid using ink that is absorbed into the paper and then appears on the reverse side as this can make it difficult to read the pages when scanned.

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. In recent years additional rubric has been added to individual questions, for example 'show all your working.' This is to ensure that candidates can demonstrate understanding of the methods even when their calculator is capable of finding the answer. For example, modulus of a vector, scalar product of two vectors, product or division of two complex numbers, modulus or argument of complex number. No credit is given to answers written directly from the calculator, without working. This will be even more important when the modified syllabus commences, since it has a new rubric 'no marks will be given for unsupported answers from a calculator'. Very strong candidates who can do some of these tasks mentally should also be encouraged to show the steps in their solution in order to gain credit.

Where answers are given after the comments on individual questions, it should be understood that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question proved to be challenging for candidates. The omission of the parameter *a* was usually the source of the problem. Many candidates failed to square the 2 outside the modulus sign or poor algebra prevented them from arriving at the correct equation. Although many candidates took the quadratic equation approach, this question is more easily solved by considering the pair of linear equations or linear inequalities. Many candidates who successfully obtained a correct quadratic equation ran into problems because they did not have the correct power of *a* where required. Factorising was generally more successful. Some candidates decided to set *a* to unity, i.e. to solve for $\frac{x}{a}$, but they did not reintroduce *a* after solving their equation. Those who did obtain $x = -\frac{a}{5}$ and $\frac{5a}{3}$ usually made the correct choice of region at the very end.

Answer: $-\frac{a}{5} < x < \frac{5a}{3}$

Question 2

This question was usually well done, however there were still many candidates who did not reach an equation of the form $ae^{2x} = b$, $ae^{x} = be^{-x}$ or $ae^{x} = \frac{b}{e^{x}}$ with the correct values of both *a* and *b*. Many arithmetic and algebraic errors were seen, as well as multiplication of indices to give an exponential term to the power x^2. Most candidates knew they then needed to take lns but some did not show this step despite the additional instruction to show all necessary working. Too many candidates had a correct equation, but with *a* and *b* negative. Then, before moving to an equation with positive terms, they attempted to take lns. Candidates should be aware that they cannot take lns of negative numbers

Answer: 0.46

Question 3

(i) This part proved challenging more many candidate, for various reasons. The graph of $y = x^3$ should be sketched for at least $-2 \leqslant x \leqslant 2$ to show its shape. Stopping at $x = -1$ does not highlight how the graph behaves for negative *x*. Similarly the graph of $y = 3 - x$ should be sketched for at least $-1 \leqslant x \leqslant 4$. However, leaving the graphs in this form is still insufficient, and adding the comment 'there is exactly one real root' is not enough. The question required a clear indication of the single point of intersection with a dot or some other mark, together with a comment that there is only one point of intersection so only one root. This final statement should be clearly made without it being left to the examiner to interpret.

(ii) Convergence, starting with either form, was usually well done. As it is a proof there should be no jumps in the working; it is better to show too many lines than too few.

(iii) Most candidates found the root to the required accuracy but occasionally candidates left their answer to 2 d.p. or 4 d.p.

Answer: **(iii)** 1.213

Question 4

(i) Most candidates knew what was required and were successful with either $\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$. However many did not apply the chain rule correctly to either $\sin 2\theta$ or $\cos 2\theta$. Although the question didn't ask for the expression in a particular form, most candidates tried to simplify it into an expression in just $\sin\theta$ and $\cos\theta$. This often introduced errors and if candidates continued to use this form into **(ii)** little progress was made in that section. Fortunately most candidates returned to their initial expression that was hopefully free of errors.

(ii) This is where candidates really ran into trouble. Most candidates believed that this question must be about finding where $\frac{dy}{dx} = 0$, 1 or ∞, with about half using 0, 10% using 1 and the remainder using ∞. If candidates cannot reason this mentally they should draw a small sketch to help themselves, as a wrong guess meant zero out of four immediately. Most of the candidates making the correct choice scored three or four marks, since some stopped after finding the value of θ. Some candidates factorised their quadratic incorrectly or had a sign error in their double angle formula.

Answers: **(i)** $-\frac{(2\sin\theta + 2\sin 2\theta)}{(2\cos\theta + 2\cos 2\theta)}$ **(ii)** $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

Question 5

This proved a high scoring question as most candidates could separate variables correctly and obtain the ln *y* term. The integral in *x* proved more difficult, since instead of considering $\frac{2}{x}$ and *x*, some candidates resorted to integrating by parts and increased the possibility of sign errors. A considerable number of candidates left the expression as ln *y* instead of obtaining an expression for *y* as requested in the question. Candidates who left the term exp(2ln *x*) in their final answer were penalised in their final mark.

Answer: $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$

Question 6

(i) Many candidates found it difficult to obtain the given expression, taking far more steps than were required. Fortunately most did achieve it, although several had sign and coefficient errors in their cos *x* and sin *x* terms. Done correctly, $R\cos\alpha$ and $R\sin\alpha$ should have been $\sqrt{3}$ and 1 respectively, and the working for *R* and α should not have had any negative signs present. However, a large number of candidates simply quoted the expressions $R = \sqrt{(a^2 + b^2)}$ and $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$, with $(-1)^2$ seen in the expression for *R*. Whilst examiners allowed this error in the evaluation of $R = 2$ they were less generous if the sign error was present in finding α. There is a similar issue arising in **Question 9(i)** when determining the partial fractions. In the present case the use of the formula for *R,* which is perfectly valid, has covered up the incorrect mathematics. A few candidates made the error of writing $\cos\alpha = \sqrt{3}$ and $\sin\alpha = 1$ (omitting *R*), leading incorrectly to the correct expression $\tan\alpha = \frac{1}{\sqrt{3}}$ and $\alpha = 30°$.

(ii) A few candidates did not spot the link between **(i)** and **(ii)** and as a result were unable to make any progress. However, most candidates readily found the values of $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$ and hence both required solutions. Some incorrectly believed that there was only one angle arising from $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$ and that the other angle should come from 180° minus their correctly found angle.

Answer: $x = 75°$ and 165°

Question 7

(i) This was a question where candidates usually scored either just a single method mark, for using the product rule, or full marks. Correct solutions required candidates to apply the chain rule accurately twice. It was still possible to proceed to a solution if the errors in the chain rule were restricted to the coefficients. Other errors such as incorrect powers of trig functions meant that it was impossible to reduce the equation to one that could be solved.

(ii) Nearly all candidates struggled with this question despite being given the substitution required. This was evidenced by the amount of working most produced, with little progress made. Common errors were poor substitution of $du = \cos x\, dx$, inability to express $\cos^2 x$ correctly in terms of u, substituting limits 0 and $\frac{\pi}{2}$ for u or substituting limits 0 and 1 for x. In fact very few candidates reached the correct integrand in terms of u, together with the correct limits.

Answers: **(i)** $x = 0.685$ **(ii)** $\frac{2}{3}$

Question 8

(a) The rubric demanded that candidates show detailed working in order to gain marks. Most candidates divided out the complex numbers then converted to polar form, as opposed to converting to polar form and then dividing out. When multiplying the numerator and the denominator by the conjugate of the denominator, candidates often had errors either in the denominator, e.g. $1 + (2i)^2$ instead of $1 - (2i)^2$, or arithmetic errors when collecting the real or imaginary parts in the numerator. Basic errors such as these meant that none of the accuracy marks could be gained. However, many candidates who performed the division correctly then left their modulus in the exact form instead of correct to 3 significant figures. Few candidates managed to obtain the correct argument, the usual answer being $\theta = -1.05$, something that candidates arrived at from their calculators without any reference to the position of the complex number $\frac{-4}{5} + \frac{7}{5}i$ in the Argand diagram. A quick sketch would have helped them.

(b) Many candidates scored full marks, although some restricted their solution to the Argand diagram part only, with no attempt at the least value of $|z|$. Several candidates finished with their circles in the wrong quadrant, whilst others did not indicate that the radius of the circle was unity.

Answers: **(a)** $1.61e^{i2.09}$ **(b)** $\sqrt{13} - 1$

Question 9

(i) This question was extremely well done by most candidates. However, a few candidates introduced a constant term into their partial fractions, something only necessary when the powers of the numerator and denominator are the same. A few others incorrectly used a linear term as opposed to a constant term for the numerator of the term with the denominator $(3 + 2x)^2$ in the 3-term partial fractions. This form will see two of the constants determined uniquely but only a combination of the other two as unique. Candidates need to check very carefully when they remove the denominator in order to find their constants, since slips are possible, e.g. multiplication by a factor from the denominator that is of the incorrect power. Any error at this point leads to errors in the three equations obtained by equating the coefficients of the powers of x and incorrect constants. However, if the candidate opts instead to construct their equations by using the roots of the factors of the denominator then in some cases the incorrect terms will vanish and the correct values of the constants will be obtained. Unfortunately, these cannot be given credit since the equations were in error and it is only by the incorrect terms dropping out that correct answers have been obtained. This feature is like that arising in **Question 6(i)** where the correct solution only resulted from the squaring of an incorrect negative sign.

(ii) Integrating the partial fractions proved testing, as many candidates had the incorrect coefficients for both their ln terms. Hence it was possible to see some candidates who had the correct forms for all three integrals gaining no marks due to mistakes in determining the appropriate coefficients. In obtaining the given answer, it is necessary for candidates to ensure that there are no gaps in their working; again too much detail is better than not enough.

Answers: **(i)** $\frac{1}{(2-x)}-\frac{1}{(3+2x)}+\frac{3}{(3+2x)^2}$ or $\frac{1}{(2-x)}-\frac{2x}{(3+2x)^2}$

Question 10

(i) All details of the working using the scalar product were required. Most candidates were able to show the scalar product was zero, but omitted to verify that one point of *l* does not lie in the plane. If, instead, the candidate substituted coordinates of a general point of *l* in the equation of the plane, it was necessary to establish and state there is no point that satisfies this equation. Again few candidates made the concluding statement.

(ii) Candidates usually showed the full details for their scalar product and moduli hence even with the presence of errors it was possible for them to gain both method marks.

(iii) Those candidates who knew the formula for the distance of a point from a plane usually quickly scored full marks, whilst most others struggled. There were a few candidates who didn't appear to know this formula yet made a reasonable attempt by finding where the normal vector from a point on the line met the plane and then setting the distance between these points equal to 2. Others appeared to think they had solved the problem when they found where the line met the plane at (5, 3, 3) and stated this as their answer. In fact they still had to find the distance between this point and a point on the line and set this equal to the length $\frac{2}{(\cos\alpha)}$, where α is the acute angle between the direction of the line and the normal vector to the plane.

Answers: **(ii)** 74.5° or 1.30 radians **(iii)** $7\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$

MATHEMATICS

Paper 9709/41
Paper 41

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf. This was often seen to affect answers to **Questions 1**, **2**, **3**, **4**, **5** and **6** in this paper, and led to particular problems in **Question 5(ii)**
- When answering questions such as **Question 7(i), 7(ii)** and **7(iii)** on this paper, where the acceleration is given as a function of time, the equations of constant acceleration cannot be used.
- In questions such as **Question 4**, where the motion of connected particles takes place, or **Question 6**, involving motion on an inclined plane, it is always advisable to draw a force diagram. In order to give the answer more clarity, candidates should also state the origin of their equations, e.g. applying Newton's law to particle *A* along the plane.

General comments

There were some candidates who produced very good answers on this paper but also a significant number who scored low marks. Overall a wide range of performance was seen.

The examination allowed candidates at all levels to show what they knew, whilst differentiating well between the stronger candidates. **Question 1** was the question candidates found easiest whilst **Question 6** proved to be the one candidates found most challenging.

One of the rubrics on this paper is to take $g = 10$ and virtually all candidates followed this instruction. In some cases it is impossible to achieve a correct given answer unless this value has been used.

Comments on specific questions

Question 1

Most candidates made a good attempt at this question. It was first necessary to use the given information to determine the acceleration of the particle. As all of the given forces were constant, the constant acceleration equations could be used. Once the acceleration was found, Newton's second law of motion could be used to enable the required value of *F* to be found.

Answer: $F = 1.58$

Question 2

(i) As the train in this question was travelling at constant speed there was no net force acting on it. This means that the driving force provided by the engine is exactly balanced by the resistance force. Using the given values of the power and the speed, and applying Resistance force = Driving force = $\frac{P}{v}$, the resistance force could be found. Some candidates made the error of thinking that the driving force was directly related to the weight of the train.

(ii) In this part of the question the train moved up a hill but maintained its speed of 85 m s^{-1} and so again the forces were balanced. Newton's second law had to be applied to the train with acceleration = 0. As the train was going up a hill there was a force acting due to the component of the weight of the train acting against the direction of motion. This force takes the form $490\,000\,g \sin\theta = 490\,000\,g \times \frac{1}{200}$. The three forces, namely the driving force, the weight component and the resistance, were balanced. Application of Newton's second law takes the form $\frac{P}{85} - 48\,000 - 490\,000\,g \times \frac{1}{200} = 0$. Some candidates forgot that the resistance force was still acting. Others did not include the weight component. Some also forgot to include the factor of g in the weight component term.

Answers: **(i)** 48 000 N (Answer given) **(ii)** 6.16 MW (to 3 sf)

Question 3

Many candidates made the incorrect assumption that the driving force provided by the van's engine was constant. This is not given in the question and so cannot be assumed. In cases such as this, it is necessary to use work and energy principles rather than Newton's second law. There are several stages to go through in order to solve this problem. Firstly there is a loss of potential energy (PE) as the van descends the hill and this could be evaluated as PE loss = $2500 \times g \times 400 \sin 4$. Secondly there is a gain in kinetic energy (KE) as the speed increases and this is given by KE gain = $\frac{1}{2} \times 2500 \times (30^2 - 20^2)$. Thirdly the work done against the constant resistance force is given by WD against resistance = 600×400. Finally these three elements had to be combined using the work-energy principle in order to find the required work done by the van's engine. This takes the form: WD by van's engine + PE lost = KE gain + WD against resistance. Substituting the values found earlier enabled the required work done to be found. Many candidates made the wrong assumption that the driving force acting on the van was constant and proceeded to use constant acceleration equations and lost some marks in doing so. Some candidates who used the work-energy principle did not include all of the terms, while others used incorrect signs within their equation.

Answer: 167 000 J

Question 4

(i) In this question involving a simple pulley it was a good idea to draw a diagram showing the situation and the forces acting on each particle. The question gave the distance travelled by the particles before particle *A* reached the ground as 0.8 m, and the speed of particle A as it reached the ground as 0.6 ms^{-1}. This enabled the acceleration of the particles to be found by using the constant acceleration equations with $u = 0$, $s = 0.8$ and $v = 0.6$ and this gave $a = 0.225$. When applying Newton's second law to particle *B* in the form $T - 3 = 0.3\,a$ where $a = 0.225$, the tension T in the string could be found. Most candidates found the acceleration but some involved the unknown mass m at this stage rather than applying the equations to the 0.3 kg particle.

(ii) With the value of T determined, the equation of motion for the particle of mass m kg in the form $mg - T = ma$ could be used with the values of a and T found in part **(i).** This enabled the value of m to be determined. Since the question stated that particle *A* reached the ground, it was clear that $m > 0.3$ and so the equations of motion took the form $T - 3 = 0.3a$ and $mg - T = ma$. Some candidates took the signs to be opposite to those stated here and found the tension and mass to be negative.

Answers: **(i)** 3.07 N (to 3 sf) **(ii)** $m = 0.314$ (to 3 sf)

Question 5

(i) Most candidates made a good attempt at this question. It is important when solving this type of problem for candidates to state the directions in which they are resolving. The majority of candidates resolved forces horizontally and vertically and found the components of the system in each of these directions. The majority of candidates also made good attempts to find the resultant force, generally using Pythagoras. Very few candidates correctly identified the direction of the force. Although an angle was found in most cases, this angle had to be correctly described and the most straightforward method was by drawing a diagram.

(ii) Candidates performed well on this part, realising that the component of the 25 N force to the left had to balance the component of the F N force to the right and that this gave an equation which enabled F to be determined. The final part of this question involved combining the vertical components of the three forces and is a very good example of making sure that full accuracy is maintained throughout the calculations. Although the individual components of the forces were large, the final answer when combining the three forces was very small and so any premature approximation made during the calculation led to an inaccurate final result. This affected a significant number of candidates. All calculations should be kept to at least 4 sf when performing these calculations.

Answers: **(i)** 12.8 N; Direction: angle of 37.7° (to 3 sf) below the negative x-axis
(ii) $F = 28.3$ (to 3 sf) The magnitude of the new force is 0.667 N (to 3 sf) in a direction vertically upwards

Question 6

(i) Most candidates found this question difficult to answer. When the result is given in the question, as it is here, candidates must be particularly careful when setting out their responses. There were several different approaches which could be taken to this question. One method was to use the constant acceleration equation $s = ut + \frac{1}{2}at^2$ applied to the section PQ and then to section PR. This produced two simultaneous equations involving the variables initial speed, u, and the acceleration, a. Solving these two equations gave the required value of u and confirmed the given value of the deceleration. Many candidates applied the equation to the sections PQ and QR but wrongly assumed that the initial speed was the same in both cases, leading to incorrect solutions. Some candidates took the given value of the deceleration and used it to find u but this did not provide a proof for the value of the deceleration.

(ii) In this part of the question a force diagram was particularly helpful. The motion was given to be on an inclined plane. Even candidates who could not prove the result from part one could now assume the given value of the deceleration and use it when applying Newton's second law. The normal reaction on the particle could be shown to be $R = mg\cos 3$ and Newton's second law takes the form $-mg\sin 3 - \mu R = m \times -\frac{2}{3}$. This equation could be solved for the required coefficient of friction, μ. Most candidates attempted this but often with incorrect signs in their equation.

Answers: **(i)** $a = -\frac{2}{3}$ m s^{-2} (Answer given) $u = \frac{23}{15}$
(ii) $\mu = 0.0144$ (to 3 sf)

Question 7

(i) This question gave the acceleration as a function of time, t. In cases such as this it is not possible to use the constant acceleration equations and calculus techniques must be used. In order to find an expression for velocity, the given expression for a had to be integrated and the given condition that the particle starts from rest used. This expression for the velocity had to be set to zero and the resulting equation solved in order to answer the question. Note that the answer had to be given as an exact fraction. Some candidates did not do this and hence lost marks.

(ii) In this part the expression found in part **(i)** for velocity had to be evaluated at $t = 10$. The velocity-time graph could be drawn using the information that $v = 0$ at $t = 0$, the value of v found at $t = 10$ and the fact that from part **(i)** it is known that the velocity is again zero at $t = \frac{20}{3}$. Also the expression for v is a quadratic function of t which means that the graph is the shape of a parabola. Using all of this information the v-t graph could be drawn.

(iii) Integration of the expression for velocity was required here to find the distance travelled. However, from the v-t graph it could be seen that the part of the region from $t = 0$ to $t = \frac{20}{3}$ consisted of a curve above the t-axis while that from $t = \frac{20}{3}$ to $t = 10$ lay below the axis. In order to find the total distance travelled over the first ten seconds these two areas had to be evaluated separately, remembering that when evaluating areas below the t-axis the answer will be negative

Answers: **(i)** $t = \frac{20}{3} = 6\frac{2}{3}$

(ii) The v-t graph is an inverted parabola, passing through the points (0,0), ($\frac{20}{3}$, 0) and (10,–27)

(iii) 80 m

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Question 1**, **Question 2**, **Question 6** and **Question 7**. Candidates would be advised to carry out all working to at least 4sf if a final answer is required to 3sf.
- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 6 parts (ii) and (iii)**.
- In questions such as **Question 5** in this paper, where acceleration is given as a function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

In **Question 4** the tangent of an angle which is used in the question was given. In questions such as this it is not necessary to calculate the angle itself as the sine and cosine of the angle which are required in order to achieve the question could be evaluated exactly. However, many candidates often proceeded to find the relevant angle to 1 decimal place and immediately lost accuracy and in some cases lost marks.

The examination allowed candidates at all levels to show what they knew, whilst differentiating well between even the stronger candidates. **Question 2** was found to be the easiest question whilst **Question 7(ii)** proved to be the most challenging for almost all candidates.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases such as **question 4(i)** it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

In this question it is vital that a force diagram is drawn so that the directions of the tension forces acting on the ring can be seen. Some candidates wrongly assumed that the tension forces in the two parts of the string were different but as it is a continuous string the forces must be the same. There are two possible approaches to this question. One method is to resolve forces horizontally and vertically. Alternatively the resolution of forces could be made along the direction of the 2.5 N force and along the direction perpendicular to this force. The horizontal resolution does not involve the mass, m, of the ring and immediately gives the value of the tension, T, in the string. Vertical resolution immediately gives the value of the mass, m, of the ring. If forces are resolved along and perpendicular to the 2.5 N force, this produces two simple simultaneous equations for T and m which can readily be solved. It is possible to solve the problem using Lami's theorem but two of the forces must be combined as it is currently a 4–force system. Some good solutions were seen to this question and the major problem was where candidates assumed that two different tensions were acting in the string

Answers: The tension in the string is 1.25 N $m = \sqrt{\frac{2}{8}} = 0.177$ (to 3sf)

Question 2

Almost all candidates made a good attempt at this question. Firstly the normal reaction must be found in the form $R = 5g \cos 6$. Since the block moves with constant speed there is no net force acting on the block. The equilibrium equation can be written as $T = 5g \sin 6 + \mu R$ where T is the required tension in the rope. On substituting the given value for μ and the expression for R this gives the required value of the tension in the rope. Errors that were seen included misreading 6° as 60° and having the wrong sign on the μR term.

Answer: The tension in the rope is given by $T = 20.1$ N (to 3 sf)

Question 3

(i) This was a very straightforward use of the given velocity-time graph and candidates had to use the fact that the gradient of the graph represents the acceleration. Some candidates appeared to mix up the signs of acceleration and deceleration since the gradient of the graph over this period is negative. This means that if acceleration is stated then it is negative whereas if deceleration is quoted then it is positive.

Answer: The acceleration of the particle over the first 2 seconds is $-1\,\text{ms}^{-2}$

(ii) Again most candidates made a good attempt at this question. It is necessary to use the graph either between $t = 4$ and $t = 10$ or between $t = 6$ and $t = 10$, since the acceleration is constant over those periods. The required result can be achieved either by use of the constant acceleration formulae or simply by use of similar triangles in the velocity-time graph such as $\frac{V}{4} = \frac{(V+2)}{6}$.

Answer: $V = 4$

(iii) Some candidates had difficulty with this part of the question, particularly in understanding how to use the one third factor that was referred to in the question. It was necessary to evaluate the area beneath the graph between the times $t = 0$ and $t = 6$, since this represents the distance that the particle has travelled, which is one third of the distance AB. Multiplying this area by 3 gives the required distance AB. At $t = 6$ the particle reaches A and its direction of travel reverses. It then moves from A to B, with the area under the triangle between $t = 6$ and $t = T$ representing the distance AB. The area of this triangle is then equated to three times the distance travelled in the first 6 seconds, leading to an equation for T. One error seen was to mix up the factors of 3 and $\frac{1}{3}$ when setting up the equation for T. Some candidates lost marks because although they found the correct value of T, they did not fully answer the question by specifically stating the distance AB.

Answers: Distance $AB = 24$ m $T = 18$

Question 4

(i) This is a case of a given answer and so care must be taken to show the full working of this problem. It is also a case where the tangent of the angle α is given and so throughout the question there is no requirement for the specific angle to be found. The sine and cosine of this angle can be evaluated exactly as $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. In addition, if candidates did not use $g = 10$ in this problem then they would not reach the given answer. As it is given that P remains stationary, the system is in equilibrium but with particle P on the point of moving down the plane. As particle Q is stationary, the tension in the string $T = 0.7g = 7$. The normal reaction at P is given by $R = 0.4g \cos\alpha = 3.2$ and the friction force $F = \mu R = 1.6$ The weight component acting down the plane is given by $0.4g \sin\alpha = 2.4$ and hence the equilibrium condition can simply be stated as $X + 2.4 = 7 + 1.6$ and hence the given result follows. Many candidates performed well on this part, with the main loss of marks being due to rounding errors when using the angle as 36.9 degrees. Some candidates misread the direction of the force X, some taking it vertically downwards and others assuming that it was acting in a direction perpendicular to the plane. Some candidates introduced acceleration into their calculations even though the question stated that the particles were stationary.

Answer: $X = 6.2$ (answer given)

(ii) In this part of the question, since X has a given value which is smaller than in 4**(i)** with $X = 0.8$, the particle P must begin to move up the plane and particle Q must move vertically downwards. It is necessary to write down the equations of motion for both P and Q or to consider the system of the two particles. If T is the tension in the string and a is the magnitude of the acceleration of the particles, then the equation for the motion of P takes the form $T - \mu R - 0.4g \sin \alpha - 0.8 = 0.4a$, while the equation for particle Q is $0.7g - T = 0.7a$. By substituting the given values of μ and α, the tension can be eliminated and the value of the required acceleration can be found. Some common errors seen were to forget to include some of the terms in the equation of motion for P or , where the system equation was used, often the forces acting on the system were equated to $0.4a$ whereas they should have been equated to $(0.4 + 0.7)a$

Answer: The acceleration of P is $2\,\text{ms}^{-2}$

Question 5

(i) In this question the acceleration, a, is given as a function of t and so calculus must be used. The question asks for the time at which maximum velocity is reached and this happens when $a = 0$. Many candidates found difficulty manipulating the fractional power which occurs in the expression for a when attempting to solve this equation. Some candidates unnecessarily found an expression for the velocity before considering the equation $a = 0$. The majority of problems encountered by candidates in this question related to algebraic errors in the manipulation of the equation.

Answer: $T = 4$

(ii) In order to find when the acceleration is a maximum, one approach is to differentiate the given expression for a and to set this to zero and solve to find the required value of t. This was the method used by most candidates, although some realised that the given expression for a is a quadratic in $\sqrt{t}$ and so the method for finding the vertex of a quadratic was also seen. Once the time at which maximum acceleration occurs has been found, the expression for a must be integrated in order to produce an expression for the velocity. Care is needed here since the velocity at $t = 0$ is given as $v = 1\,\text{ms}^{-1}$ and this must be used to find the constant of integration. The value of t at which the maximum acceleration takes place can then be substituted into the expression for v in order to complete the answer. Most candidates found the correct expression for v by using integration. Some forgot to use the given condition to determine the constant of integration. Many did not differentiate a in order to find the time at which maximum acceleration occurred but instead used the value of t found in 5**(i)**.

Answer: The velocity of the particle when the acceleration is maximum is $1.5\,\text{ms}^{-1}$

Question 6

(i) In this problem the car is moving at a constant speed and so there is no net force acting on it. This means that the driving force exactly balances the resistance force. In order to find the required power of the car's engine, use must be made of the formula $P = Fv$ where the driving force F is the resistance force 350 N and v is the given speed. Almost all candidates successfully solved this problem.

Answer: The power of the car's engine is 5250 W

(ii) In this part of the question the car is accelerating under the effect of three forces, namely, the driving force, the component of the weight of the car and the resistance force. The equation of motion for this situation can be written as $P/15 + 1200g \sin 1 - 350 = 1200 \times 0.12$ and by rearranging this equation, the required power P could be found. Some errors seen included either ignoring the weight component or taking the wrong sign for the weight component term, and forgetting to include the resistance force. However, most candidates performed well on this part.

Answer: The new power of the car's engine is 4270 W (to 3 sf)

(iii) There are two distinct methods which can be applied to solve this problem, either being acceptable. As the only forces acting are constant, namely the weight component and the resistance, Newton's second law can be applied in the form $1200g \sin 1 - 350 = 1200a$ to determine the deceleration of the car as $a = -0.117$. Once this has been found, the constant acceleration equations can be applied with the values $u = 20$, $v = 18$ and $a = -0.117$. Use can be made of the equation $v^2 = u^2 + 2as$ and this can be solved for the distance, s, travelled. Alternatively the problem can be solved using work-energy methods. The PE loss can be shown to be $1200g \times s \sin 1$, the KE loss is $\frac{1}{2} \times 1200 \times (20^2 - 18^2)$ and the work done against the resistance force is $350s$. In this situation the work energy principle can be written as PE loss + KE loss = WD against friction. Both of these methods were seen. Errors that occurred when using Newton's laws included missing terms, a driving force still included in the equations and the factor of g missing in the weight term. When using the energy method some candidates used the correct terms but with wrong signs and others mixed up units by including the resistance force 350 N in the energy equation rather than the work done of $350s$ J.

Answer: The distance travelled = 324 m (to 3 sf)

Question 7

(i) This question specifically requests an energy approach and so any candidate who did not use this method was penalised. There are several approaches to the problem. One method is to find the speed of the particle as it reaches the surface of the liquid. This can be done by equating the loss in PE to the gain in KE. Alternatively the initial height of the particle above the liquid surface can be found. Once either of these values has been found, energy principles can be used either for the motion from the liquid's surface to the bottom of the tank or for the whole motion from the instant when the particle is released until it reaches the bottom of the tank. In both methods PE is lost, KE is gained and work is done against the resistance force while the particle is in the tank. The work-energy principle can be used in either method to find the speed at which the particle reaches the bottom of the tank. Some candidates found the acceleration in the tank by assuming a constant resistance force. This assumption was incorrect and also did not use an energy method and so scored few marks. Some candidates who used energy methods missed out some of the terms within the energy equation or combined the terms incorrectly.

Answer: The speed of the particle as it reaches the bottom of the tank is $9\,\text{ms}^{-1}$

(ii) This question proved to be by far the hardest on the paper for the majority of candidates. This was caused in part by a misunderstanding of what was happening in the problem. Once the particle rebounded from the bottom of the tank, almost all candidates assumed that it would come to rest within the liquid. However, once the acceleration in the liquid was found, it could be shown that the particle was still moving after it had risen 1.25 m and reached the surface of the liquid. The majority of candidates only scored the marks available for finding the acceleration in the liquid by using $-3g - 1.8 = 0.3a$ and finding $a = -16$. From this point, the most common incorrect approach was to use the equation $v = u + at$ with $u = 7$, $v = 0$ and $a = -16$, which gave an answer of $t = \frac{7}{16}$. A correct approach, once the acceleration in the liquid is found as $a = -16$, is to find the speed at the liquid surface using the equation $v^2 = u^2 + 2as$ with $u = 7$, $a = -16$ and $s = 1.25$. This gives the speed of the particle at the liquid surface as $3\,\text{ms}^{-1}$. Using $v = u + at$ in the liquid with $u = 7$, $v = 3$ and $a = -16$ leads to a time of 0.25 seconds which the particle takes to reach the surface of the liquid. The particle leaves the liquid with a speed of $3\,\text{ms}^{-1}$ and slows down to rest under gravity. Using the equation $v = u + at$ with $u = 3$, $v = 0$ and $a = -g = -10$ leads to $0 = 3 - 10t$ and solving this shows that the additional time taken for the particle to come to rest once it has left the liquid is 0.3 seconds. Adding the time spent in the liquid and the time taken to come to rest outside the liquid gives the final answer. There are several alternative methods which could be used including energy based methods.

Answer: The value of t is $t = 0.25 + 0.3 = 0.55$ s

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

Candidates are reminded to maintain sufficient accuracy in their working to achieve 3 significant figure accuracy in their final answers e.g. **Question 1** and **Question 6** parts **(ii)** and **(iii)**.

When forming an equation of motion or an energy equation, candidates are reminded to check that all relevant forces or all energies have been considered e.g. **Question 3** and **Question 6** parts **(ii)** and **(iii)**.

General comments

As usual much of the work seen was of a very high standard. The paper provided the opportunity for candidates of all levels to demonstrate what they knew. The easiest questions were found to be **Question 4** part **(i), Question 5** part **(i)** and **Question 6** part **(i)**. The least well answered were **Question 3** part **(ii)**, **Question 6** part **(iii)** and **Question 7** part **(iii)**.

Comments on specific questions

Question 1

This was a straightforward question for candidates who realised that the tension in the **one** string was the same throughout. However, many resolved with different tensions for parts *AR* and *BR* of the string and thus formed two equations in three unknowns. Some solutions were complicated by using angles, other than the given 70° and 45°, which were not always calculated correctly. A few attempted to resolve along and perpendicular to either *AR* or *BR* which had more scope for error than the usual choice to resolve horizontally and vertically. The value of *P* was sometimes given as 0.44, correct to two, rather than the required three, significant figures.

Answer: Tension in string = 1.21 N   P = 0.443

Question 2

Whilst there were many correct solutions seen, the main difficulty for candidates was in determining the normal reaction. Some used $R = mg - 50\sin 20^{\circ}$ rather than $R = mg + 50\sin 20^{\circ}$ leading to a mass of 17.4 kg. Others used $R = mg$, omitting to consider the vertical component of the 50 N force and leading to a mass of 15.7 kg. A few obtained the equation $50\cos 20^{\circ} = 0.3 \times 50\sin 20^{\circ} + mg$ either following $R = 50\sin 20^{\circ}$ or possibly from omitting brackets.

Answer: 14.0 kg

Question 3

The majority of candidates appropriately attempted an energy solution for both parts of this question and thus avoided a solution which depended on a constant resistance.

(i) In equating the change in kinetic energy to the work done against the resistance, it was common to see $\frac{1}{2} \times 1.2(v^2 - 7.5^2) = 25$ rather than $\frac{1}{2} \times 1.2(7.5^2 - v^2) = 25$. The resulting 9.90 ms^{-1} suggested an **increase** in velocity as a result of the resisting force. This example provides a reminder that some errors may be avoided by checking that an answer is suitable for the given situation. A few candidates ignored the initial kinetic energy and mistakenly solved $\frac{1}{2} \times 1.2v^2 = 25$.

Answer: 3.82 ms^{-1}

(ii) In the expected four term work/energy equation the usual errors were: to omit a term such as the original kinetic energy; to duplicate a term e.g. potential energy included also as work done by the component of weight down the plane; or to make a sign error, usually resulting in a negative distance. Some calculated '*h*' the vertical distance without involving $\sin 30^\circ$ to obtain the required distance *AB*.

Answer: 6.64 m

Question 4

(i) Most candidates sketched a correct graph and confirmed the distance 96 m either by calculating the area of the trapezium or by totalling the distances for the three stages of motion. It was expected that the *v–t* sketch would show three straight lines joining the four points (0,0), (5,6), (17,6) and (20,0) with the key values 5, 17 and 6 seen on the relevant axes. A few graphs showed the acceleration and/or the deceleration with a curve rather than a straight line. Occasionally a value was missing or incorrect such as 1.2 ms^{-1} shown as the maximum velocity.

Answer: 96 m (AG)

(ii) This part was more challenging. A variety of methods were attempted, the most successful being to use the area property to calculate the maximum velocity before applying $v = u + at$. The cyclist's journey was sometimes misinterpreted to include a section of constant speed either in place of the acceleration or the deceleration, and sometimes showing a journey time of 30 s rather than an end time of 30 s. Candidates who attempted a '*suvat*' solution often produced erroneous equations such as $96 = \frac{1}{2}a(20)^2$ or $96 = \frac{1}{2}a(10)^2$.

Answer: 0.96 ms^{-2}

Question 5

(i) This was a routine pulley problem which was solved confidently by most candidates. Any errors were usually due to an incorrect sign in setting up or in solving simultaneous equations, or due to an incorrect system equation such as $0.5g - 0.3g = 0.5a$. A very few candidates oversimplified and used $a = g$ as the acceleration.

Answer: a = 2.5 ms^{-2} h = 0.45

(ii) Candidates often applied an appropriate '*suvat*' formula to find the required velocity of particle *P* using the acceleration found in part **(i)**. The majority of candidates understood that the acceleration of *P* changed to *g* ms^{-2} when *Q* reached the ground and calculated the time taken to reach a maximum height, usually either by using $v = u - gt$ or by using a longer two stage method of finding the maximum height and then applying $s = ut - \frac{1}{2}gt^2$. Candidates often misunderstood or misread the total time required, with common final answers of 0.3 s (the total time with the string slack) or 0.75 s (the total time for *P* in upward motion).

Answer: Velocity of P when Q reaches floor = 1.5 ms^{-1} Total time = 0.9 s

Question 6

Whilst part **(i)** was found to be one of the easiest questions, part **(iii)** differentiated between candidates and included the highest proportion of non-attempts.

(i) The application of $P = Fv$ and Newton's Second Law to this situation was usually carried out successfully, although the resistance was sometimes given as $1800 + 3200 \times 0.2$ (= 2240 N) instead of $1800 - 3200 \times 0.2$ (= 1160 N).

Answer: 1160 N

(ii) Whilst most candidates attempted to resolve forces along the incline, some omitted to include either the component of weight down the hill or the resistance. Others appeared to be unaware of the constant speed and equated to 'ma' as in part **(i)**. 60 000 W was sometimes given as the answer, correct to two, instead of three, significant figures.

Answer: 59900 W

(iii) As in part **(ii)**, some candidates oversimplified the situation by forming an equation of motion with two instead of three terms. Some overlooked the component of weight down the hill and solved $\pm 1160 = 3200a$. The application of $v^2 = u^2 + 2as$ sometimes suggested an unawareness that the van was **decelerating up the hill to rest** with an unexplained negative value found for s.

The alternative work/energy approach was regularly seen. The common errors with this method were: to omit one term; to include an extra term (e.g. potential energy duplicated as work done by the component of weight down the hill); to use 1160 instead of 1160s or to make a sign error. 720 m was sometimes seen as the answer, either following premature approximation in the working or from a final approximation correct to two significant figures.

Answer: 721 m

Question 7

(i) Part **(i)** was frequently well answered with accurate integration of $a(t)$ and substitution to find the maximum velocity. Many found the velocity when $t = 5$ as expected, but a common error was to evaluate $v(9)$, assuming that the maximum velocity occurred for the greatest value of t.

Answer: $83.3\,\text{ms}^{-1}$

(ii) Whilst a majority of candidates recognised that a further integration was needed, a significant number of candidates applied $s = \frac{1}{2}(u + v)t$ often using $v = 83.3$ and $t = 5$ from part **(i)** to obtain a total distance of 208 m. Others used integration but with an upper limit of 9 instead of 5. Errors in integration usually involved the cubic term e.g. $\int \frac{t^3}{3}\,dt = \frac{t^4}{4}$ instead of $\frac{t^4}{12}$.

Answer: 260 m

(iii) A complete solution for this part depended on a consideration of the limits or the constant of integration. Many candidates ignored the constant of integration (C) or assumed C=0 without considering both functions when $t = 9$ to obtain C. Some found $v(9) = -18$ and stated erroneously $C = -18$, leading to a velocity of $-48\,\text{ms}^{-1}$. The integration of $-3t^{-\frac{1}{2}}$ caused a few problems, mainly with the coefficient but also with the index e.g. $-\frac{3t^{\frac{1}{2}}}{2}$ or $\frac{3t^{-\frac{3}{2}}}{1.5}$.

Answer: $-30\,\text{ms}^{-1}$

MATHEMATICS

Paper 9709/51
Paper 51

General comments

The paper proved to be harder than the one set last November.

The presentation of a few candidates was rather untidy and difficult to read.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures. Very few candidates gave answers to 2 significant figures.

Most candidates now use $g = 10$ as requested on the question paper.

Occasionally candidates used an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, they should refer to the booklet.

The questions candidates found easier proved to be **1**, **3(i)**, **4(i)**, **5(i)**

The questions candidates found harder proved to be **2(i)**, **2(ii)**, **6(ii)**, **7(i)** and 7**(ii)**

Comments on specific questions

Question 1

This question was generally well done by candidates. The vertical velocity, v, of the ball at B was 20 $\text{m}s^{-1}$. Because it was the second time, $v = -20$ had to be used.

Answer: 4.6(0)

Question 2

(i) Many candidates found this part of the question quite difficult. If θ is the angle between the horizontal and the axis of the cone, then $\cos\theta = \dfrac{0.2}{0.3}$ and so $\tan\theta = \dfrac{x}{0.3}$, where x is the required distance.

(ii) This part of the question proved to be too difficult for many candidates. It was necessary to realise that the centre of mass would be at the centre of the base of the hemisphere. To solve this part of the question, it was necessary to take moments about the point A. The resulting equation would be $\left(\dfrac{\pi 0.3^2 h}{3}\right) \times \left(\dfrac{h}{4}\right) = \left(\dfrac{2\pi \times 0.2^3}{3}\right)\left(\dfrac{3 \times 0.2}{8}\right)$ where h was the height of the cone.

Answers: **(i)** 0.335 **(ii)** 0.231

Question 3

(i) This part of the question was generally well done. Candidates needed to find the extension at the equilibrium position, which is where the greatest speed would occur. From here a 3 term energy equation could be set up.

(ii) Many candidates scored well on this part of the question. A 2 term energy equation was required, and from this a three term quadratic equation could be found.

Answers: **(i)** 3.32 ms^{-1} **(ii)** 0.932 m

Question 4

(i) This part of the question was usually well done.

(ii) Many candidates made a good attempt to solve this part of the question. The first step was to equate the acceleration to zero. The next step was to integrate. This led to an equation in *v* and *x*. When the correct values were substituted, the required value of *v* could be calculated.

Answers: **(i)** $\frac{\text{vdv}}{dx} = 32 - 40x - 48x^2$ **(ii)** 1.5

Question 5

(i) Many candidates made a good attempt at this part of the question. The value of T could be calculated by using Newton's Second Law horizontally for P. The value of R was then found by resolving vertically for P.

(ii) This part of the question was generally well done. Candidates needed to resolve vertically for P and then to use Newton's Second Law horizontally.

Answers: **(i)** T = 0.703 R = 0.578 **(ii)** 3.54 $rads^{-1}$

Question 6

(i) This part of the question was well done by many candidates. The required distances could be found by taking moments about BC and AB.

(ii) Very few candidates scored well on this part of the question. Because the prism is going to topple about D, candidates were required to take moments about D. If W is the weight then 2cos45 × (0.7– 0.32) = 2cos45 × (0.3 – 0.276) + W(0.3 – 0.276) is the required moment equation. This proved far too difficult for many candidates to set up.

Answers: **(i)** 0.32 m **(ii)** 21(.0) N

Question 7

(i) Many candidates were able to express x and y in terms of t. Unfortunately very few candidates realised that *x* = *y*. With this information, an equation in t could be found and then solved to give the required answer.

(ii) This part of the question proved to be too difficult for many of the candidates. The total height above the ground at time *t* is $24\sin 60t - \frac{1}{2}gt^2$ and the height to the plane is 24cos60*t*, so the required height, *h,* was $24\sin 60t - \frac{1}{2}gt^2 - 24\cos 60t$. By differentiating to find $\frac{dh}{dt}$ and equating to zero, the required time could be found and hence the greatest height.

Answers: **(i)** $x = 24\cos 60t$, $y = 24\sin 60t - \frac{1}{2}gt^2$ t = 1.76 s **(ii)** 3.86 m

MATHEMATICS

Paper 9709/52
Paper 52

General comments

The paper proved to be much harder than the one set last November.

The presentation of a few candidates was rather untidy and difficult to read.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures. Very few candidates gave answers to 2 significant figures.

Most candidates now use $g = 10$ as requested on the question paper.

Occasionally candidates used an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, candidates should refer to the booklet.

The questions candidates found easier proved to be 1, 3**(i)**, and 4**(ii)**

The questions candidates found harder proved to be 2**(ii)**, 3**(iii)**, 6**(i)**, 6**(ii)** and 7**(i)**

Comments on specific questions

Question 1

This question was generally well done by candidates. A number of candidates reached the final answer using circuitous methods. The quickest and most efficient method was to say initial horizontal velocity = the final horizontal velocity i.e. $v\cos 20 = 38 \cos 30$, where v is the required speed, and solve for v.

Answer: 35(.0) $\text{m}s^{-1}$

Question 2

(i) This part of the question proved to be rather difficult due to the complex expression resulting from the moment equation. Candidates were required to take moments about the vertex of the cone.

(ii) Only a few candidates successfully solved this part of the question. Candidates were required to take moments about a point on the circumference of the base of the cone. The resulting equation is $kW\cos 30 \times 0.3 + kW \sin 30 \times 0.8 = 0.3W$

Answers: **(i)** 0.525 m **(ii)** 0.455

Question 3

(i) This part of the question was generally well done.

(ii) A number of candidates were unable to integrate the $-5e^{-x}$ term. The result should have been $5e^{-x}$. After the integration it was necessary to use the correct limits of 0.5 and 0.

(iii) Only a handful of candidates were able to solve this part of the question. The answer was found by using $8x - 4 = T = \frac{\lambda(x - 0.5)}{0.5}$ with $x = 0$ used.

Answers: **(i)** $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 20x - 10 - 5e^{-x}$ **(ii)** 5.2(0) **(iii)** 4 N

Question 4

(i) Some candidates did not follow the instruction to express *x* and y in terms of *t*. These candidates simply used the trajectory equation quoted in the formula booklet.

(ii) This part of the question was generally well done. Candidates needed to substitute the two given points into the equation found in part **(i)** and then solve the two equations to find a.

(iii) This part of the question was generally well done.

Answers: **(i)** $y = -\dfrac{5x^2}{V}$ **(ii)** $a = 4$ **(iii)** Height = 100 m

Question 5

(i) Many candidates realised that it was necessary to set up a 4 term energy equation. Sign errors occurred on a number of occasions.

(ii) Candidates needed to realise that the greatest speed occurred at the equilibrium position. The first step was to use $T = \dfrac{\lambda x}{L}$ with T = 0.7 g. Having found *x*, a 5 term energy then had to be set up.

Answers: **(i)** 0.424 m **(ii)** 2.06 ms^{-1}

Question 6

(i) Very few candidates were able to solve this part of the question. Quite often the first mistake was in finding the centre of mass of the quadrilateral OAB, simply because the wrong formula was used. Candidates needed to take moments about O. The resulting equation was rather complicated and proved to extremely difficult to manipulate.

(ii) Only a few candidates were able to give a satisfactory answer to this part of the question.

Answers: **(i)** *x* = 1.01 r **(ii)** Within quadrant as the square will be smaller than the rectangle

Question 7

(i) Most candidates realised they had to set up two equations in the two tensions. The two equations were set up by resolving vertically and by using Newton's Second Law horizontally. Errors occurred when trying to solve the two equations.

(ii) This part of the question proved to be very difficult for many candidates. Candidates were required to resolve vertically and to use Newton's Second Law horizontally before using F = μR.

Answers: **(i)** 1.66 N **(ii)** 0.556

MATHEMATICS

Paper 9709/53
Paper 53

General comments

The paper proved to be harder than the one set last November.

The presentation of a few candidates was rather untidy and difficult to read.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures. Very few candidates gave answers to 2 significant figures.

Most candidates now use $g = 10$ as requested on the question paper.

Occasionally candidates used an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, they should refer to the booklet.

The questions candidates found easier proved to be **1**, **3(i)**, **4(i)**, **5(i)**

The questions candidates found harder proved to be **2(i)**, **2(ii)**, **6(ii)**, **7(i)** and 7**(ii)**

Comments on specific questions

Question 1

This question was generally well done by candidates. The vertical velocity, v, of the ball at B was 20 ms^{-1}. Because it was the second time, $v = -20$ had to be used.

Answer: 4.6(0)

Question 2

(i) Many candidates found this part of the question quite difficult. If θ is the angle between the horizontal and the axis of the cone, then $\cos\theta = \frac{0.2}{0.3}$ and so $\tan\theta = \frac{x}{0.3}$, where x is the required distance.

(ii) This part of the question proved to be too difficult for many candidates. It was necessary to realise that the centre of mass would be at the centre of the base of the hemisphere. To solve this part of the question, it was necessary to take moments about the point A. The resulting equation would be $\left(\frac{\pi 0.3^2 h}{3}\right) \times \left(\frac{h}{4}\right) = \left(\frac{2\pi \times 0.2^3}{3}\right)\left(\frac{3 \times 0.2}{8}\right)$ where h was the height of the cone.

Answers: **(i)** 0.335 **(ii)** 0.231

Question 3

(i) This part of the question was generally well done. Candidates needed to find the extension at the equilibrium position, which is where the greatest speed would occur. From here a 3 term energy equation could be set up.

(ii) Many candidates scored well on this part of the question. A 2 term energy equation was required, and from this a three term quadratic equation could be found.

Answers: **(i)** 3.32 ms^{-1} **(ii)** 0.932 m

Question 4

(i) This part of the question was usually well done.

(ii) Many candidates made a good attempt to solve this part of the question. The first step was to equate the acceleration to zero. The next step was to integrate. This led to an equation in *v* and *x*. When the correct values were substituted, the required value of *v* could be calculated.

Answers: **(i)** $\frac{\text{vdv}}{dx} = 32 - 40x - 48x^2$ **(ii)** 1.5

Question 5

(i) Many candidates made a good attempt at this part of the question. The value of T could be calculated by using Newton's Second Law horizontally for P. The value of R was then found by resolving vertically for P.

(ii) This part of the question was generally well done. Candidates needed to resolve vertically for P and then to use Newton's Second Law horizontally.

Answers: **(i)** T = 0.703 R = 0.578 **(ii)** 3.54 rads^{-1}

Question 6

(i) This part of the question was well done by many candidates. The required distances could be found by taking moments about BC and AB.

(ii) Very few candidates scored well on this part of the question. Because the prism is going to topple about D, candidates were required to take moments about D. If W is the weight then 2cos45 × (0.7– 0.32) = 2cos45 × (0.3 – 0.276) + W(0.3 – 0.276) is the required moment equation. This proved far too difficult for many candidates to set up.

Answers: **(i)** 0.32 m **(ii)** 21(.0) N

Question 7

(i) Many candidates were able to express x and y in terms of t. Unfortunately very few candidates realised that $x = y$. With this information, an equation in t could be found and then solved to give the required answer.

(ii) This part of the question proved to be too difficult for many of the candidates. The total height above the ground at time *t* is $24\sin60t - \frac{1}{2}gt^2$ and the height to the plane is 24cos60*t*, so the required height, *h,* was $24\sin60t - \frac{1}{2}gt^2 - 24\cos60t$. By differentiating to find $\frac{\mathrm{d}h}{\mathrm{d}t}$ and equating to zero, the required time could be found and hence the greatest height.

Answers: **(i)** $x = 24\cos60t$, $y = 24\sin60t - \frac{1}{2}gt^2$ t = 1.76 s **(ii)** 3.86 m

MATHEMATICS

Paper 9709/61
Paper 61

Key messages

Candidates should be aware of the need to work to at least 4 significant figures to achieve the required degree of accuracy. Efficient use of a calculator is expected, but candidates should be encouraged to show sufficient workings in all questions to communicate their reasoning.

General comments

Candidates would be well advised to read the question again after completing their solution to ensure they have included all the relevant details.

Candidates who presented their work in a clear and ordered fashion often avoided careless mistakes.

A significant numbers of candidates did not perform well on this paper and had correspondingly low marks.

Comments on specific questions

Question 1

Many candidates did not recognise that this was a 'selection' question which required the use of combinations to be answered successfully. Good solutions presented a logical working progression, identifying the three groups separately with linked expressions before finding the product of their three expressions. Most candidates used the question order of the groups, but this was not necessary to be successful.

Answer: 1260

Question 2

This question was the best answered question on the paper.

(i) Many candidates produced an expression from the table and equated the sum of the probabilities to 1. They then successfully solved the equation. A common error was to omit the 0.1 in the expression for the sum of the probabilities.

(ii) Although E(X) was stated, a reasonable number of candidates recalculated from the initial data, not always accurately. Good solutions provided an unsimplified expression for the variance and then used the calculator efficiently to evaluate this expression. Weaker solutions failed to square the mean in the variance formula.

Answer: **(i)** 0.15 **(ii)** 1.93

Question 3

The best solutions included a representation of the context. For example, a list of all of the possible scenarios or a diagram to illustrate the question restrictions.

(i) Many partial solutions were seen, where not all possible scenarios were identified. Many solutions only used the boundary values stated within the question to obtain one value. Candidates needed to have a good understanding of the meaning of ‘at least’ and ‘no more than’ to be successful. Many solutions omitted the group containing four violinists and two cellists. As the value obtained was exact, candidates should not round their answer to 3 significant figures.

(ii) Here, a simple diagram, showing the four violinists sitting together plus a separate cellist and double bass player, helped clarify the restrictions. Strong responses demonstrated understanding that this was an arrangement problem with two conditions: the arrangement of violins within their group and the arrangement of the three different instruments. However, weaker responses were seen which started to list the possible arrangements, but did not consider the different ways the violins could sit together.

Answers: **(i)** 12 210 **(ii)** 144

Question 4

(a) This question required the use of the standardisation formula. Strong responses demonstrated both correct interpretation of the notation and correct substitution to find the required standard deviation. Many candidates stated the complement of the required answer. Drawing a simple diagram would have helped to avoid this error. Candidates should be aware of how to use the normal tables provided accurately.

(b) Candidates with a good understanding of the normal distribution performed well on this question. On this unstructured question, good solutions demonstrated logical progress through the information to produce two simultaneous equations and then calculate the mean and standard deviation. The accurate use of the normal tables was essential to reach an acceptable final answer. Weak solutions often solved the simultaneous equations formed by equating the standardisation formula with the calculated probabilities. Setting out work in a structured way is advisable in order to avoid the transfer errors which were seen in some candidates’ algebraic manipulation.

Answers: **(i)** 0.146 **(ii)** $\mu = 15.8$, $\sigma = 3.03$

Question 5

Most candidates recognised that this was a context which would be modelled by the binomial distribution.

(i) Solutions generally contained at least one correct binomial term, although not always with the probabilities stated in the question. Again, candidates needed to have a clear understanding of ‘at most’ as a common error was to interpret this as ‘less than’ and omit the probability of 6 students passing. Good solutions stated an appropriate expression to calculate and then evaluated efficiently with the calculator, which reduced the possibility of working to insufficient accuracy.

(ii) Many candidates did not progress from part **(i)**. Good solutions recognised that, as the data were discrete, a continuity correction was required. These solutions went on to evaluate the mean and variance accurately and often included a simple sketch of the normal curve to help identify the required probability area. Weaker solutions omitted the continuity correction or found the complement probability.

(iii) Few successful solutions were seen. Where candidates are asked to justify the use of the normal distribution as an approximation for the binomial distributions, *np* and *nq* **both** need to be evaluated and stated as fulfilling the required condition.

Answers: **(ii)** 0.262 **(ii)** 0.125 **(iii)** $np = 160$ $nq = 40$, both > 5 so normal approximation holds

Question 6

(i) Although most candidates attempted the cumulative frequency graph, few complete solutions were seen. Candidates needed to use a scale which allowed them to plot points accurately, to label axes fully including units, and to draw a smooth curve. Most candidates plotted at the upper class boundary, although some introduced an unnecessary continuity correction.

(ii) Very few solutions interpreted the question correctly. Almost all candidates stated their reading for a cumulative frequency of 100, rather than (250–100) as required.

(iii) Again few fully correct answers were seen. The unstructured nature of the question required candidates to work logically, applying their knowledge of mean and variance from grouped data. Good solutions often copied and extended the original data table to include 'frequency (f)', 'mid-point (m)', '$m \times f$' and '$m^2 \times f$'. They then used these values to calculate both the mean and the standard deviation. In many solutions inaccurate midpoints were used throughout, which limited progress. A number of candidates calculated the mean but did not attempt the standard deviation. A few candidates did not find the square root of their variance to obtain the standard deviation. Some candidates did not use the square of their mean when calculating the variance.

Answers: **(ii)** 42 **(iii)** $\mu = 39.9$ $\sigma = 23.2$

Question 7

(i) Almost all candidates successfully used the data table to state the required probability. Some candidates unsuccessfully attempted to simplify or convert their answer to a decimal. Candidates should be aware that using fractions in probability generally leads to more accurate answers.

(ii) Although many candidates attempted this question, the use of $P(B) \times P(M) = P(B \cap M)$ as the required condition was limited. A number of solutions produced circular numerical arguments rather than comparing data from the original table. The best solutions identified the probabilities required and made an appropriate numerical comparison before reaching a conclusion. The weakest solutions presented a logic argument without evidence.

(iii) The most successful solutions used the original data table to identify the groups that fulfilled the conditional probability conditions. Where candidates attempted to use the alternative approach of substituting probabilities into the standard conditional probability formula, some produced quite complex and incorrect calculations.

(iv) Few fully successful solutions to this part were seen. Good solutions identified the scenarios that fulfilled the question conditions, recognised that once a candidate had been selected they could not be chosen again, and accounted for different orders, since order was not important. The most common error seen was finding the products of probabilities with the same denominator, indicating that replacement had occurred.

Answers: **(i)** $\frac{13}{40}$ **(ii)** Not independent **(iii)** $\frac{27}{64}$ **(iv)** 0.201

MATHEMATICS

Paper 9709/62
Paper 62

Key messages

In order to gain full credit, candidates should show sufficient method to justify their conclusions.

Candidates are reminded that only non-exact answers should be rounded to 3 significant figures.

Candidates need to consider whether their scale will enable their graphical solutions to be plotted to an appropriate degree of accuracy.

General comments

The majority of candidates presented their solutions in a logical manner. However, there was sometimes a lack of structure which made determining the final answer difficult for examiners. In **Question 7**, this was possibly the cause for the number of solutions that did not provide the required information.

Many good solutions were seen for **Questions 3** and **5**. The context in **Question 6** was found challenging by many. Sufficient time seems to have been available for candidates to complete all the work they were able to. However, a significant number of candidates made little or no attempt at the final parts of **Question 7**.

Candidates should read the question again once they have finished their solution to ensure that they have provided the required information.

Comments on specific questions

Question 1

This was a standard permutations and combinations question, which was attempted by almost all candidates

(i) Good solutions simply stated the standard result to calculate the number of arrangements. Weaker candidates did not divide by all the terms required to remove the effect of repeated letters. A few candidates provided solutions which assumed that the repeated letters could be identified, so gained little credit. There appeared to be some inefficient use of calculators, as a number of solutions displayed the correct numerical formula, evaluated inaccurately.

(ii) A number of different approaches were seen to this question. The most efficient considered the three probabilities for selecting the double letters and summed the results. A common error was to treat the situation as if replacement occurred and not reduce the denominator. The alternative efficient approach was to consider the number of selections for each of the double letter scenarios and then convert the total into the required probability by dividing by the total number of possible selections. Some candidates assumed that the letters were individually identifiable here. In this approach, weaker solutions simply stated either the numerator or denominator value.

Answers: **(i)** 34 650 **(ii)** $\frac{13}{55}$

Question 2

The majority of candidates correctly interpreted the key given for the back-to-back stem-and-leaf data and then used it appropriately throughout. Candidates did not always use a scale that would enable the box-and-whisker plot to be drawn accurately using their values.

(i) The median was identified correctly by almost all candidates. Good solutions stated the candidate's values for the upper and lower quartiles before their calculation of the inter-quartile range. There was less consistency in identifying the upper quartile, with the weakest solutions stating the position of the piece of data, rather than its value. When finding the interquartile range, some candidates incorrectly found the mean of their upper and lower quartile values or restated the median value as the interquartile range.

(ii) Few good comparison box-and-whisker plots were seen overall. Many candidates plotted only one, and failed to identify which data they were using. Good solutions had a linear scale that used the graph grid well, with 2 cm = 0.01 seconds allowing for accurate representation. Good solutions also labelled the axis both with the item being recorded and measurement units and identified the two plots. The use of a ruler is expected at this level to draw straight lines.

A number of candidates used the notation for the mean when plotting their median value.

Some candidates used separate axes for each plot, which only gained full credit if they aligned exactly, as otherwise they did not allow visual comparison of the data as required.

Answers: **(i)** median = 0.225 IQR = 0.021

Question 3

(i) This was a standard binomial distribution probability question, and almost all candidates used the appropriate method. Candidates should be aware that 'at least 3 times' means that '3 times' is included in the solution set, as omission of this value was the most common error. Good solutions stated the unsimplified expression and then used a calculator efficiently to evaluate the probability. Some candidates who calculated each value did not work to sufficient accuracy, which resulted in premature approximation affecting their final answer.

(ii) The use of a tree diagram assisted many candidates to identify the outcomes required to fulfil the stated condition and link the different probabilities successfully. The most common approach was to calculate the probabilities of being successful on both days, on Monday only and on Tuesday only and sum the results. A few candidates identified that this was equivalent to 1 – P(unsuccessful on both days). Common errors were to link the probabilities from the question inaccurately, or to omit P(successful on Monday, unsuccessful on Tuesday).

Answers: **(i)** 0.896 **(ii)** 0.92

Question 4

Candidates were generally more successful with **(i)** and it was encouraging to more explanation of the candidates' logic than in previous papers.

(i) Many good solutions were seen, with almost all realising that each person could be identified. 86 400 was a common incorrect answer, where candidates did not allow the boy and girl blocks to change places.

(ii) The common feature in many good solutions was a simple diagram which interpreted the requirement for no boy to stand next to another boy. A number of solutions appeared not to allow a boy to stand at the end of the row, and so eliminated two possible positions. A few candidates only stated a non-exact answer in this question.

Answers: **(i)** 172 800 **(ii)** 1 814 400

Question 5

(i) The best solutions for this question were well structured, laying out the steps involved clearly. They identified that Σx for the Junior members was required and presented a single expression to sum the two age totals and divide by the total number of members. Weaker solutions were often poorly structures and included information that was not required for the answer. Candidates used the incorrect method of calculating the mean of the Senior members and then finding the average of the two means. A number of candidates correctly found the total of all the ages, but only divided this by the number of Junior members. At this level candidates should not accept final answers which are unrealistic, such as ages over 100. Candidates should be aware that exact answers should not be rounded in this context.

(ii) Many candidates answered this question with confidence, applying the variance formula appropriately for the Juniors to calculate Σx^2 and then using all the appropriate data to calculate the variance for the whole group. A small number of candidates did not continue to finally state the standard deviation. Common errors were either to use a 3 significant figure mean from **(i)**, which led to inaccurate answers, or to use an incorrect value from **(i)** accurately. Weaker candidates calculated the standard deviation for each group of members and averaged their values.

Answers: **(i)** 34.25 **(ii)** 16.0

Question 6

There was evidence that some candidates did not read the question fully and assumed that the random variable X was the difference between the spinners rather than the score on the red spinner minus the score on the blue spinner.

(i) Almost all solutions contained a clear probability distribution table. Good solutions often included a sample space table as well, which enabled candidates to complete the PDF without further calculation. Many candidates showed clear calculation to obtain each of their required probabilities. In a few solutions, 4 was incorrectly included as a value for the random variable.

A significant number of solutions only considered positive values, using the difference between the spinners. A few candidates correctly calculated the probabilities of the positive random variable values and omitted the negative values as if scores could not be negative.

(ii) Good solutions clearly presented unsimplified expressions using the information from their probability distribution table, and calculated first the mean and then the variance. Inaccurate transfer of probabilities was seen, often where presentation was not clear. A common error was not squaring the mean within the variance formula.

(iii) Many candidates found this question challenging, and it was omitted by a significant number. Candidates who had used a sample space table in **(i)** were often able to simply write down the required conditional probability by identifying the outcomes fulfilling the conditions. The most common method was to apply the standard conditional probability formula and calculate the required values for the numerator and denominator. A very common incorrect solution was simply stating their P(1) value.

Answers: **(i)**

–2	–1	0	1	2	3
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

(ii) $\frac{23}{12}$ **(iii)** $\frac{1}{3}$

Question 7

This was a fairly standard normal distribution question. However, many candidates used the normal tables inaccurately. Many solutions were poorly structured, and some candidates did not show understanding of the purpose of standardisation.

(a) (i) The majority of candidates applied the normal standardisation formula correctly, without continuity correction, and found the required probability. A few candidates did not take account of the statement in the question that the variable had a normal distribution and used a binomial distribution. Good solutions realised that the probability was not a final answer and continued to work at 4 significant figure accuracy to calculate the expected number of days. The context required an integer answer, and candidates needed to interpret their calculation rather than simply round.

(ii) Although a fairly standard question, good solutions presented the solution in a logical progression, linking the standardisation to the *z*-value and manipulating the algebra accurately. Weaker solutions often used an inaccurate *z*-value. A number of solutions equated with the stated probability, or treated the probability as a *z*-value and found a new probability to use. The use of a probability in this question could gain little credit.

(iii) This question acted as a discriminator and enabled candidates with a good understanding of the principles of the normal distribution to work efficiently.

Good solutions often had a simple sketch of the normal curve with a visual interpretation of the required conditions. The symmetry properties were identified, and there was an appreciation that 'within 1.5 standard deviations of the mean' provided the required *z*-value without further calculation. This led to an expression for the required probability area which was then evaluated. Weaker solutions calculated the mean $\pm$ 1.5 x standard deviation as stated in the question, then used their expression to calculate the *z*-values which, with the extra stages, often became inaccurate. These candidates were usually able to calculate the correct probability area from their values. Candidates should be aware that probabilities cannot be greater than 1, so their final value should meet this requirement.

(b) Many candidates found this question challenging, and no attempt was made by a significant proportion. Good solutions recognised that this was a generalisation of the standardisation formula, with 0 as the value to be standardised. Rearrangement of the question condition was performed before substitution into the formula, which was followed by appropriate cancellation to produce the *z*-value. A simple diagram often helped candidates appreciate the probability area that was required.

Some candidates did not achieve the required accuracy, either rounding their decimal equivalent to $-\frac{4}{3}$ to 3 significant figures or stating their probability as a 2 significant figures value. A number of solutions introduced *x* as a variable to replace 0, which limited progress.

Answers: **(a)(i)** 286 or 287 **(ii)** 4.05 **(iii)** 0.866 **(iv)** 0.909

MATHEMATICS

Paper 9709/63
Paper 63

Key messages

A few questions in this paper could be solved in several ways and a number of candidates chose to do more than one method to arrive at the final answer. Candidates need to be aware that we will only mark one solution when more are presented – normally the 'more complete' solution. However, if more than one 'complete solution' is offered, we will mark the last solution.

Those who choose to use a calculator rather than tables when finding answers using the Binomial or Normal distributions need to remember to write down the unsimplified binomial or standardisation expression before writing the final answer. This advice is even more important when working on a question like **Question 5i** where the answer is given.

General comments

Candidates need to remember that all non-exact numerical answers must be given correct to 3 significant figures. This requires them to work with numbers to at least 4 significant figures before the final answer is reached. Premature approximation was apparent in several questions, particularly the probabilities in **Question 2i**, where some candidates chose to work in decimals rather than fractions and **Question 5i**, where the answer was given to 3 significant figures and we required them to work with probabilities to 4 decimal places. Candidates also need to be aware that a decimal rounded to 3 decimal places which results in a zero in the third decimal place cannot be treated as an exact decimal and truncated as was seen frequently in **Question 5ii**.

Comments on specific questions

Question 1

This question was answered accurately by many candidates, with about the same number choosing to subtract the number of ways with two women next to each other from the total number of ways (7! – 2 × 6!) as finding the number of ways to position two women apart once five men have been arranged (5! X 6P2). Of those who chose the first method, not all candidates remembered to multiply 6! by 2 i.e. they forgot that the two women together could be ordered in two ways. Among those who chose the second method, 5! frequently appeared on its own or multiplied by other factorials. A few chose a longer method, considering the five different scenarios where the women are separated by one, two, three, four or five men and summing the number of ways.

Answer: 3600

Question 2

(i) This question was well answered by most candidates. Just a few candidates appeared not to understand what a probability distribution table was and produced a possibility table instead. Of those who slipped up with the probability distribution, the most common error was to omit one of the X values, usually 3.

Answer:

x	–2	–1	0	1	2	3
P(X = x)	$\frac{2}{18}$	$\frac{4}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{2}{18}$	$\frac{1}{18}$

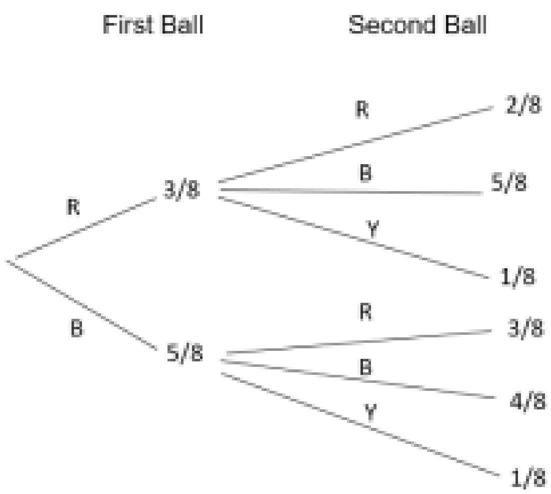
(ii) The strongest candidates appreciated that they should show their method for obtaining both the mean and the variance, and presented clear unsimplified expressions for both before writing down their final answer. We insisted on the probabilities from part **i** summing to 1 to justify the first method mark and most candidates made sure that this was the case. The most common error was to subtract the mean, rather than the mean squared, in the variance formula.

Answer: $\frac{65}{36}$ (1.81)

Question 3

(i) This question was well answered by most candidates despite the introduction of a third colour possibility for the second ball. Most knew to label the branches with both the colour and the assigned probability. The few who made mistakes either introduced 9 as the denominator for the First Ball or, more commonly, introduced 7 as the denominator for the Second Ball. Some of those using 7 still considered the outcomes with 8 balls, making their numerators sum to 8 and the probabilities to more than 1.

Answer:



(ii) Most candidates realised that they needed to consider the two possibilities of both balls being red or both being blue and knew to multiply the probabilities in each of these cases. Only a few misunderstood the question and gave two separate answers instead of summing the products. Even those with incorrect tree diagrams usually earned the method mark as long as the probabilities in each branch summed to 1.

Answer: $\frac{13}{32}$ (0.406)

(iii) The majority of candidates were able to use the Conditional Probability formula correctly and find the correct answer. Of those who found the question more challenging, some calculated the probability of red then blue and presented it as their final answer, others found both required probabilities but did not put them in a fraction and still others calculated two probabilities to put in a fraction but one or both were incorrect.

Answer: $\frac{3}{7}$ (0.429)

Question 4

(i) This question was successfully answered in a number of different ways. Most successful candidates divided the number of ways of choosing 6 people from the 11 after one particular boy is removed (11C6 = 462) by the total number of ways of choosing 7 people out of 12 (12C7 = 792). Some took a lot longer to arrive at the numbers 462 and 792 by summing five products of combinations for one or both. Very few used the probability method to find the probability that the particular boy was chosen by multiplying six fractions $\left(\frac{11}{12}\times\frac{10}{11}\times\ldots\ldots\times\frac{5}{6}\right)$ and subtracting the answer $\left(\frac{5}{12}\right)$ from 1.

The most commonly seen incorrect approaches were either to assume constant probability and use a Binomial distribution or to forget what the question asked and give the number of ways that the particular boy could be chosen in a group of seven rather than a probability.

The few candidates who offered $\frac{7}{12}$ as the answer with no working or justification for their answer were not given the marks.

Answer: $\frac{7}{12}$ (0.583)

(ii) This question proved to be more challenging than part **i** and was approached in a variety of ways. As in part **i**, a number of candidates incorrectly assumed constant probability and tried to use the Binomial distribution. Of those who made a good attempt at the question, most chose to find the number of ways 2, 3 or 4 girls could be chosen (with 5, 4 or 3 boys) using products of combinations and then summing them. A simpler way was to subtract the number of combinations with 0 or 1 girl (and 7 or 6 boys) from 12C7. A surprising number of candidates then presented the total (672) as their answer and forgot to find the probability of this happening by dividing by the total number of ways.

Those who attempted the Probability methods were generally less successful. The most common slip when attempting '1 – Probability (0 or I girl)' was to forget to multiply by 7 when finding the probability of 1 girl. Only a handful of candidates who tried calculating the Probability (2, 3 or 4 girls) had any success, as candidates generally omitted the combination term (7C2, 7C3 and 7C4 respectively) in the three probability calculations.

Answer: $\frac{28}{33}$ (0.848)

Question 5

(i) Most candidates appreciated that in a 'Show that' question where the answer is given, all stages of the working must be shown. The majority knew to standardise and they generally obtained the z-values $\frac{5}{6}$ and $-\frac{5}{4}$. At that point some did not know how to proceed and abandoned their working. Others drew useful diagrams to demonstrate the area that they needed and explained their working using phi notation, before they found the probabilities from the tables or a calculator. Only a few 'fudged' their numbers to arrive at the given answer using incorrect numbers. Some candidates who used calculators rather than tables lost marks by jumping to the final answer too quickly. However, a pleasing number of candidates presented detailed and careful solutions, appreciating that they needed to write down probabilities to at least 4dp if their final answer was to be correct to 3dp.

Answer: *0.692* AG

(ii) Most candidates recognised this as a Binomial Distribution question and produced at least one binomial term with $n = 4$ and $p = 0.692$. A few used an incorrect probability from part **i**, despite having been given the correct answer. The words 'at least two' caused confusion, with some candidates interpreting it as 'more than 2', but most appreciated that they either had to sum the probabilities of 2, 3 or 4 medium apples or subtract the probabilities of 0 or 1 medium apple from 1. Some forgot the combination element in a binomial term and a few did not use two probabilities that summed to exactly 1 in each term. A surprising number did not realise that a zero as the final digit in a non-exact decimal is still significant. 0.91 was frequently presented as the answer without showing a longer answer before the rounding.

Answer: 0.910

Question 6

(i) This question was well answered. Most candidates knew to find the z-value corresponding to 0.96 and equate it to a standardised expression. A few used the tables the wrong way round or equated the result of their standardisation to 0.96 or 0.04, and some tried to deal with the negative z-value by subtracting 1. Those who drew a diagram generally dealt well with the negative value of the standardised expression. However, a significant number of candidates either gave their final answer as a negative value or crossed out the negative sign when they realised that a standard deviation has to be positive.

Answer: 114

(ii) Most candidates recognised this as a Normal approximation from the Binomial distribution (300, 0.2) and correctly found the mean and variance to be 60 and 48. A pleasing number knew to apply a continuity correction when standardising, although 70.5 was frequently seen instead of 69.5. A more common mistake was to divide by 48 instead of the square root of 48, either because they thought the variance was 48 squared or because they did not know to divide by the standard deviation.

Answer: 0.915

(iii) Surprisingly few candidates understood what was being asked here. Some had quoted the conditions in part **ii** and still did not recognise that it was those conditions being asked for in this question. Of those who did understand the question, only a few realised that quoting the theory was not enough and that they needed to evaluate np and nq as 60 and 240, and state that they are both greater than 5. Many thought that it was the mean and variance that had to be greater than 5.

Answer: $np = 60 > 5$; $nq = 240 > 5$

Question 7

(i) Candidate who scored well on this question knew to read the question carefully and to make sure that they followed the instructions. They needed to put the Anvils on the left and the Brecons on the right in a back-to-back stem and leaf with both the diagram and the key having clear labels and the 'leaves' being lined up carefully to demonstrate the shape of the data. For three digit data like this, the first two figures appear in the 'stem' and the 'leaves' are single digits. Many candidates omitted the '1' in the 'stem' but usually 'recovered' in the key. Marks were deducted for the presence of commas, lack of labels, data the wrong way round, misalignment of the 'leaves', splitting the 'leaves' into two rows, units and labels omitted from the key, or the key being presented separately for the Anvils and Brecons.

Answer:

Anvils		Brecons
8	15	
9 5	16	6
5 3 2 0	17	0 1 2 2 8
4 1 0	18	1 2 3 3
6	19	2
		Key: 5|16|6 means 165 cm for Anvils and 166 cm for Brecons

(ii) This question was well answered by most candidates. Most knew that the median from 11 items of data is the 6th item and identified this to be 173. Just a few divided 11 by 2 and took the average of the 5th and 6th items. Those who did not identify the median correctly usually made a similar mistake with the quartiles, but gained a mark if they calculated the difference between them. A few candidates found the quartiles but did not do the subtraction.

Answer: median = 173; IQ range = 12

(iii) This question was well answered by most candidates. They knew to add the three extra heights to 337221 to find Σx for 14 members, although a few tried to sum the 14 numbers from scratch. Calculating the new Σx^2 presented a greater challenge. Some examples of incorrect thinking were 337221^2 and $(1923^2 + 166^2 + 172^2 + 182^2)$.

Most candidates used the correct variance formula and knew to write their unsimplified expression for the variance in full before writing the final answer. Only a few forgot to subtract the mean squared, or gave the variance as their final answer, rather than the standard deviation.

Answer: 9.19

MATHEMATICS

Paper 9709/71
Paper 71

Key messages

As stated on the front of the paper, non-exact numerical answers are required to 3 significant figures. Sometimes this requires working to be carried out to more than 3 significant figures, as in **Question 6(iii)** and in **Question 7**.

The conclusion to a significance test should be expressed in the context of the question, as in **Question 2(ii)** and in **Question 7(i)**.

General comments

Many candidates presented their solutions in a clear and logical manner.

When standardising within a normal distribution the appropriate form for the numerator was " $x - \mu$".

In **Question 4(ii)** this was 59.5 – 61. In **Question 5(i)** this was 6 – 5 and in **Question 5(ii)** this was 0 – 0.16.

Comments on specific questions

Question 1

(i) In order to score full marks here it was necessary to provide the correct form for the interval and the correct value of z (2.24). Many candidates found both of these and calculated correctly. Other candidates gave an incorrect value for z, such as 1.96.

Other candidates did not use the variance $\frac{7.2^2}{200}$ correctly.

Also the answer for the confidence interval had to be given as an interval. Acceptable forms were "175 to 177" and "(175, 177)".

(ii) The necessary condition for the calculations to be valid was that a random sample was obtained.

Many candidates suggested different conditions which were not appropriate such as "the heights must be normally distributed", "n must be large", "the data must be continuous" and "the standard deviation must be the same".

Answers: (i) 175 to 177 (ii) random sample

Question 2

(i) Many candidates stated the correct hypotheses in terms of p. Other candidates incorrectly used $\mu = \frac{1}{3}$ instead of $p = \frac{1}{3}$, omitted p = altogether, or tried μ = 50.

(ii) Some candidates proceeded correctly by comparing the given probability (0.0084) with the significance level (1%) and then making the correct conclusion about p from this significant result. Other candidates incorrectly compared 0.0084 with $\frac{1}{3}$ or wrote general phrases without using the values.

(iii) The largest number of children was 150. Other values such as 50, 36 and 114 were suggested by some candidates.

Answers: (i) $H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$ (ii) 0.0084 < 0.01 There is evidence that p has decreased
(iii) 150

Question 3

Many candidates correctly found the value of n to be 600, or at least reached the value 600.25, though the final answer did need to be the whole number value. It was necessary to use the variance ($\frac{2.5^2}{n}$) in a correct form, to use the correct value for z (1.96), and to rearrange the equation accurately. Some candidates tried an incorrect form for the variance, such as $\frac{2.5}{n}$ or an incorrect value for z, such as 2.24. Some candidates made algebraic errors, such as reaching $\sqrt{n} = 24.5$ but then taking the square root instead of squaring.

Answer: 600

Question 4

(i) Many candidates found the combined Poisson distribution with parameter 6.1 and used the relevant terms to obtain the correct answer 0.857. Some candidates used λ = 0.61. Other candidates added an extra Poisson term (P(4)) or omitted the first term (P(0)). Most candidates used $P(\geqslant 4) = 1 - P(\leqslant 3)$. A few candidates attempted to use a Binomial distribution or a normal distribution. These were not appropriate.

(ii) As the Poisson distribution for the total number of drops in 100 cm^3 of air had a parameter of 61 which was large (> 15), the appropriate approximating distribution was the normal distribution N(61, 61). Many candidates realised this and used the necessary continuity correction (59.5) to find the probability. Some candidates found λ = 61, but then used an incorrect variance such as 23.79 (from npq). Other candidates omitted the continuity correction factor and used 60, or used an incorrect value such as 60.5. A diagram can assist in deciding on the value of this continuity correction factor and also in choosing the correct area (the tail) for the probability.

Answers: (i) 0.857 (ii) 0.424

Question 5

(i) Many candidates found the distribution for the total time ($T_1 + T_2$) correctly as N(5.0, 0.41), standardised correctly and found the probability. Some candidates inserted a continuity correction factor. This was not appropriate. Other candidates incorrectly used 0.4 + 0.5 = 0.9 as the variance. Other candidates found 0.41 but used it as the standard deviation. The larger area was required here for the probability.

(ii) Some candidates used the appropriate new variable "Y" = $T_2 - 1.2 \times T_1$ and found the probability that Y > 0. Some candidates found the correct distribution N(0.16, 0.4804) for this.

Other candidates incorrectly multiplied T_2 by 1.2 or made errors when trying to find the variance. These included multiplying by 1.2 instead of 1.2^2 or subtracting instead of adding the separate variances (common errors for the variance were 0.0196 and 0.442).

A diagram can be helpful for identifying the correct area and probability for $Y = T_2 - 1.2 \times T_1 > 0$.

This is especially so when the value 0 is not the mean.

Answers: (i) 0.941 (ii) 0.591

Question 6

(i) Many candidates correctly integrated kx^{-1} between the limits 2 and 6 and equated the result to 1. The combination of the ln 6 and ln 2 was carried out to obtain the given answer for k. Some candidates used incorrect limits such as 0 and 6. Other candidates wrongly combined the ln terms to $\ln \frac{6}{\ln 2}$.

(ii) Many candidates correctly integrated $x.kx^{-1}$ and applied the limits 2 and 6 to the result of the integration (kx). The result was $\frac{4}{\ln 3}$ which then gave 3.64 to the required accuracy. Some candidates multiplied the x and the x^{-1} incorrectly. Other candidates omitted the x or tried to integrate the x and the x^{-1} separately. This was usually unsuccessful as the approach did not become a correct form of "integration by parts".

(iii) Many candidates attempted to find the median and then the probability. The median required the integration of kx^{-1} for the limits 2 to m (or m to 6) and the use of 0.5. Some candidates followed this route correctly and found the median to be 3.46(4) (the exact value was $\sqrt{12}$). Other candidates used incorrect limits such as 0 to m or used limits 2 to 3.64 and incorrectly equated the result to m. The required probability was found by using the limits $\sqrt{12}$ to E(X). To obtain an accurate answer the earlier working needed to be carried out to more than 3 sig. figs.

Some candidates used the method listed on the left hand side of the mark scheme. First they found P(2 < X < E(X)). Then they noted that P(2 < X < m) = 0.5 so subtracted this 0.5 from 0.545(08) to obtain the probability.

A few candidates found the median, but then found the variance and attempted to use a normal distribution. The second part of this was not valid.

Answers: (i) $k = \frac{1}{\ln 3}$ (ii) 3.64 (iii) 0.045 (2 sig. figs.)

Question 7

(i) Many candidates followed a logical route through this significance test. The first stage required the hypotheses and the unbiased estimate of the variance of the masses. Some candidates stated the hypotheses incorrectly in terms of x or "the mean". Some candidates found only the sample biased variance. The second stage required the standardisation of the sample mean (49.8667) using the normal distribution N(51, $\frac{46.962}{150}$). Some candidates omitted the 150.

The third stage required the comparison and conclusion. The comparison could involve the values of z ($-2.025 < -1.96$) or the probabilities ($0.0215 < 0.025$) or the critical value. It was necessary to show the comparison very clearly. Some candidates used inconsistent signs, while others compared incorrect probability tails. The conclusion should have been given in the context of the question. Some candidates wrote contradictory statements.

(ii) It was necessary to find the critical value for the rejection (or acceptance) region for the normal distribution N(51, $\frac{6.856^2}{150}$) as defined by H_0. As this critical value was very close to the sample mean value, it was necessary to provide this value to several sig. figs. (49.903). To find the Type II error it was necessary to work with this critical value in the new distribution with population mean mass 49 kg. Some candidates successfully followed this procedure. Some candidates reversed the means and some candidates omitted the 150. Other candidates used +1.96 instead of –1.96. Some of these errors created difficulties in selecting the relevant probability tail. Some candidates omitted the first stage and attempted only a standardisation for the second stage.

A range of answers was allowed to score full marks here.

Answers: (i) there was evidence that the population mean mass of sacks was less than 51 kg
(ii) 0.0534

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

There were places on this paper where answers indicated that candidates needed to read the question more carefully (see below).

Candidates need to know how to round answers to 3.s.f. Candidates need to be aware of the difference between rounding to a specified number of significant figures, and a specified number of decimal places.

In a 'show that' question, all steps in the calculation need to be shown (see comments below). If essential stages of the working out are missing, full marks will not be obtained.

All of the steps required for a Hypothesis test should be fully presented (see comments below on **Question 5(ii)** and **6(i)**. It is important that the conclusion to the test is fully justified.

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. Script from across the complete attainment range were seen. In general, candidates scored well on **Questions 1, 2(i), 4(i) and 7(i),** whilst **Questions 5(ii), 6 and 7(ii)** proved particularly challenging.

Most candidates kept to the required level of accuracy, though if a question requires the final answer to be to a specified level of accuracy (e.g. to 1 decimal place as in **Question 2(i)** this instruction must be adhered to.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

In general, this was a well answered question. Most candidates correctly chose to calculate P(2,3,4) with $\lambda = 2.3$. There were only a few cases where P(5) was incorrectly included or P(2) was omitted from the calculation. Weaker candidates separated the inequalities and attempted to calculate separate probabilities and then subsequently combine. This approach was seldom successful.

Answer: 0.585

Question 2

Many candidates correctly found the required confidence interval in part **(i)**. Some candidates did not read the question correctly and did not give the end-points of their interval correct to 1 decimal place, as requested. Hence they could not gain the final answer mark. Other errors included use of an incorrect z value and confusion between standard deviation and variance. It is important that a confidence interval is written as an interval and not as two separate values.

Part **(ii)** proved challenging as candidates were not precise enough in their responses. It had to be clear that the candidate knew that it was the population that was not stated to be normal (therefore the CLT was required). Many candidates did not specify this clearly, indicating a possible confusion between the random sample and the population. Some candidates did not give a clear yes or no answer, and some did not state a reason.

Answers: (i) 329.4 to 330.8 (ii) Yes, because volume of all cans (i.e. the parent population) was not stated to be normal

Question 3

Candidates were mostly aware of how to approach this question, though errors were made in calculating the variance. Calculation of the mean and the standardisation calculation were usually very well attempted despite the often incorrect value used for the variance. A small number of candidates found the probability of less than 1310 rather than greater than.

Answer: 0.108

Question 4

In part **(i)** candidates were required to show that the value of k was $\frac{(a+1)}{a}$. In questions of this kind it is important that candidates show all relevant working. There were occasions here where full marks were not awarded due to lack of essential working. There were also occasions when candidates incorrectly reached the required answer, offering a solution containing an error or errors (sign errors were particularly noted here). For full marks to be awarded, no errors should be seen. Other candidates did not evaluate the lower limit of the integration, or incorrectly assumed it was zero.

An answer in the context of the question was required in part **(ii)**. Questions of this type are not always well answered. Many candidates merely said that ‘a’ was the maximum value of the pdf. This statement on its own was not sufficient to gain the marks as no context was given. The maximum time for the runners to finish would have given the required context.

Part **(iii)** was not always well answered. Candidates did not always use the information given in the correct way. An attempt to integrate f(x) between 0 and 0.5, and then equate to 0.75, was required. Some candidates indicated a lack of understanding by interchanging the 0.5 and 0.75, while some candidates equated their integral to 1, and many candidates used incorrect limits.

Answers: (ii) Maximum time allowed by the model for the runners to finish (iii) 0.8

Question 5

Part **(i)** produced a mixed response. There was confusion in finding the values required from the given table (Σfx, Σfx^2 and Σf were required, but these were not always correctly found). There was also confusion between the two alternative formulae for the unbiased estimate for the population variance. Candidates would be advised to consistently use just one of these formulae to avoid this confusion.

Part **(ii)** proved to be challenging. Candidates should be aware of all the steps required to carry out the test. The null and alternative Hypotheses should be stated; the calculation of test statistic clearly shown, and a comparison made between the test statistic and the critical value in order to justify the conclusion. Candidates often gave no Hypotheses or incorrect ones. Calculation of the test statistic was not always done well. Even the candidates who realised they needed to use the values found in **(i)** often did not use $\sqrt{70}$. A significant number of candidates did not show the comparison clearly; often diagrams were seen that referred to ‘RR’ or ‘AR’ without defining what was meant by this. A clear inequality statement or a diagram with both figures for comparison clearly marked was required. (The comparison here could either be between z values or between areas). The conclusion to the test should be non-definite and in context. Some candidates drew a correct conclusion but then made a contradictory statement.

In part **(iii)** many candidates realised that they needed to check whether H_o had or had not been rejected. A few candidates did not make the reason for their choice clear.

Answers: (i) 0.858 (ii) No evidence that mean number of courts in S is less than in N
(iii) Type II because H_o was not rejected

Question 6

As in **Question (5)**, all steps for the Hypothesis test need to be clearly followed. Some candidates did not state their Hypotheses, or gave incorrect ones. Use of N(9,7.65), or equivalent, was not always seen and candidates again did not show their comparison clearly, meaning that their conclusion was not justified. As an alternative method, some candidates correctly used Bin(60,0.15) and were able to successfully carry out the test, though errors here included calculating P(<6) rather than P(⩽6). Conclusions to the test should again be non-definite and in context and should not contain contradictory statements.

In part **(ii)** the main error noted was using 0.15 rather than $\frac{6}{60}(=0.1)$. The calculation required to find α from the z value, using the symmetry of the confidence interval, was not always successfully done. The question specified that the answer should be given to the nearest integer. This was not always done, illustrating the need for candidates to read the question carefully.

Answers: (i) No evidence that the train is late less often (ii) 80

Question 7

Most candidates used the correct value of λ (5.6) and successfully found the required probability in part **(i)**.

Part **(ii)**, however, was not well attempted, with many candidates not appreciating that the question required a conditional probability. Thus, there were few candidates who attempted the required calculation of $\frac{P(X = 2 \text{ and } Y = 1)}{P(X + Y = 3)}$.

Part **(iii)** was reasonably well attempted with the use of N(2.1, $\frac{2.1}{100}$), or equivalent, often seen. Common errors included the omission of 100, and confusion between different methods. A continuity correction, if included, was often incorrect.

Answers: (i) 0.108 (ii) 0.264 (iii) 0.245

MATHEMATICS

Paper 9709/73
Paper 73

Key messages

As stated on the front of the paper, non-exact numerical answers are required to 3 significant figures. Sometimes this requires working to be carried out to more than 3 significant figures, as in **Question 6(iii)** and in **Question 7**.

The conclusion to a significance test should be expressed in the context of the question, as in **Question 2(ii)** and in **Question 7(i)**.

General comments

Many candidates presented their solutions in a clear and logical manner.

When standardising within a normal distribution the appropriate form for the numerator was " $x - \mu$".

In **Question 4(ii)** this was 59.5 – 61. In **Question 5(i)** this was 6 – 5 and in **Question 5(ii)** this was 0 – 0.16.

Comments on specific questions

Question 1

(i) In order to score full marks here it was necessary to provide the correct form for the interval and the correct value of z (2.24). Many candidates found both of these and calculated correctly. Other candidates gave an incorrect value for z, such as 1.96.

Other candidates did not use the variance $\frac{7.2^2}{200}$ correctly.

Also the answer for the confidence interval had to be given as an interval. Acceptable forms were "175 to 177" and "(175, 177)".

(ii) The necessary condition for the calculations to be valid was that a random sample was obtained.

Many candidates suggested different conditions which were not appropriate such as "the heights must be normally distributed", "n must be large", "the data must be continuous" and "the standard deviation must be the same".

Answers: (i) 175 to 177 (ii) random sample

Question 2

(i) Many candidates stated the correct hypotheses in terms of p. Other candidates incorrectly used $\mu = \frac{1}{3}$ instead of $p = \frac{1}{3}$, omitted p = altogether, or tried μ = 50.

(ii) Some candidates proceeded correctly by comparing the given probability (0.0084) with the significance level (1%) and then making the correct conclusion about p from this significant result. Other candidates incorrectly compared 0.0084 with $\frac{1}{3}$ or wrote general phrases without using the values.

(iii) The largest number of children was 150. Other values such as 50, 36 and 114 were suggested by some candidates.

Answers: (i) $H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$ (ii) 0.0084 < 0.01 There is evidence that p has decreased
(iii) 150

Question 3

Many candidates correctly found the value of n to be 600, or at least reached the value 600.25, though the final answer did need to be the whole number value. It was necessary to use the variance ($\frac{2.5^2}{n}$) in a correct form, to use the correct value for z (1.96), and to rearrange the equation accurately. Some candidates tried an incorrect form for the variance, such as $\frac{2.5}{n}$ or an incorrect value for z, such as 2.24. Some candidates made algebraic errors, such as reaching $\sqrt{n} = 24.5$ but then taking the square root instead of squaring.

Answer: 600

Question 4

(i) Many candidates found the combined Poisson distribution with parameter 6.1 and used the relevant terms to obtain the correct answer 0.857. Some candidates used λ = 0.61. Other candidates added an extra Poisson term (P(4)) or omitted the first term (P(0)). Most candidates used $P(\geqslant 4) = 1 - P(\leqslant 3)$. A few candidates attempted to use a Binomial distribution or a normal distribution. These were not appropriate.

(ii) As the Poisson distribution for the total number of drops in 100 cm^3 of air had a parameter of 61 which was large (> 15), the appropriate approximating distribution was the normal distribution N(61, 61). Many candidates realised this and used the necessary continuity correction (59.5) to find the probability. Some candidates found λ = 61, but then used an incorrect variance such as 23.79 (from npq). Other candidates omitted the continuity correction factor and used 60, or used an incorrect value such as 60.5. A diagram can assist in deciding on the value of this continuity correction factor and also in choosing the correct area (the tail) for the probability.

Answers: (i) 0.857 (ii) 0.424

Question 5

(i) Many candidates found the distribution for the total time ($T_1 + T_2$) correctly as N(5.0, 0.41), standardised correctly and found the probability. Some candidates inserted a continuity correction factor. This was not appropriate. Other candidates incorrectly used 0.4 + 0.5 = 0.9 as the variance. Other candidates found 0.41 but used it as the standard deviation. The larger area was required here for the probability.

(ii) Some candidates used the appropriate new variable "Y" = $T_2 - 1.2 \times T_1$ and found the probability that Y > 0. Some candidates found the correct distribution N(0.16, 0.4804) for this.

Other candidates incorrectly multiplied T_2 by 1.2 or made errors when trying to find the variance. These included multiplying by 1.2 instead of 1.2^2 or subtracting instead of adding the separate variances (common errors for the variance were 0.0196 and 0.442).

A diagram can be helpful for identifying the correct area and probability for $Y = T_2 - 1.2 \times T_1 > 0$.

This is especially so when the value 0 is not the mean.

Answers: (i) 0.941 (ii) 0.591

Question 6

(i) Many candidates correctly integrated kx^{-1} between the limits 2 and 6 and equated the result to 1. The combination of the ln 6 and ln 2 was carried out to obtain the given answer for k. Some candidates used incorrect limits such as 0 and 6. Other candidates wrongly combined the ln terms to $\ln\frac{6}{\ln 2}$.

(ii) Many candidates correctly integrated $x.kx^{-1}$ and applied the limits 2 and 6 to the result of the integration (kx). The result was $\frac{4}{\ln 3}$ which then gave 3.64 to the required accuracy. Some candidates multiplied the x and the x^{-1} incorrectly. Other candidates omitted the x or tried to integrate the x and the x^{-1} separately. This was usually unsuccessful as the approach did not become a correct form of "integration by parts".

(iii) Many candidates attempted to find the median and then the probability. The median required the integration of kx^{-1} for the limits 2 to m (or m to 6) and the use of 0.5. Some candidates followed this route correctly and found the median to be 3.46(4) (the exact value was $\sqrt{12}$). Other candidates used incorrect limits such as 0 to m or used limits 2 to 3.64 and incorrectly equated the result to m. The required probability was found by using the limits $\sqrt{12}$ to E(X). To obtain an accurate answer the earlier working needed to be carried out to more than 3 sig. figs.

Some candidates used the method listed on the left hand side of the mark scheme. First they found P(2 < X < E(X)). Then they noted that P(2 < X < m) = 0.5 so subtracted this 0.5 from 0.545(08) to obtain the probability.

A few candidates found the median, but then found the variance and attempted to use a normal distribution. The second part of this was not valid.

Answers: (i) $k = \frac{1}{\ln 3}$ (ii) 3.64 (iii) 0.045 (2 sig. figs.)

Question 7

(i) Many candidates followed a logical route through this significance test. The first stage required the hypotheses and the unbiased estimate of the variance of the masses. Some candidates stated the hypotheses incorrectly in terms of x or "the mean". Some candidates found only the sample biased variance. The second stage required the standardisation of the sample mean (49.8667) using the normal distribution N(51, $\frac{46.962}{150}$). Some candidates omitted the 150.

The third stage required the comparison and conclusion. The comparison could involve the values of z (–2.025 < – 1.96) or the probabilities (0.0215 < 0.025) or the critical value. It was necessary to show the comparison very clearly. Some candidates used inconsistent signs, while others compared incorrect probability tails. The conclusion should have been given in the context of the question. Some candidates wrote contradictory statements.

(ii) It was necessary to find the critical value for the rejection (or acceptance) region for the normal distribution N(51, $\frac{6.856^2}{150}$) as defined by H_0. As this critical value was very close to the sample mean value, it was necessary to provide this value to several sig. figs. (49.903). To find the Type II error it was necessary to work with this critical value in the new distribution with population mean mass 49 kg. Some candidates successfully followed this procedure. Some candidates reversed the means and some candidates omitted the 150. Other candidates used +1.96 instead of –1.96. Some of these errors created difficulties in selecting the relevant probability tail. Some candidates omitted the first stage and attempted only a standardisation for the second stage.

A range of answers was allowed to score full marks here.

Answers: (i) there was evidence that the population mean mass of sacks was less than 51 kg
(ii) 0.0534