Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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5896421517

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 3 - \mathrm{f}(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

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2 **(a)** Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

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(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

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3 **(a)** Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

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(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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4 The first term of an arithmetic progression is 84 and the common difference is −3.

(a) Find the smallest value of n for which the nth term is negative. [2]

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It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

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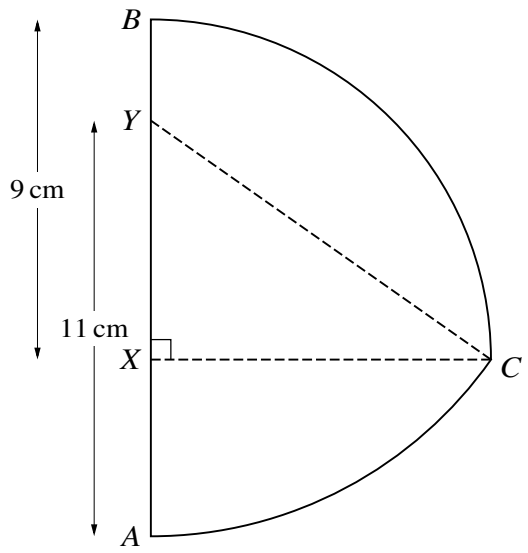

In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

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(b) Find the perimeter of ABC. [6]

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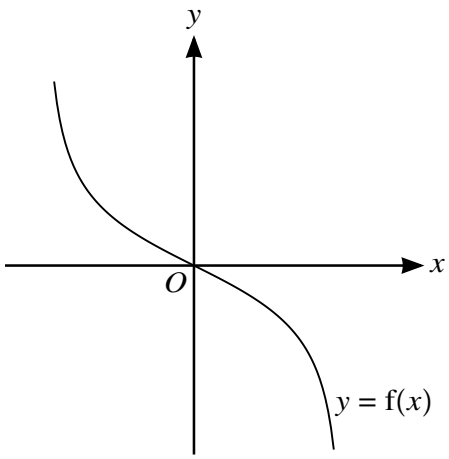

The diagram shows the graph of $y = \mathrm{f}(x)$.

(a) On this diagram sketch the graph of $y = \mathrm{f}^{-1}(x)$. [1]

It is now given that $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [4]

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The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

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(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

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7 **(a)** Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k + 1)\sin^2 x + (k - 1)\sin x - (k + 1) = 0.$$ [4]

(b) Hence solve the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

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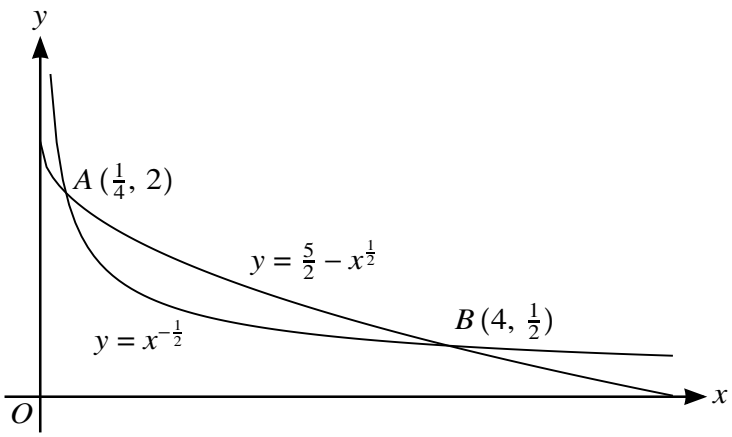

The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

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(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point (1, 1) intersects the y-axis at the point $(0, p)$.

Find the value of p. [4]

9 The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

10 A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $\mathrm{f}''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

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It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $\mathrm{f}(x)$. [4]

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(c) Find the coordinates of the other stationary point and determine its nature. [4]

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Additional Page

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