MATHEMATICS

Paper 9709/11
Paper 1 Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that ‘no marks will be given for unsupported answers from a calculator.’ This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. For quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the quadratic formula, candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

General comments

Some very good responses were seen but the paper proved challenging for a number of candidates. For this paper, the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

In both **parts (a)** and **(b)**, it was necessary to evaluate the terms; in some cases this process led to errors in coefficients or in the position of the x in the middle term in **(a)**. Some candidates omitted the first term, 1, in **part (b)**.

In **part (c)**, the best answers identified the three combinations of terms resulting in terms in x, added them together then isolated the coefficient. Weaker responses contained a full multiplication of the two expansions, but this tended to introduce errors and required the desired terms to be identified.

Question 2

It was essential to equate the curve and the line first, then use the discriminant to form a quadratic equation in k and solve the inequality. Sign errors were common, leading to the wrong quadratic equation. Some candidates chose the wrong regions for the inequality while others omitted to solve an inequality, giving only the critical values.

Question 3

This question required candidates to factorise the trigonometric expression to form two brackets, the simplest method being to factorise in its original form. However, most candidates replaced $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$ which, although possible to factorise, resulted in a lot more work and few were successful in reaching the two solutions.

Question 4

In **part (a)**, candidates needed to use the sum to infinity for a geometric series, equate it to the sum of 8 terms for an arithmetic series then solve for a. Some sign and arithmetical errors were seen and it was common for candidates to use the 8^{th} term formula instead of the formula for the sum.

In **part (b)**, a method mark could be awarded for using the correct formula for the 8th term with their value of *a*, if there had been errors in **part (a)**.

Question 5

The majority of candidates attempted this question and some fully correct answers were seen. Some candidates showed working but could not identify numerical values for *a*, *b* or *c* in **part (a)**. The horizontal stretch proved to be the most challenging of the three transformations.

In **parts (b)** and **(c)**, few candidates produced correct answers. There was evidence that some candidates had used the graph to help them, adding straight lines corresponding to $\left(\frac{6}{\pi}\right)x$ and $6-\left(\frac{6}{\pi}\right)x$. However, many candidates did not do so, perhaps because they did not realise they needed to consider the intersection of straight lines with the curve.

Question 6

In **part (a)**, candidates needed to recognise that the angle in the sector/triangle was $\frac{\pi}{3}$ then calculate the perimeter as the length of two arcs plus 6. A number of candidates were unable to find an angle from the information given in the question, hence could not make progress.

Part (b) involved using the angle found in **part (a)** and devising a strategy for calculating the area. There were several ways to do this: sector + segment, triangle + 2 segments or 2 sectors – triangle. A reasonable number of fully correct responses were seen for each of these strategies, with candidates generally making it clear what their chosen strategy was. Those who had no angle in **part (a)** often made little progress. A common error was to calculate half the area of the triangle rather than the whole area. Some candidates misquoted the area formula for the triangle.

Question 7

Many candidates calculated the radius correctly from the points given, then formed an equation for the circle. Errors seen involved omitting to square the radius or including a negative sign rather than a positive one in the circle equation. Some candidates found an equation of a straight line which was incorrect.

In **part (b)** it was necessary to eliminate one variable by substituting the equation of the line, then form and solve a 3-term quadratic equation to find the coordinates of the two points. Some errors were seen in manipulating and solving this equation. The final part of this question required candidates to calculate the length of a line segment, a standard procedure which some of them could not carry out as they had no coordinates.

Question 8

Many candidates were able to complete the square in **part (a)** and obtain correct values for *a*, though fewer had *b* correct also. Although the majority of candidates had $(x - 2)$ in their bracket, they did not all relate this to the maximum value of *k* in **part (b)**.

Correct answers to **part (c)** often used a sketch graph to establish that it was possible to calculate the range by substituting $x = -1$ into the function, or realised it was one-to-one from the coordinates of the vertex. Common errors included using the wrong inequality sign or incorrect notation to denote the range.

In **part (d)**, finding an inverse function is a standard algebraic procedure and many candidates were able to demonstrate this successfully. However, in this case, they needed to realise that the range of $f^{-1}(x)$ had a maximum of 2 since it was equal to the domain of f(*x*), hence the positive square root should be discounted. Few candidates had this expression fully correct.

In **part (e)**, many correct responses were seen that replaced *x* with $x + 3$ in either the original form of f(*x*) or its completed square form, and added 1 to *y*. A significant number of candidates either did not know how to modify the equation or made algebraic errors and could not obtain the correct answer.

Question 9

In **part (a)**, many candidates successfully found the equation of the curve, including evaluating the constant of integration. Those candidates who omitted the constant could not be awarded the final two marks. Weaker responses contained sign errors.

Part (b) required candidates to form a quartic equation in x which was a hidden quadratic equation and could therefore be solved using quadratic methods. Candidates who did not show how they solved the equation could not be awarded a method mark. Successful candidates either factorised the quartic itself or substituted another variable for x^2. Surprisingly, it was common for arithmetic errors to creep in when candidates calculated y-coordinates. In some cases this was because the candidate had omitted to calculate c in **part (a)** while others used f'(x) instead of f(x).

In **part (c)** many candidates had a correct 2nd derivative, but some had an incorrect power in the second term and others introduced errors when simplifying their expression.

'Hence' in **part (d)** was a clue for candidates to use the 2nd derivative test with the x-values they had found in **(b)**. It was necessary to evaluate the expression, give correct values and draw an appropriate conclusion about whether each of the points was a maximum or minimum. Alternatively, candidates could have used the first derivative to check a point either side of each of the stationary points; again values were necessary to justify their conclusions.

Question 10

Some fully correct responses were seen but a number of candidates had difficulty working with the indices in this question. In **part (a)**, common errors were omitting the 3 in the denominator when integrating or introducing errors when attempting to simplify their result. Some candidates could not rewrite the expression for y in a form that they could integrate. Few candidates could complete the solution: using the limits to show that the first term disappeared and so the value of the integral was in fact $\frac{2}{3}$.

In **part (b)**, most candidates omitted to square the expression for y, as required by the formula for integration to find a volume of revolution. Those who did square the expression often continued to a correct solution or one with just a coefficient error.

Some fully correct responses were seen in **part (c)**. As in **part (a)**, a number of candidates could not rewrite the expression for y in a form that they could differentiate. Others could not reduce the power to $-\frac{5}{2}$. Other candidates progressed to finding the gradient when $x = 1$ but omitted to use the gradient of the normal to find the y-intercept of this line.

MATHEMATICS

Paper 9709/12
Paper 1 Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. For quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the quadratic formula, candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

It is very important that a sufficient level of accuracy is used in all calculations. If a result is asked to be shown to 3 significant figures, then any intermediate calculations need to be given to at least 4 significant figures. For integer answers candidates should ensure that they simplify them as much as possible rather than leaving them as $\frac{18}{3}$ for example.

Questions need to be read very carefully and the phrase 'it is given instead that' means that the previous answers obtained cannot be used in the next part of the question.

It is important that the correct terminology is used when answering transformation questions. Stretch (not squeeze or compression, for example) should always be given with the correct scale factor and with the direction clearly indicated. Translation (not move) should be given with a column vector.

General comments

The paper was reasonably well received by candidates and a reasonable number of very good responses seen. Some candidates though seemed to have struggled with the time available for this particular paper, it is important that candidates plan their time to allow sufficient time on the later questions. Presentation of work was mostly good, although some of the answers were written in pencil and then superimposed with ink, which makes it difficult to read. Candidates are strongly advised not to do this.

Comments on specific questions

Question 1

Most candidates were generally able to make some correct progress and obtain at least one of the two correct answers for this question. Weaker responses often did not form a 3-term quadratic or did not obtain the solution -60°.

Question 2

In **part (a)** many good descriptions of the transformations were given. Some responses showed confusion in the transformation terms, as some used the word factor in describing a translation or gave the stretch with a factor of 2 rather than $\frac{1}{2}$, or in the wrong direction.

Part (b) was found to be more challenging for candidates with a significant number applying the transformations to the point given rather than finding the original point on the curve. Many did not seem sure how to proceed. Candidates should ensure they ready the question carefully and make sure their answer fits the requirements.

Question 3

Both parts of this question were generally well answered. Some candidates made **part (a)** more complicated by working out and attempting to simplify ff(x), then substituting 5, rather than finding f(5) then f(2). The standard technique for finding the inverse for the type of function in **part (b)** was generally well known.

Question 4

This question was well done by most candidates. The integration was usually good with most candidates remembering to divide by 3 as well as –1 and the constant of integration was usually found successfully. Weaker candidates sometimes ignored the word curve and used the equation of a straight line instead of integrating.

Question 5

Many candidates found this question to be particularly challenging. The trigonometric form of the terms of the arithmetic progression meant that some candidates were unable to make sufficient progress although the vast majority did attempt to do so. Some candidates only considered two of the three terms and so did not obtain the two different expressions or equations needed. Some considered the gaps between the terms as d rather than $2d$ and although this was condoned within the method in **part (a)** it led to incorrect answers in **part (b)**. **Part (b)** was often omitted because no answers had been obtained in **part (a)** and occasionally the 25th term was found rather than the sum.

Question 6

Many fully correct answers were seen for this question with candidates realising the need to form two equations from the given information, although several candidates did not show their method of solution of the quadratic equation which was obtained. Weaker responses often confused by the statement that r was greater than $\frac{1}{2}$ and tried to use this as a value in their expressions. The term 'exact' was not noticed by some candidates, giving their answer as a decimal. 10 – 1 should have been evaluated to 9 if the final answer was given in index form.

Question 7

A significant number of candidates omitted this question, in particular **part (b)** where the given answer from **part (a)** could have been used. In **part (a)** a variety of approaches were taken involving finding different lengths and angles, but often intermediate working was not given to at least 4 significant figures, and hence did not sufficiently justify the given answer. In **part (b)** most candidates attempted to find the area of the sector BCQ although weaker responses sometimes gave 9 or 15 as the radius. The area of the triangle, which needed to be subtracted from the sector, could usually be found but some issues of premature approximation occurred. A few candidates attempted the area of triangle PQC plus the segment.

Question 8

Part (a) was well attempted by many candidates, but **part (b)** proved more challenging with a significant number omitting it. In **part (a)** most candidates realised that two terms from the expansion would give a coefficient of x^2 and were able to form a quadratic equation in a. Weaker responses often obtained only one term. Sign errors in the expansion were common with negative signs sometimes discarded for no obvious reason, but the subsequent method marks were still available. In **part (b)** the phrase 'it is given instead that' was misunderstood or ignored in weaker responses that attempted to use the answers from **part (a)** to answer this part. Those who understood what was required were usually able to form a quadratic in a and use the discriminant to find the required value of k and subsequently of a.

Question 9

Many candidates found this question challenging, in particular **part (b)** and a sizeable minority omitted it completely. In **part (a)** most candidates realised the need to differentiate V and this was usually done correctly although a few left the –1 on the end of their differential. Many realised that they needed to apply the chain rule, but this was often done incorrectly with candidates not recognising the components correctly from the given information. A few candidates appear confused confused by the reference to a 'circular' mound and tried to use formulae for the volume of a sphere or hemisphere, even though the formula for the volume of the mound was given.

A significant number of candidates tried to link their answers to **part (a)** with **part (b)**. It is important for candidates to realise that in this situation it is only the stem part of the question which refers to both parts.

Question 10

The first two parts of this question proved to be straightforward for most candidates. Nearly all realised the need to differentiate the given function in **part (a)** and set it to 0, although some weaker responses set the original function equal to 0 instead. Most managed to solve the resulting equation but there were a number of errors. In **part (b)** the vast majority of those who attempted it tried to make use of the second differential. A small number did not substitute $x = 2$ into their second differential. **Part (c)** proved more challenging with many candidates not realising the significance of the given information. Some put the range equal to a single value and others made it $>$, $<$ or $\leqslant$ f(2).

Question 11

Many fully correct solutions were seen to this question, especially **part (a)**. In **part (a)** most realised the need to differentiate, and this was often done correctly although some failed to realise the need to substitute $x = 3$. In **part (b)** many candidates realised the need to integrate and this was often done well but the upper limit was often incorrect with the answer to **part (a)** common. Many then ignored the triangular part of the area or made extra work by integrating the normal. A few only integrated the normal.

Question 12

In **part (a)** many good solutions were seen with candidates realising the need to find the centre of the circle, the gradient of the normal and then the gradient of the tangent. Some attempted implicit differentiation and others completed the square and re-arranged to find $y =$ then differentiated, with mixed success. Weaker responses sometimes used the origin rather than the centre of the circle. Some stopped when they had found A and B.

Many found **part (b)** very challenging with almost half of candidates omitting it. Of those who did attempt it, many realised the need to find the radius but were unsure about how to proceed. Those who used the given diagram to see which lengths were needed were often most successful, with a number of different approaches possible.

MATHEMATICS

Paper 9709/13
Paper 1 Pure Mathematics 1

Key messages

Candidates should be encouraged to draw appropriate sketch diagrams, which can aid them in answering certain types of questions, for example **Question 9**.

When a question asks candidates to "show" a given result all stages of the process leading to that result need to be shown.

Candidates are expected to solve quadratic equations by factorisation, formula or completing the square and their method of choice should be shown clearly in their solution.

General comments

For this paper, the knowledge and use of basic algebraic, including manipulating and form algebraic expressions, and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

In general, candidates used mathematical language well to describe the two transformations. Those who viewed the sequence as a reflection or stretch by scale factor –1 followed by a translation often scored full marks. Those who viewed the sequence as a translation followed by a reflection were rarely able to describe the translation correctly. When describing a translation it should be noted that use of a vector is encouraged as it avoids the ambiguities of verbal descriptions. Reflections should be accompanied by a clear statement of the line in which the reflection takes place.

Question 2

(a) This part was well answered by most candidates. Occasionally the first term was omitted and the power of *a* was omitted in the third term.

(b) The requirement to obtain two terms in x^2 was seen by most candidates and the majority of these went on to form a correct quadratic equation and find both answers correctly. It is a requirement of the syllabus that solutions are fully justified so it is important that candidates showed a clear method to obtain and solve the quadratic equation for this question.

Question 3

(a) Various techniques for completing the square for this type of quadratic were evident and most were used successfully to find the two required values.

(b) Whilst most candidates appreciated the need for differentiation of f(x) and completed this correctly only stronger responses showed why this must always be positive. It was expected that the result from **part (a)** would be adapted for this purpose but complete methods using the discriminant of the quadratic in x^2 were acceptable although rarely seen. Candidates should realise that showing a function is positive for a selection of x values does not prove it is positive for all values of x.

Question 4

(a) This part was solved in a variety of ways including trial and improvement and use of the *n*th term formula. Many correct answers were seen when the correct inequality was formed and solved.

(b) The use of the main two forms of the formula for the sum of *n* terms was nearly always seen. The two sums were often formed and equated correctly leading to many correct answers.

Question 5

(a) Almost all candidates were able to correctly find the angle in radians, though a few found it necessary to obtain the angle in degrees and then convert it to radians.

(b) This part was also often answered completely correctly. Use of rounded intermediate answers was rarely seen and candidates were able to effectively deal with angles presented in radians both in the arc length formula and their use of trigonometric functions.

Question 6

(a) This proved to be the most challenging question part for many candidates. Candidates appreciated a reflection of the given graph was required and a lot of reflections of the graph were seen but few were reflections in $y = x$. It was rare to see the line $y = x$ on the candidates' sketches.

(b) The need to change the subject of the function was appreciated by most but the algebraic skills necessary to complete the procedure were not always evident. Those who cleared the fraction before squaring tended to be more successful than those who squared the given function. Errors in squaring the negative term were frequent enough to be mentioned as were the sign errors in the gathering of terms. The final correct inverse with the negative alternative of $2x$ in the numerator was very rarely seen.

(c) This was answered well by those candidates who appreciated a value of *a* rather than *x* needed to be given.

(d) The correct order of functions for the composite was nearly always used. The initial substitution of $2x$ was usually carried out well but the factorisation of the denominator and subsequent simplification was only seen in the better responses.

Question 7

(a) There were several equivalent acceptable variations. Working from the initial statement to the given result or the reverse of this or showing that the given value of *k* could be substituted into the final result to give zero. No matter which method was chosen candidates should be advised that in a 'show that' question it is necessary to show all steps taken to obtain the solution. The candidate is being asked to show, step-by-step, the path from one equation to the other. Some candidates showed the first three lines in the necessary working and then the final result, which was not enough. It was important that the use of both $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and the standard trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ and the gathering of terms prior to factorisation was present. Candidates are advised to work from the given statement to the given result as it will usually be the easiest route. Those who followed this usually obtained the given result correctly.

(b) Candidates generally followed the instruction of 'hence' and used $k = 4$ in the final equation from **part (a)**. Candidates are again advised to show a full method for solving the quadratic, in this case using the formula or completing the square. Many complete solutions were seen with both answers quoted to the required degree of accuracy.

Question 8

(a) Most candidates found this question to be relatively straightforward. Some combined the two equations and used one integral whilst others calculated two integrals separately. The fractional indices were mostly correctly integrated and there were few errors when substituting $x = \frac{1}{4}$. Most candidates appreciated that the substitution of the limits should be made clear before the use of a calculator. A very small minority attempted to find a volume of revolution or used limits of 2 and $\frac{1}{2}$.

(b) This part was also well attempted by the majority of candidates. Most were able to differentiate the fractional index, gaining the first two marks. A large proportion went on to find the gradient and equation of the normal and the required value of p.

Question 9

(a) The method for finding the x coordinates of A and B was understood by most candidates and the correct quadratic equation was usually reached. It should be clear that to obtain the roots in surd form use of the quadratic formula or completion of the square is required. Completion of the square provided the most direct route to answers in simplified surd form. The formula for the distance between two points was well known and rarely misquoted leading to many correct answers in suitable surd form.

(b) This was the most omitted question part. Very few diagrams were seen even though candidates this would have greatly benefited candidates due to their usefulness in selecting suitable methods. This was particularly true for this question which could be solved algebraically or through the use of geometry or trigonometry. Most attempts involved solving the circle equation with the tangent equation and setting the discriminant of the resulting equation to zero. Although this produced some more difficult algebraic manipulation correct solutions were often seen. The quickest route to the solution via the angle the tangent made with the x-axis was rarely seen.

Question 10

(a) The majority of the candidates saw the need for differentiation. The best responses showed that the differentiation of $-(1 + k)^{-2}$ with respect to x led to zero and not $2(1 + k)^{-3}$. The use of $f''(x) > 0$ was often seen as was the resulting inequality solved by inspection or simple algebra.

(b) Most candidates answered this part very well. Those who did not integrate the constant term with respect to k usually found the equation correctly. A few candidates chose to work with k rather than substituting it, which made it slightly more challenging. It was expected that the complete equation would be stated as the final answer.

(c) A significant number of candidates were able to set the first derivative from **part (a)** to zero and find the two solutions. They often reached the correct value of x, found the corresponding y value and then the nature of the point. A small number of successful candidates did not attempt to find the y-coordinate and a few did not determine the nature of the point found, as requested by the question.

MATHEMATICS

Paper 9709/21
Paper 2 Pure Mathematics 2

Key messages

Candidates should be reminded of the need to ensure that they have read the question carefully and met the full demands of the question to the required level of accuracy. Candidates should ensure they understand the requirement of an exact answer. Candidates should also recognise that in some questions, work they have done in previous parts is meant to help them with a solution; for example, in **Question 2** and **Question 7**.

General comments

Some candidates did not appear to be well prepared for the examination, with incomplete questions or questions that were not attempted. There appeared to be no issues with timing and the majority of candidates had sufficient room on the paper to answer the questions.

Question 1

Most candidates realised that the form of the integral involved $k_1\mathrm{e}^{2x} + k_2\mathrm{e}^{-2x}$. There were errors with the values of k_1 and k_2. Most candidates who obtained an integral of this form were able to apply the limits correctly although some used their calculators to give a rounded answer rather than the exact answer as required.

Question 2

(a) Most candidates were able to sketch a V shaped graph together with a straight-line graph. The positioning of the V shaped graph was occasionally incorrect. It was essential that the gradient of the straight-line graph was such that there was only one point of intersection with the graph of the modulus function, this being in the first quadrant. The gradient of the straight-line graph had to be greater than the gradient of the graph of the modulus function for $x > 3$. It was evident that many candidates did not consider this when sketching the straight-line graph.

(b) Many correct solutions were seen, with candidates who chose to form two linear equations usually having more success. Some candidates who opted for the method of squaring did not square the $3x$ term. For those that did, sometimes sign errors or poor algebra resulted in incorrect solutions. It should also be noted that **part (a)** was meant to be a guide for **part (b)** together with the wording of the question which stated that there was only one point of intersection. Many candidates obtained two solutions, but some did not relate them to the graph they had sketched in **part (a)** and did not go on to discard one of them. Some candidates omitted finding the *y*-coordinate.

(c) This part asked for candidates to deduce the solution of the inequality, however some candidates chose to solve the inequality again rather than use their answer to **part (b)**.

Question 3

Very few correct solutions were seen. The main difficulty for candidates was identifying the gradient of the straight-line to $\frac{1}{\ln a}$, having written the given relationship in logarithmic form. Problems also arose when attempting to substitute values in order to find $\ln k$ and hence k. It was also acceptable to use the alternative method of using exponentials but very few responses using this method were seen.

Question 4

(a) Provided candidates recognised the need to differentiate the term xe^{2x} as a product, they were usually able to obtain a method mark. Some candidates had difficulties differentiating the exponential terms correctly. Most realised that they had to equate their derivative to zero and attempt to rearrange the equation obtained to the given form. It was essential that the equation be multiplied throughout by e^x in order to obtain terms in e^{3x}. Few completely correct rearrangements were seen.

(b) Most candidates were very familiar with solving equations using iterations and this question part was usually well-answered. Candidates had to start with 0.35 and only a few iterations were needed for convergence. Too many candidates obtained the correct number of iterations to the correct level of accuracy and gave a final answer of 0.356, therefore did not gain the final accuracy mark.

Question 5

(a) Many candidates were unable to find $\frac{dx}{dt}$ correctly with the most common mistake being the omission of 2 in the numerator. Most candidates realised that the quotient rule had to be used in order to obtain $\frac{dy}{dt}$. Most candidates used a correct approach in order to subsequently find $\frac{dy}{dx}$.

(b) Most candidates realised that they needed to find the value of *t* when $x = 0$ in order to find the gradient of the curve at the point *A*.

(c) Most candidates realised that they needed to find the value of *t* when $y = 0$ in order to find the gradient of the curve at the point *B*.

Question 6

(a) Very few incorrect approaches were seen, although some candidates chose to attempt algebraic long division or synthetic division rather than a straight-forward application of the factor theorem. Those who chose to use algebraic long division or synthetic division, were usually less successful. Candidates are advised to check their working to ensure that their solutions they obtain are realistic or possible.

(b) Very few incorrect approaches were seen to this part, but the best and most accurate solutions were obtained when the remainder theorem was used rather than algebraic long division or synthetic division.

(c) Few correct solutions were seen with candidates recognising that $x + 3$ is a factor of both $f(x)$ and $g(x)$. If the common factor was identified, the final result was often given as $(x+3)(4x^2-1)$ rather than as a product of linear factors. For those candidates who had incorrect solutions for **part (a)**, credit was given if they attempted to find a quadratic factor, using, $x + 3$ and their values from **part (a)**.

(d) It was essential that candidates make use of their answer from **part (c)**, ensuring that $\operatorname{cosec}\theta = k$ where $k < -1$. Some candidates did not recall that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$. Few fully correct solutions were seen although some candidates were able to obtain one valid solution.

Question 7

(a) Most candidates were able to use the correct expansion for $\cos(2\theta + \theta)$ although some had incorrect signs in their expansion. Most made use of $\sin 2\theta = 2\sin\theta\cos\theta$ but did not use a correct identity for $\cos 2\theta$. It was essential that sufficient detail be shown in order to gain full marks.

(b) Very few correct solutions were seen. Candidates needed to make use of **part (a)** and write the given expression as $\frac{1}{2}\cos 3\left(\frac{5\pi}{18}\right)$. This could then be evaluated but an exact answer was needed for full marks.

(c) Very few correct solutions were seen, with some candidates making no attempt at this part. Candidates needed to make use of **part (a)** and write the given integral as $\int (9\cos x - \cos 9x)\,dx$ after simplification.

MATHEMATICS

Paper 9709/22
Paper 2 Pure Mathematics 2

Key messages

Candidates should be reminded of the need to ensure that they have read the question carefully and met the full demands of the question to the required level of accuracy. Candidates should also recognise that in some questions, work they have done in previous parts is meant to help them with a solution; for example, in **Question 2** and **Question 7**.

General comments

It was evident that many candidates were well prepared for the examination and generally attempted most, if not all the questions. There appeared to be no issues with timing and most candidates had sufficient room on the paper to answer the questions.

Comments on specific questions

Question 1

(a) Most candidates recognised the need to use both the factor theorem and remainder theorem to form two equations. The majority of these equations were correct, but some candidates made simple errors when solving their correct equations simultaneously. Candidates are advised to check their working to ensure the solutions they obtain are realistic.

(b) Provided the solutions obtained in **part (a)** were correct, most candidates made a valid attempt to obtain a quadratic factor making use of the linear factor given in **part (a)**. In most cases this was done either by inspection or by algebraic long division. Some candidates gave their final answer as the product of the correct linear factor and the correct quadratic factor, not realising that the quadratic factor could be factorised further. For those candidates who had incorrect solutions for **part (a)**, credit was given if they attempted to find a quadratic factor, using the given linear factor, and their values from **part (a)**.

Question 2

(a) Most candidates were able to sketch a V shaped graph together with a straight-line graph. The positioning of the V shaped graphs was occasionally incorrect. It was essential that the gradient of the straight-line graph was such that there were two points of intersection with the graph of the modulus function. It was evident that many candidates did not consider this when sketching the straight-line graph. There should be a point of intersection in the second quadrant and a point of intersection, or implied point of intersection in the first quadrant.

(b) Many correct solutions were seen, with candidates who chose to form two linear equations usually having more success. Some candidates who opted for the method of squaring did not square the $x+3$ term. For those that did, sometimes sign errors or poor algebra resulted in incorrect solutions. It should also be noted that **part (a)** was meant to be a guide for **part (b)**. Many candidates obtained two solutions but did not relate them to the graph they had sketched in **part (a)**. If this had been done, then perhaps the sketch in **part (a)** could have been amended to give two points of intersection in some cases.

(c) Many candidates related this part of the question with the work they had done in **part (b)** and made use of their positive answer to **part (b)**. Some candidates chose to start again but invariably made algebraic errors.

Question 3

A reasonable attempt was made by most candidates to differentiate the given equation with respect to *x*. However, errors often occurred in the differentiation of $2\tan 2x$. These errors were usually with the coefficient of $\sec^2 2x$ or the coefficient of *x* itself. Although most realised that they had to equate their $\frac{dy}{dx}$ to zero and solve, some were unsure of how to deal with the trigonometric term correctly, not realising that $\sec^2 2x = \frac{1}{\cos^2 2x}$. Occasionally the *y*-coordinate was omitted, hence underlining the importance of making sure that the question has been answered fully. Many candidates did obtain a fully correct solution.

Question 4

The given form of the question did help most candidates realise that logarithms were involved in the integration. Most errors occurred with the coefficient of the logarithmic term, with many candidates not having the coefficient of $\frac{1}{3}$. The rules involving logarithms were usually applied correctly but it should be noted that some candidates obtained a fortuitously correct answer by stating that $\ln(a+14) - \ln a$ was the same as $\frac{\ln(a+14)}{\ln a}$, for which accuracy marks were not awarded.

Question 5

Candidates found this question to be quite demanding; candidates had to realise that not only implicit differentiation was needed but also that a product was involved. Several errors were noted, often which usually involved omissions of terms or coefficients in the differentiation of the product. Some candidates equated their result to 6 rather than zero. Most candidates attempted to substitute the given x and y values into their derivative and fine the gradient of the tangent to the curve. This question highlighted the need for candidates to make sure that they had answered the question fully, checking to ensure they have fulfilled all of the requirements. Having obtained the gradient of the tangent, many omitted to find the gradient of the normal as required. Of the candidates that did find the gradient of the normal, most did leave it in an exact form as required. When attempting implicit differentiation, candidates should be encouraged not to start their work with $\frac{dy}{dx} = \ldots$ as very often this extra derivative gets considered in the overall working and thus constitutes an incorrect method.

Question 6

(a) Very few correct sketches were seen. Many candidates did not attempt a sketch. It was intended that a sketch of the graph of $y = \ln x$ together with a sketch of the graph of $y = 2e^{-x}$ be produced and the single point of intersection be acknowledged as representing a single solution to the given equation.

(b) Most candidates considered either $\ln x - 2e^{-x}$ or $2e^{-x} - \ln x$ with substitutions of 1.5 and 1.6 to show that a change of sign occurred and so the root lies between 1.5 and 1.6.

(c) Very few correct solutions were seen, with many candidates misunderstanding what was required. It was intended that the iterative formula given be reduced to $x = e^{2e^{-x}}$ followed by the application of logarithms to obtain $\ln x = 2e^{-x}$.

(d) Most candidates seemed very familiar with solving equations using iterations and this question part was usually well-answered. Those candidates who started with 1.55 required fewer iterations than those who started with either 1.5 or 1.6. Many candidates obtained the correct number of iterations to the correct level of accuracy and gave a final answer of 1.53, thus not gaining the final accuracy mark.

Question 7

(a) Many candidates realised that the use of the double angle identities was needed and obtained correct solutions. It is essential in questions of this type that sufficient detail is shown. It was intended that candidates deal with the left-hand side of the given identity and manipulate it correctly to obtain the right-hand side of the identity. A correct solution starting with the right-hand side was also acceptable but more difficult to show.

(b) It was essential that candidates realised that they needed to use the result from **part (a)** to re-write the integrand before attempting integration. Of those candidates that did, many made reasonable attempts at integration with errors usually involving signs or coefficients. Most candidates attempted to give their final answer in exact form.

(c) For this part it was also essential that candidates realised that they needed to use the result from **part (a)** to re-write the left-hand side of the equation. It was also important that candidates realised that they were dealing with $4y$ not $2y$ in order to obtain a correct solution. Correct solutions in exact form were seen.

MATHEMATICS

Paper 9709/23
Paper 2 Pure Mathematics 2

Key messages

Candidates should be reminded of the need to ensure that they have read the question carefully and met the full demands of the question to the required level of accuracy. Candidates should also recognise that in some questions, work they have done in previous parts is meant to help them with a solution; for example, in **Question 2** and **Question 7**.

General comments

It was evident that many candidates were well prepared for the examination and generally attempted most, if not all the questions. There appeared to be no issues with timing and most candidates had sufficient room on the paper to answer the questions.

Comments on specific questions

Question 1

(a) Most candidates recognised the need to use both the factor theorem and remainder theorem to form two equations. The majority of these equations were correct, but some candidates made simple errors when solving their correct equations simultaneously. Candidates are advised to check their working to ensure the solutions they obtain are realistic.

(b) Provided the solutions obtained in **part (a)** were correct, most candidates made a valid attempt to obtain a quadratic factor making use of the linear factor given in **part (a)**. In most cases this was done either by inspection or by algebraic long division. Some candidates gave their final answer as the product of the correct linear factor and the correct quadratic factor, not realising that the quadratic factor could be factorised further. For those candidates who had incorrect solutions for **part (a)**, credit was given if they attempted to find a quadratic factor, using the given linear factor, and their values from **part (a)**.

Question 2

(a) Most candidates were able to sketch a V shaped graph together with a straight-line graph. The positioning of the V shaped graphs was occasionally incorrect. It was essential that the gradient of the straight-line graph was such that there were two points of intersection with the graph of the modulus function. It was evident that many candidates did not consider this when sketching the straight-line graph. There should be a point of intersection in the second quadrant and a point of intersection, or implied point of intersection in the first quadrant.

(b) Many correct solutions were seen, with candidates who chose to form two linear equations usually having more success. Some candidates who opted for the method of squaring did not square the $x+3$ term. For those that did, sometimes sign errors or poor algebra resulted in incorrect solutions. It should also be noted that **part (a)** was meant to be a guide for **part (b)**. Many candidates obtained two solutions but did not relate them to the graph they had sketched in **part (a)**. If this had been done, then perhaps the sketch in **part (a)** could have been amended to give two points of intersection in some cases.

(c) Many candidates related this part of the question with the work they had done in **part (b)** and made use of their positive answer to **part (b)**. Some candidates chose to start again but invariably made algebraic errors.

Question 3

A reasonable attempt was made by most candidates to differentiate the given equation with respect to *x*. However, errors often occurred in the differentiation of $2\tan 2x$. These errors were usually with the coefficient of $\sec^2 2x$ or the coefficient of *x* itself. Although most realised that they had to equate their $\frac{dy}{dx}$ to zero and solve, some were unsure of how to deal with the trigonometric term correctly, not realising that $\sec^2 2x = \frac{1}{\cos^2 2x}$. Occasionally the *y*-coordinate was omitted, hence underlining the importance of making sure that the question has been answered fully. Many candidates did obtain a fully correct solution.

Question 4

The given form of the question did help most candidates realise that logarithms were involved in the integration. Most errors occurred with the coefficient of the logarithmic term, with many candidates not having the coefficient of $\frac{1}{3}$. The rules involving logarithms were usually applied correctly but it should be noted that some candidates obtained a fortuitously correct answer by stating that $\ln(a+14) - \ln a$ was the same as $\frac{\ln(a+14)}{\ln a}$, for which accuracy marks were not awarded.

Question 5

Candidates found this question to be quite demanding; candidates had to realise that not only implicit differentiation was needed but also that a product was involved. Several errors were noted, often which usually involved omissions of terms or coefficients in the differentiation of the product. Some candidates equated their result to 6 rather than zero. Most candidates attempted to substitute the given x and y values into their derivative and fine the gradient of the tangent to the curve. This question highlighted the need for candidates to make sure that they had answered the question fully, checking to ensure they have fulfilled all of the requirements. Having obtained the gradient of the tangent, many omitted to find the gradient of the normal as required. Of the candidates that did find the gradient of the normal, most did leave it in an exact form as required. When attempting implicit differentiation, candidates should be encouraged not to start their work with $\frac{dy}{dx} = \ldots$ as very often this extra derivative gets considered in the overall working and thus constitutes an incorrect method.

Question 6

(a) Very few correct sketches were seen. Many candidates did not attempt a sketch. It was intended that a sketch of the graph of $y = \ln x$ together with a sketch of the graph of $y = 2e^{-x}$ be produced and the single point of intersection be acknowledged as representing a single solution to the given equation.

(b) Most candidates considered either $\ln x - 2e^{-x}$ or $2e^{-x} - \ln x$ with substitutions of 1.5 and 1.6 to show that a change of sign occurred and so the root lies between 1.5 and 1.6.

(c) Very few correct solutions were seen, with many candidates misunderstanding what was required. It was intended that the iterative formula given be reduced to $x = e^{2e^{-x}}$ followed by the application of logarithms to obtain $\ln x = 2e^{-x}$.

(d) Most candidates seemed very familiar with solving equations using iterations and this question part was usually well-answered. Those candidates who started with 1.55 required fewer iterations than those who started with either 1.5 or 1.6. Many candidates obtained the correct number of iterations to the correct level of accuracy and gave a final answer of 1.53, thus not gaining the final accuracy mark.

Question 7

(a) Many candidates realised that the use of the double angle identities was needed and obtained correct solutions. It is essential in questions of this type that sufficient detail is shown. It was intended that candidates deal with the left-hand side of the given identity and manipulate it correctly to obtain the right-hand side of the identity. A correct solution starting with the right-hand side was also acceptable but more difficult to show.

(b) It was essential that candidates realised that they needed to use the result from **part (a)** to re-write the integrand before attempting integration. Of those candidates that did, many made reasonable attempts at integration with errors usually involving signs or coefficients. Most candidates attempted to give their final answer in exact form.

(c) For this part it was also essential that candidates realised that they needed to use the result from **part (a)** to re-write the left-hand side of the equation. It was also important that candidates realised that they were dealing with $4y$ not $2y$ in order to obtain a correct solution. Correct solutions in exact form were seen.

MATHEMATICS

Paper 9709/31
Paper 3 Pure Mathematics 3

General comments

Some candidates showed a good understanding of the topics examined. The majority of candidates offered no response to several items and showed only limited understanding of basic algebra and arithmetic.

Key messages for candidates

- Candidates are reminded to ensure they are confident with the basic rules of algebra and arithmetic and use brackets when appropriate.
- Practise the basic methods in calculus to ensure that you are familiar with all the standard techniques and patterns.
- Ensure that working out is displayed clearly in the answer space and that all the stages of the working is shown.
- In the case where candidates wish to replace their solution, neatly cross out the solution and re-attempt the question below this. Candidates should ensure that they do not overwrite one solution with another.
- If a question asks for an exact answer, then decimal working is not appropriate and exact numbers are expected throughout the working.
- Candidates are reminded to read the question carefully and make sure that their response matches what is being asked.

Comments on specific questions

Question 1

The majority of candidates started their solution by squaring the equation and multiplying out the brackets. Errors such as $4\times5^x = 20^x$ and $\left(5^x\right)^2 = 25^{2x}$ were common. Candidates who got as far as $3\times5^x = 4$ did not always obtain a correct value for 5^x.

Question 2

(a) Some candidates were familiar with the processes required here and there were some correct solutions. Those candidates who found the value of α first and used that to find *R* were not often able to state the exact value of *R*.

(b) This question was testing the candidates' knowledge of the function $R\sin(x-a)$. A few wrote down the correct answers but the majority either did not respond or offered irrelevant working.

Question 3

(a) There were several correct attempts to use the product rule to differentiate this function. Obtaining the correct coordinates for the turning point required both the correct derivative and correct algebra.

(b) The question states that the curve has one stationary point, so the candidates had a wide choice of methods here. They could consider the sign of the second derivative, which was a popular option, or they could consider the value of *y* or the value of the gradient at another point on the curve. Candidates who chose one of the latter two approaches often scored no marks because although they calculated values, they did not say whether these were values of y or values of the gradient, and they did not draw clear conclusions.

Question 4

Some candidates were clearly familiar with the method required, they started with a statement of $\frac{du}{dx}$, or equivalent, and attempted to form an integral in terms of u. Some candidates made algebra slips in the process and some stopped part way through. Very few obtained $2\tan^{-1}u$.

Question 5

(a) The candidates who attempted this question usually made good progress in forming an equation in $\tan\theta$ and rearranging this to form a quadratic in $\tan\theta$. However, this was not always correct, often due to slips in the algebra and arithmetic.

(b) Candidates with a correct quadratic equation usually obtained at least one correct angle. Some candidates who obtained a negative angle did not go on to use the periodic nature of $\tan\theta$ to obtain the obtuse angle.

Question 6

The majority of candidates made no attempt to expand $\sqrt{1+4x}$. Those candidates who did attempt the expansion usually obtained a relevant pair of equations in a and b which they were able to solve.

Question 7

(a) The answer here was given, so candidates needed to present convincing evidence that they understood how to obtain the result. The minimum expected here was $\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{\ln x} = \frac{1}{x\ln x}$.

(b) Those candidates who started by substituting the boundary condition did not get as far as appreciating the relevance of **part (a)**. A small minority of candidates did start by separating the variables correctly and making some progress with the integration.

(c) Most candidates offered no solution to **part (b)** or they had a solution of an incorrect form, therefore found it difficult to answer this part as it required them to interpret their solution.

Question 8

(a) A small number of candidates recognised the need to use integration by parts. There were some errors in the coefficients, but these candidates usually made good progress towards demonstrating the given result.

(b) The most successful candidates compared the value of a with the value of $\exp\left(\frac{1}{\sqrt{a}} + 2\right)$ for $a = 9$ and $a = 11$ and drew the correct conclusion. Some candidates mentioned making an appropriate comparison but did not provide any values to support their answer.

(c) Many candidates did not respond to this part, which suggested that these candidates were not familiar with using an iterative process. This particular formula worked equally quickly with 9, 10 or 11 as the starting point. A small number of candidates used a starting point of 10.12, which suggested inappropriate use of calculator functions to solve the equation. Candidates who did not work to the level of accuracy requested did not score the accuracy marks.

Question 9

(a) Those candidates who demonstrated correct use of the scalar product of the direction vectors of the two lines usually scored both marks.

(b) Some candidates were familiar with the method for finding the point of intersection of two lines. The two common errors were to overlook the need to verify that the equations for all three components were satisfied, or to show that the two lines do intersect but not state the point of intersection.

(c) To make any progress with this part of the question, a candidate needed to start by considering a general point *P* on the line and using the scalar product to determine the position vector of *P*; very few candidates used a strategy that did this.

Question 10

(a) The first three parts of this question are all inter-related. The approach taken by the minority of candidates who attempted the question was to find the square and the cube of *u* and substitute these into the equation $\mathrm{p}(u) = 0$. With this approach it was then necessary to compare the real and imaginary parts of the equation and solve the resulting simultaneous equations to obtain *a* and *b*.

(b) Some correct answers were seen to this part. The common incorrect answer was $-1-2\mathrm{i}$.

(c) Very few candidates demonstrated knowledge of how to use the pair of complex roots to form a quadratic factor with real coefficients.

(d) (i) Candidates who knew how to represent the two regions on an Argand diagram usually drew a good, clear diagram.

(ii) In order to obtain the correct answer candidates needed to understand where to look on the diagram, and that the answer would be negative. There were a small number of correct answers.

MATHEMATICS

Paper 9709/32
Paper 3 Pure Mathematics 3

General comments

Many candidates offered solutions to all eleven questions, however the standard of the responses seen was very variable, with some responses demonstrating a clear lack of understanding of the content of the specification.

Questions which are seen to be generally be more straightforward for candidates, such as solution of a modular inequality (**Question 2**), partial fractions (**Question 4**) and use of an iterative formula (**Question 11(c)**) did not earn many marks. In questions where the first part gives an indication of how to solve the second part, such as complex numbers (**Question 3**), integration of a trigonometric function (**Question 6**), and solution of a trigonometric equation (**Question 8**), many responses did not suggest that the link between the parts had been recognised.

Key messages for candidates

- Candidates are reminded to ensure they are confident with the basic rules of algebra and arithmetic and use brackets when appropriate.
- Practise the basic methods in calculus to ensure that you are familiar with all the standard techniques and patterns.
- Ensure that working out is displayed clearly in the answer space and that all the stages of the working is shown.
- In the case where candidates wish to replace their solution, neatly cross out the solution and re-attempt the question below this. Candidates should ensure that they do not overwrite one solution with another.
- If a question asks for an exact answer, then decimal working is not appropriate and exact numbers are expected throughout the working.
- Candidates are reminded to read the question carefully and make sure that their response matches what is being asked.

Comments on specific questions

Question 1

The majority of candidates recognised the need to use logarithms, and many solutions contained some correct use of the rules of logarithms. Several candidates reached the correct solution. A common error was to start by claiming that $3\left(2^{1-x}\right) = 6^{1-x}$. Some candidates with correct working got as far as $x = \frac{\ln 2 + \ln 3}{\ln 2 + \ln 7}$ and stopped without completing the question. A few who reached this stage then 'cancelled' the $\ln 2$ to obtain an answer of the required form, which was not a correct step.

Question 2

The most common approach was to start by squaring both sides of the inequality to obtain an expression free of modulus signs. Many candidates obtained the correct quadratic equation for the critical values. A few went on to obtain the critical values of *x* in terms of *a*. Some did not reach the correct final answer as they thought that since *a* is a positive constant then the critical value $x = -\frac{3}{5}a$ was not possible. The majority of candidates who obtained the correct quadratic equation made no further progress because they tried to solve $b^2 \geqslant 4ac$ rather than to solve for *x*. The most common reasons for not reaching the correct quadratic were to forget to square the 2, or to make errors in squaring the brackets and simplifying.

A small number of candidates tried the alternative approach of considering two linear equations, some of which were successful. The main errors in using the alternative method were slips in the arithmetic and the algebra.

Question 3

(a) Candidates found this question to be particularly challenging, with many candidates not communicating a clear proof. Some candidates did not understand the notation u^* for the conjugate of *u*. Some responses made no mention of the conjugate at all, and claimed incorrect statements such as $a + \mathrm{i}b + c + \mathrm{i}d = a - \mathrm{i}b + c - \mathrm{i}d$. Many solutions appeared to assume the given result and to be working on both sides of the equation at the same time, rather than working on the two separately and concluding that they were equal.

(b) A minority of candidates realised that they needed to substitute $z = x + \mathrm{i}y$ and then use the result from **part (a)**. Those candidates who did this often went on to consider the real parts and the imaginary parts of their equation and complete correctly. A few equated the real part to *x* and the imaginary part to *y*, which was an incorrect approach. Many candidates did a lot of work trying, unsuccessfully, to form a solvable equation in *z*.

Question 4

Candidates who recognised this as a top-heavy fraction often answered the question correctly. For some, the most common errors were slips in the arithmetic.

The majority of candidates started with the incorrect form $\frac{4x^2 - 13x + 13}{(2x-1)(x-3)} = \frac{A}{2x-1} + \frac{B}{x-3}$ and only scored a maximum of 2 marks. It is possible to solve the problem in two stages by starting with $\frac{A}{2x-1} + \frac{Bx+C}{x-3}$. Some candidates completed the first stage correctly, but few of them went on to split the top-heavy fraction.

Question 5

(a) Several fully correct Argand diagrams were seen. Some candidates made the task more difficult by using different scales on the two axes, whilst some candidates used inconsistent scales. The most common errors were to draw a circle in the wrong quadrant, to draw a circle when the scale required an ellipse, or to draw the wrong straight line. On correct diagrams the shading was usually correct, if it was shown.

(b) Many candidates offered no response to this item. There were a few fully correct solutions, usually following a clear diagram with a clearly marked tangent. Some candidates made errors in the trigonometry, using $\tan^{-1}\frac{1}{\sqrt{13}}$ in place of $\sin^{-1}\frac{1}{\sqrt{13}}$. The question asked for an answer in degrees, so an answer in radians was not awarded the final mark.

Question 6

(a) Most candidates started with a correct expansion, usually for $\sin(2x + 3x)$. Several then followed the expected route of substituting the two expansions in $\frac{1}{2}(\sin 5x + \sin x)$ and deriving the given result within two or three lines of working. A significant minority started by trying to use $\sin(2x + 3x) = \sin(2x - 3x)$.

(b) The result in **part (a)** and the use of 'hence' in the question were intended to help candidates with this integral, however many did not make use of this. There were some errors with the coefficients and some errors in using the limits, but several reached the given answer correctly. When the answer is given, candidates are expected to show sufficient working to make it clear that they have reached the answer correctly; some candidates went direct from the first stage of substitution to the given answer with no interim working. A large number of candidates tried to use integration by parts to evaluate the integral, but none of them succeeded in completing this.

Question 7

The correct approach here is to separate the variables, integrate both sides of the resulting equation and then use the boundary condition. A large minority of candidates started by trying to use the boundary condition and were unable to make any progress. Several candidates recognised the correct form for $\int \frac{1}{y^2}\,dy$. The common incorrect answers were $\ln y^2$ and $\frac{1}{2y}\ln y^2$. The majority of candidates recognised the need to use integration by parts for the second integral. Many had the correct approach, but there were errors in the coefficients, and e^{-2x} often became e^{2x}. Some candidates had no constant of integration, and there were several errors in taking the reciprocal to find the expression for *y*.

Question 8

(a) Candidates found this part to be particularly challenging. Many candidates equated $\left(\cos^2\theta + \sin^2\theta\right)^2$ to zero. Only a small minority started by equating it to one, and a few more concluded that it was 1 after expanding and rearranging the terms. Some candidates did succeed in justifying $2\sin^2\theta\cos^2\theta = \frac{1}{2}\sin^2 2\theta$, but many derived the result from a succession of incorrect statements.

(b) The majority of candidates did try to use the result from **part (a)**. The candidates found potential for error at each stage of rearranging the resulting equation. 2θ often became θ during the working, and some candidates thought that $\sin^2 2\theta = \sin\left(2\theta\right)^2$, so they obtained an angle and then took the square root. It was common for candidates to give more than one decimal place in their answers, often rounding incorrectly. Most candidates obtained at least one correct value, but it was unusual to see all four values. Some candidates ignored the 'hence' in the question. They were often able to reach a correct equation in $\cos\theta$ or $\sin\theta$, but completely correct solutions were rare.

Question 9

(a) Most candidates used the correct method in answering this part. Candidates are reminded to be careful to ensure the correct use of brackets and to pay careful attention to signs to ensure marks can be gained. Candidates with errors in their working often claimed to have reached the given answer without trying to find the error in their working. Several of the weaker responses demonstrated that they were not familiar with implicit differentiation.

(b) Many candidates attempted to find the point where the gradient was equal to zero, and several looked for the point where the gradient was equal to 1. Those candidates who interpreted the question correctly often made good progress. Those who formed and solved an equation in *y* were often more successful than those who found the value of *x* first. The unsimplified answer $x = \frac{1}{3}\ln 8$ was common and made the task of solving for *y* more complex than necessary.

Question 10

(a) The majority of candidates started correctly by forming a vector in the direction of the line. Several reached an incorrect **k** component because of the error $1-\left(-1\right)=0$. Most candidates attempted to use the correct structure for the equation of the line, but many responses concluded 'line = ...' or 'equation = ...' or ' *l* = ...' rather than the correct form '**r** = ...'.

(b) Many candidates formed a correct expression for $\overrightarrow{AC}$ but a lot did not go on to use this to find the position vector $\overrightarrow{OC}$.

(c) Many candidates found the lengths of various vectors but did not have a correct strategy to find *P*. Some candidates who gave a correct expression for $\overrightarrow{OP}$ in component form did not go on to use the length $\sqrt{14}$. Most of the candidates who formed a correct equation in λ did complete the question correctly, but there were several errors in the algebra and arithmetic between the initial equation and the final answers.

Question 11

(a) Those candidates who started by writing $\sqrt{\tan x}$ as $(\tan x)^{\frac{1}{2}}$ often then used the chain rule to obtain a correct form of the derivative. Some used $\sec^2 x = 1 + \tan^2 x$ to obtain an answer in $\tan x$, as asked for in the question. The final mark, for verifying that the derivative was equal to 1 when $x = \frac{\pi}{4}$ was available from any correct form of the derivative. Many candidates did not score this mark because, despite showing their expression with $x = \frac{\pi}{4}$ substituted, they did not show any evidence of working to obtain the given answer.

(b) Candidates found this part to be particularly challenging. Rather than starting with the equation $\frac{dy}{dx} = 1$ and squaring this to form an equation in *t*, most candidates spent a lot of time trying to manipulate the given cubic equation. Obtaining the given answer to **part (b)** does depend on starting with a correct expression from **part (a)**, but most candidates did not adopt a strategy where this became relevant.

(c) The question does not specify a starting value for the iteration. Although the graph given in the question clearly indicates that *a* lies between 0 and $\frac{\pi}{4}$, many candidates used a starting value outside this interval. For most of the values chosen, the iteration does eventually converge. Some candidates were not working in radians, and many candidates keyed in the formula incorrectly. Candidates with a correct sequence of iterations often rounded the final answer incorrectly.

MATHEMATICS

Paper 9709/33
Paper 3 Pure Mathematics 3

General comments

The standard of work on this paper was extremely high. A considerable number of candidates performed well on many of the questions.

It was clear that most candidates had taken notice and understood the instructions in the rubric and including sufficient detail in their solutions to demonstrate that they had not used a calculator.

Candidates should be reminded to set out their work clearly to ensure it is legible and presented in a logical manner.

Key messages

- Candidates should be aware of how to use a sketch of an inequality graph to aid them in checking their solutions when solving inequalities, see **Question 2(a) and (b)**,
- Candidates should ensure they are familiar with basic techniques, specifically using the chain rule which was particularly relevant for **Question 7(a) and 9(a)**,
- Candidates should be aware of the meaning of the word 'hence', this is used to indicate that a solution or information from previous part should be used to answer the question.
- Candidates should ensure that they show adequate detail when solving equations, in particular when solving the equation $a^x = b$,
- It is recommended that candidates use the same scale on both axes when sketching an Argand diagram.

Comments on specific questions

Question 1

Most candidates opted to solve by long division as opposed to inspection using $2x^4 + 1 = (x^2 - x + 2)(Ax^2 + Bx + C) + Dx + E$. Many excellent solutions were seen, but a considerable number of candidates either did not correctly obtain the constant term in the quotient or made an arithmetical error in one of the terms in the remainder.

Question 2

(a) The sketch required symmetry, both arms of approximately the same length and the points (0, 3) and $\left(\frac{3}{2}, 0\right)$ shown. A large number of candidates believed that since it was $|2x - 3|$ then $x < 0$ was not possible and stopped their sketch at the y-axis.

(b) The sketch in **part (a)** could have been used in **part (b)** to determine which of the two lines to solve with $y = 3x + 2$, and hence to establish the correct region. Instead, most candidates either created a quadratic equation or two pairs of linear equations. As this led to having two critical points $x = \frac{1}{5}$ and $x = -5$, this often resulted in candidates making the wrong choice.

Question 3

Some candidates found this question to be particularly challenging. Candidates who scored zero did not express 4^{x-2} as $\frac{4^x}{16}$ and obtain an expression for 4^x. These candidates often tried to apply log laws as opposed to laws of indices. Having $4^x = \frac{16^2}{15}$ candidates needed to proceed with $x\ln 4 = \ln(\frac{16^2}{15})$ and then obtain x to 3 d.p.

Question 4

This question required integration by parts and was generally well answered. However, many candidates did not integrate $\sin\left(\frac{1}{2}x\right)$ correctly, with the 2 in the denominator instead of the numerator or made a sign error in integrating $\cos\left(\frac{1}{2}x\right)$. Substitution of limits into the trigonometrical functions needed sufficient detail to establish that the correct trigonometrical values such as zero, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$ and 1 had been used. Only a few candidates gave a decimal solution instead of the requested exact answer.

Question 5

Most candidates gained either 4 or 5 marks on this question. Those who gained 4 marks often either forgot the solution from $\sin\theta = -1$ or did not realise that the solution from the other $\sin\theta$ value also had an answer in the second quadrant: 180° – 56.4°.

Question 6

(a) Many candidates produced fully correct solutions to this trigonometrical problem. It was rare to see an incorrect expansion of $\cos(x - 60°)$ and values of sin(60°) and cos(60°) were almost always exact and correct. The main error seen was expressing R as a decimal rather than an exact value.

(b) Candidates found this part to be particularly challenging with many incorrect responses seen. Many candidates believed that $\cos(x - \alpha)$ was least when the argument was 0°, 90°or 270°, instead of at 180°. Candidates are recommended to use a sketch of the cosine function, which could have helped to avoid these errors.

Question 7

(a) Most candidates found using the chain rule within the implicit differentiation of the original equation, $\frac{\left(\frac{1+\mathrm{d}y}{\mathrm{d}x}\right)}{(x+y)}$, to be challenging. Some candidates removed the ln by exponentiating so that the chain rule was only required on e^{2y}, and this proved slightly easier. In addition, many candidates manipulated the original equation into other alternative forms before differentiating. However, this approach often led to errors which could have been avoided had they differentiated the equation in its original form.

(b) Candidates generally found this part to be straightforward. Nearly every candidate successfully set the given answer from **part (a)** to zero and substituted it in the original equation of the curve. Realising ln(1) was zero meant that most candidates scored full marks for the coordinates.

Question 8

(a) Several candidates found this question to be challenging. Many candidates thought that the vector line equation for *MN* was in fact only the vector ***MN***. Candidates should also be reminded that the left side of the vector line equation of *MN* should be a vector and symbolised by the vector **r**.

(b) The majority of candidates realised that setting the scalar product of **r** from **part (a)** and the vector ***MN*** to zero and solving the resulting equation produced the required point on the line *MN*. Most candidates then found the length of this vector by Pythagoras to obtain the given answer. Alternative approaches were either to use a scalar product to find the projection of *OM* or *ON* onto *MN*, followed by Pythagoras, or to find either $\cos PMO$ or $\cos PNO$ then determine $OM \sin PMO$ or $ON \sin PNO$.

Question 9

(a) Solutions were possible via the chain rule using $\left(9x^{\frac{1}{2}} - x^{\frac{3}{3}}\right)^{-1}$, the quotient rule using $\dfrac{1}{\left(9x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)}$ or the product rule using $(9-x)^{-1}x^{-\frac{1}{2}}$. All these were methods regularly seen. However, many attempts using the product or quotient rules saw errors by having incorrect signs in the middle. In the case of the quotient rule, some candidates did not square the denominator of the formula when substituting. The three different approaches then required different amounts of algebra to determine the *x*-coordinates of the stationary point. Some solutions needed only a couple of lines, others considerably more. When undertaking the extra algebra, care must be taken if squaring since this often introduces values that are not solutions of the original equation and need to be rejected.

(b) Most candidates obtained $\dfrac{du}{dx}$ correctly, but many then made errors in substituting and hence did not obtain the correct integrand in terms of *u*. Of those that were successful about half realised this was an integral in the formulae book and so they did not need to show all the partial fraction details. However, if quoting from the formula book care is needed, especially in this case. Instead of seeing $\left(\dfrac{1}{3}\right)\ln\left(\dfrac{(3+u)}{(3-u)}\right)$ with limits $u = 0$ and $u = 2$, it was common to see $\left(\dfrac{1}{3}\right)\ln\left(\dfrac{(3+x)}{(3-x)}\right)$ with limits $x = 0$ and $x = 4$ or limits $x = 0$ and $x = 2$, or $\left(\dfrac{1}{3}\right)\ln\left(\dfrac{(u+3)}{(u-3)}\right)$. The latter led to the appearance of ln of negative numbers which needed to be handled carefully using modulus signs.

Question 10

(a) Many candidates were unable to change the proportionality information into an equation involving an arbitrary parameter, hence were unable to generate the given equation. Those that could do so had no difficulty using the two pieces of information.

(b) Most candidates were able to separate variables correctly and to integrate with respect to *t*. Many candidates divided throughout by 19 and hence finished with $\dfrac{(20-x)}{19x}$ as opposed to $\dfrac{(20-x)}{x}$. The 19 appearing in two terms led to considerable confusion and many candidates were unable to integrate correctly with respect to *x*. Several other candidates did not see that $\dfrac{(20-x)}{x}$ could be expressed as $\dfrac{20}{x-1}$, producing the easiest of integrals, instead using integration by parts. This meant that they needed to know how to integrate $\ln x$ and ensure that their work was error free otherwise the two $x\ln x$ terms did not cancel out. Some candidates who did integrate correctly forgot to introduce a constant of integration. Candidates needed to reach a correct solution to their differential equation and then manipulate that solution into the given form to gain the final mark, however this was not often seen. Candidates needed to realise that being given $x = e^{0.9 + 0.05x}$ meant that it was necessary to write $\ln x = f(x)$ and then exponentiate.

(c) This iteration was almost always error free, with convergence clearly shown. As the initial value was given in the question, this needed to be used by candidates for all of available marks to be awarded.

(d) Of the candidates that had obtained a correct solution of their differential equation in **part (b)**, many did not set $x = 20$ and solve for t.

Question 11

(a) Whilst most candidates were able to obtain the modulus correctly, this was not the case with the argument, with $\frac{\pi}{6}$ far more common than $\frac{5\pi}{6}$. Most candidates' solutions resulted in a choice of two answers, so they needed to sketch an Argand diagram and plot the given point u in order to select the correct value.

(b) This part started with the word 'Hence', this means that the candidate need to continue with the result they found in **part (a)**. The question is requesting that $\left(2e^{\frac{i5\pi}{6}}\right)^6$ becomes $2^6 e^{i5\pi} = -64$.

Candidates should not restart with the binomial expansion of $(-\sqrt{3} + i)^6$, as this was not answering the question being asked.

(c) (i) To represent complex numbers satisfying equations and inequalities on an Argand diagram it is essential that the scales on the Real and the Imaginary axes are identical. For the locus Re $z \leqslant 2$, the line $x = 2$ needed to be shown and the point (2, 0) clearly labelled. For the locus $0 \leqslant \arg(z - u) \leqslant \frac{\pi}{4}$, half lines needed to be drawn from point u at $\arg z = 0$ and $\arg z = \frac{\pi}{4}$, with the coordinates of u and the angle $\frac{\pi}{4}$ clearly indicated.

(ii) Of the candidates who obtained the correct shaded region in **part (c)(i)** many, despite realising that the shaded triangle was isosceles, omitted to add unity to the imaginary coordinate of z. Hence, gave the greatest $|z|$ as $\sqrt{(2^2 + (\sqrt{3} + 2)^2)}$ instead of $\sqrt{(2^2 + (\sqrt{3} + 3)^2)}$.

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Key messages

- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper and cases where this was commonly not adhered to were seen in **Questions 3**, **4**, **5** and **7**. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable in **Questions 4** and **7**.

- In questions such as **Question 6** in this paper, where velocity is given as a quadratic function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

- In cases such as **Question 3**, where an angle is given in the form $\sin\alpha = \frac{3}{5}$ then there is no need to determine the angle explicitly since it is simple in this case to show that $\cos\alpha = \frac{4}{5}$ and these values can be used in any calculations. If the angle is evaluated this may lead to rounding errors. The angle given in **Question 5** can also be treated similarly.

General Comments

There were some excellent responses seen for each of the questions on paper. Overall, a wide range of performance was seen but the paper was generally well answered.

Some candidates did not give answers to 3 significant figures as requested on the front of the question paper, mainly due to prematurely approximating within their calculations leading to the final answer. In **Question 3** the sine of an angle is given and in **Question 5** the angle is given in terms of inverse sine. As mentioned in the Key Messages, in such questions it was not necessary to determine the actual angle as this often leads to premature approximation and to a loss of accuracy.

On the front cover of the question paper it tells candidates take g = 10 and most candidates are correctly following this instruction.

Comments on Specific Questions

Question 1

(a) The most straightforward approach to this problem is to draw and use a velocity-time graph. This consists of a trapezium shaped graph. Since the total distance travelled corresponds to the area under the graph, the given distance of 585 m can be equated to the area of the trapezium and this information can be used to set up an equation in V, the constant speed. This method was adopted by most candidates. Solving this equation gives the required speed directly. This question was well answered.

(b) The required deceleration can be found as the gradient of the final stage of the velocity-time graph. Since the magnitude of the deceleration is required, the value stated must be positive.

Question 2

(a) This part involves the use of the principle of conservation of momentum. The two particles are moving in opposite directions and so the momentum of the particles before collision will have opposite signs. In the resulting collision the particles coalesce and so their velocity after collision will be the same. Most candidates were able to write down the required momentum equation and solve this to find k. A common error was to use the momentum of each particle before collision with the same sign.

(b) This question required candidates to evaluate the kinetic energy of the particles before and after collision. Use of the formula $\text{KE} = \frac{1}{2}mv^2$ will enable the required kinetic energies to be found. Most candidates made a successful attempt at this part of the question.

Question 3

In this question it is necessary to express the the forces are in equilibrium by resolving forces horizontally, along the direction of the 20 N force, and vertically, perpendicular to the 20 N force. This will produce two equations in the unknown variables P and θ. The value of $\sin\alpha$ is given in the question and so the actual angle is not required to be found. If the equations are rearranged to make $P\sin\theta$ and $P\cos\theta$ the subjects of the equations, then the value of θ can be found by dividing these two expressions and using trigonometry. P can then be determined by using the angle found for θ. Most candidates used this method. Common errors were to mix sine and cosine of the angles and also some sign errors were seen. Accuracy was sometimes lost if the actual angle α was found rather than to use the given value of $\sin\alpha$ and the corresponding value of $\cos\alpha$.

Question 4

(a) This part of the question asked for a force diagram. All forces and any angles must be shown in such a question. Here this means showing the weight as a vertically downwards force of magnitude $12g$, the force P N acting parallel to the plane, the friction force, F, also acting parallel to the plane and the normal reaction, R, acting perpendicular to the plane. The 25° angle must also be shown. All forces should be shown with arrows indicating their direction of action. The friction force, F, could be shown either parallel to the plane acting upwards or downwards as this depends on the direction of motion of the particle. Many candidates did not show enough detail in terms of the directions and in some cases forces were omitted.

(b) The question asked for the least possible value of P. This will be when P is applied up the plane with the aim of stopping the particle from slipping down the plane and so if the motion began it would be down the plane. This means that the friction force, F, will be acting up the plane in the same direction as the force P. Many candidates incorrectly thought that F acted down the plane but this leads to the other limiting case of the particle being about to slip up the plane. It is necessary to resolve forces along and perpendicular to the plane. As the particle is in limiting equilibrium the friction force is given by $F = 0.35R$. Most candidates made a good attempt at this question with common errors being a mix of sine and cosine and the direction of friction taken to be in the wrong sense.

Question 5

(a) This question required candidates to firstly determine how far the car travels up the plane in 30 seconds. Once this distance, d, has been determined then the potential energy gained in this time can be found as $\text{PE} = 1600 \times g \times d \times 0.12$. Most candidates used the correct definition and made a good attempt at this part of the question.

(b) In this part of the question the work-energy conservation property is required using the PE gain found in **part (a)**. The equation takes the form work done (WD) by the engine = WD against friction + PE gain. This enables the WD against friction to be found and then the constant frictional force can be found by dividing this WD by the distance, d, travelled in 30 seconds. Some good attempts were seen but errors in sign or forgetting to divide by d were most common.

(c) In this part of the question the power can be found in a variety of ways although the most straightforward method is to use the formula Power = WD by the engine ÷ Time. Most candidates made a good attempt at this question.

(d) In this part of the question Newton's second law must be used along the direction of the road. Once the new reduced power is found the driving force, DF, can be found by using DF = $P \div v$. Other forces acting are the resistance force and the component of the weight along the direction of the incline. The effect of these three forces causes the deceleration. Some errors seen here included missing forces and incorrect evaluation of the new power.

Question 6

(a) In this question the velocity is given as two different functions of t over the two given time ranges. A condition is given that the acceleration at $t = 2$ is zero. In order to determine the values of the unknowns p and q it is necessary to set up two simultaneous equations. The first of these comes from using the condition on the acceleration. Differentiating the velocity in the range $0 < t < 6$ and using the condition that $a = 0$ at $t = 2$ gives the first equation. The second equation comes from matching the velocities at $t = 6$ since it is given that there is no instantaneous change in velocity. These two equations can then be solved for p and q. Most candidates performed well on this question. Errors were sometimes seen in the manipulation and the solving of the simultaneous equations.

(b) In this part of the question a velocity-time graph is required. Once p and q are found it is clear that the shape of the two graphs are quadratic over the first six seconds and linear from $t = 6$ to $t = 14$. It is necessary to determine the time at which $v = 0$ which shows when the quadratic crosses the t-axis. Most candidates correctly sketched the linear part, but the quadratic was often incorrectly positioned.

(c) Here the total distance is required and it is important to use the velocity time graph found in **part (b)**. Use can be made of the fact that the area under the v-t graph represents the distance travelled provided that care is taken with areas above and below the t-axis. The quadratic graph is seen to be below the t-axis from $t = 0$ to $t = 4$ and above the t-axis from $t = 4$ to $t = 6$. The linear graph is above the t-axis from $t = 6$ to $t = 14$. The distance travelled from $t = 0$ to $t = 6$ must be split into two parts. Integration of v gives the displacement and this will be negative from $t = 0$ to $t = 4$ and positive from times $t = 4$ to $t = 6$. Provided that the signs of these two values are considered the distance travelled from $t = 0$ to $t = 6$ can be found. The area under the graph from $t = 6$ to $t = 14$ is readily found by evaluating the area under the triangle formed by that part of the v-t graph. The error most often seen was to integrate the quadratic from $t = 0$ to $t = 6$ without considering the sign of v.

Question 7

This question involves two connected particles, one on an inclined plane and the other hanging freely and there is no friction involved. One method of approach is to apply Newton's second law parallel to the plane for particle B and in the vertical for particle A. For both particles there are two forces acting. For particle A it is the weight of the particle and the tension in the string. For B it is the component of the weight along the direction of the plane and the tension in the string. The tension is the same throughout the string but some wrongly used different tensions acting on each particle. When the two equations of motion are stated, they produce two equations each involving the tension T and the acceleration a. These equations can now be solved simultaneously to find the value of a. Alternatively, since only a was needed, the equation for the system can be found and many candidates took this approach. Once the value of this acceleration is determined, the constant acceleration equations can be used to find the speed of the particles as A reaches the ground. At this point A remains at rest but B continues to move up the plane until it comes to instantaneous rest. The only force acting on B during this second phase of motion is its weight. Applying Newton's second law enables the deceleration to be found. Once this is known, the equations for constant acceleration can again be applied and the distance travelled up the plane until B comes to instantaneous rest can be found. The main cause of error was candidates using the wrong component of weight parallel to the plane namely $3g\cos 18$ for particle B or not using a component but using $3g$ parallel the plane when finding a in the first part of the motion. For the motion after A reaches the ground, an error seen was to use the deceleration as g rather than a component of g. Most candidates used this method.

An alternative approach to this problem is to use energy methods for either or both parts of the motion. Some candidates attempted this successfully. Some tried to solve the problem by considering only particle *B* but if this was attempted then the work done by the tension also needed to be considered.

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Key messages

- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper and cases where this was not adhered to were seen in **Questions 5**, **6** and **7**. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable in **Questions 5** and **7**.

- In questions such as **Question 4** in this paper, where displacement is given as a cubic function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

There were some excellent candidates who produced very good answers on this paper.

Some candidates did not give answers to 3 significant figures as requested on the front of the question paper, mainly due to prematurely approximating within their calculations leading to the final answer. This was often seen in **Questions 5**, **6** and **7**. In **Questions 5** and **7** the sine of an angle is given in the question. In such questions it was not necessary to determine the actual angle to 1 decimal place as this often leads to premature approximation and a loss of accuracy.

On the front cover of the question paper it tells candidates take g = 10 and most candidates are correctly following this instruction. In some cases it is impossible to achieve a correct given answer, such as in this paper in **Question 3**, unless this value is used.

Comments on specific questions

Question 1

(a) The majority of candidates gained full marks for this part of the question. It was necessary to use the fact that the acceleration is given by the gradient in a velocity-time graph. The accelerations in the two stages from t = 0 to t = 5 and the from t = T to t = 50 must be equated to form an equation in T. Some candidates set up the equation but then made errors with their algebra so did not obtain the correct value for T.

(b) Almost all candidates correctly attempted to find the area beneath the velocity-time graph with many different approaches taken on this part. The most common approach was to find the area of the six different stages and add. Some tried to combine areas by finding areas of trapeziums. An error seen was to wrongly consider the area under the curve from t = 0 to t = 30 as a trapezium. A very small number of candidates calculated the area of the large trapezium, from t = 0 to t = 50, and subtracted the area of the smaller trapezium, from t = 25 to t = 50; this was the best approach.

Question 2

(a) In this question most candidates attempted to write down the equation of motion either for the van or the trailer or for the system. Two of these equations were needed in order to solve for the required tension. Some errors seen included using the resistance for the trailer into the equation for the van or vice versa. Another error seen was to include a tension into the equation for the whole system.

(b) In this part of the question many candidates did not use the fact that that $T = 0$ and hence could not make any progress. In fact, many used values obtained in **part (a)** but this was no longer applicable.

Question 3

(a) In this question many candidates incorrectly used constant acceleration formulae which cannot be used for motion which does not take place on a straight line. Conservation of energy methods must be used here and those candidates who realised this generally achieved full marks.

(b) In this part of the question energy methods must be used. There is an initial kinetic energy and during the motion this is lost to friction and to a gain in potential energy. Most candidates who used energy methods included these three effects but in several cases these were combined with incorrect signs.

Question 4

(a) This part of the question was well done by the majority of candidates who differentiated the given expression for s in order to find the velocity. It was necessary to set this expression to zero to find when the cyclist comes to rest. Particular care must be taken in questions such as this where the answer is given, making sure that full working is shown. Most candidates found the time at which the cyclist came to rest and then used this in the given expression for s in order to find the distance AB. A few errors were seen in the differentiation but overall most candidates obtained the given answer without error.

(b) Most candidates realised that the maximum velocity took place when the acceleration is zero and that this required further differentiation of their expression for velocity. Some used the symmetry property of the quadratic expression for velocity in order to find the time at which maximum velocity occurred. Again, this part was well answered by the majority of candidates.

Question 5

(a) From the given information many candidates correctly used the formula $F = P \div v$ to find the constant driving force acting on the engine. The sine of the angle of inclination is given and so the angle need not be found. However, some wrongly interpreted the angle and used sin0.01 in their working. Some wrongly attempted to include an acceleration in their working. Most candidates correctly used the equilibrium of the three forces, namely, driving force, the effect of the component of the weight and the required resistance force.

(b) In this part of the question many candidates did not identify that an energy approach must be used. The work done by the engine in 60 seconds can be found using WD = Power $\times$ Time. This work done by the engine has two effects. It has to overcome the given WD against resistance and also to increase the kinetic energy. The work energy equation is used to express this fact. Some candidates incorrectly attempted to find a resistance force and use Newton's second law but it is given that the resistance is not constant and so this method cannot be used.

Question 6

(a) Many good responses were seen to this part of the question. Using the equilibrium of forces horizontally and vertically was a common approach. This led to two simultaneous equations involving the two tensions and these can be solved to find the required tension. A commonly seen error was to assume that the two tensions were equal. By considering the equilibrium in the vertical it is clear that this cannot be the case. Some candidates attempted to use Lami's equations but there are four forces acting here and so it is not possible unless two of the forces are combined. Since only the tension in the lower string is required then a very good method for solving this problem is to resolve forces perpendicular to the upper string which gives the required result in one equation. A few candidates did adopt this approach.

(b) Candidates found this part of the question to be particularly challenging. This was mainly due to not realising that the lower tension must be set to zero in this case. Very few candidates successfully solved this problem. The solution can be found by resolving vertically and horizontally. However, since the lower tension is zero this means that in this case it is possible to solve using Lami's equations.

Question 7

(a) This question involves application of Newton's second law to particle *Q* and including the friction effect. Care must be taken here to show sufficient detail within the working as the answer is given in the question. One of the common errors made here was to mix the *m* and 2*m* masses of the particles within the same equation. The specific angle of the plane need not be found as $\sin\alpha = 0.8$ is given and $\cos\alpha$ can easily be shown to be $\cos\alpha = 0.6$. Most candidates made a reasonable attempt at this part, including the two force terms, the weight component and friction and equating to mass $\times$ acceleration.

(b) In this part almost all candidates did not realise that particle *Q* comes to rest and then moves down the plane before the two particles meet. Particle *P* moves down the plane from rest and it can be shown that it has acceleration 4.4 m s^{-2} and this can be used to find the distance s_P travelled by particle *P* until collision. It is necessary to find the distance s_Q travelled by particle *Q* until collision. Once this has been done use of the equation $s_P + s_Q = 6.4$ enables the required time to be found. Since many candidates misinterpreted this question, a special case mark scheme was used to ensure that these candidates were not unduly disadvantaged by this error.

(c) In this part it is necessary to determine the velocities of the two particles at the instant at which the collision takes place. Once these have been found then use of the conservation of momentum equation in the form $mu_P + 2mu_Q = 3mv$ enables the required speed, *v*, of the combined particle to be found. When candidates correctly found u_P and u_Q, the majority were able to correctly apply conservation of momentum. A number of candidates incorrectly used the initial velocities of zero and 10 as their values of u_P and u_Q.

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Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the front of the question paper rather than correct to 2 decimal places as sometimes seen for example in **Question 4(a)(iii)**.
- When resolving forces in equilibrium or when forming an equation of motion, a clear and complete force diagram can be helpful in ensuring that all relevant forces have been considered for example in **Question 1(b)**, **Question 4(a)(iii)** and **4(b)**.
- When two separate strings are attached to a particle, the two tensions should be assumed to be different unless the situation suggests otherwise, e.g. **Question 2** and **Question 6(a)** and **(b)**.

General comments

Many responses of a high standard and clearly presented solutions were seen. There were also several candidates who had difficulty attempting some questions. **Question 1(a)** and **Question 4(a)(i)** and **(a)(ii)** were found to be the most straightforward questions for candidates whilst **Question 6(b)** and **(c)** were the most challenging.

Comments on specific questions

Question 1

(a) Nearly all candidates applied 'conservation of momentum' correctly to solve this problem. Very few omitted the momentum of the object after impact, $8 \times 120 = 40v$, or mistakenly attempted to apply 'conservation of energy'.

(b) A suitable method was usually attempted, either applying Newton's second law and a '*suvat*' formula or forming and solving a work/energy equation. A common error was to omit the weight $160g$ when forming the equation of motion, $-4800 = 160a$. In some solutions, either the mass of the object or the mass of the post was used instead of the combined mass, e.g. $120g - 4800 = 120a$ or $120g - 4800 = 40a$. When applying $v^2 = u^2 + 2as$ to calculate the distance required, the initial velocity was sometimes taken to be 8 ms^{-1}, which was the speed before rather than after the impact. For the work/energy method the main errors were to omit a term from the equation for example $4800s = \frac{1}{2} \times 160 \times 6^2$ (no potential energy), or to include an extra term or a sign error, e.g. $\frac{1}{2} \times 160 \times 6^2 - \frac{1}{2} \times 160 \times 8^2 = 1600s - 4800s$.

Question 2

(a) Whilst many complete force diagrams were seen, there were also a variety of incorrect or incomplete diagrams. The weight was sometimes either omitted or labelled as mass 8 kg. A normal reaction was occasionally seen even though the particle was suspended. Some diagrams indicated the same tension in each string. Others omitted to show the direction if each force.

(b) The majority of candidates resolved the forces in equilibrium to find the two different tensions accurately even following incomplete diagrams in **part (a)**. A few tried to resolve assuming that the tension in each string was the same leading to $T\sin45 + T\sin60 = 80$ in this oversimplified situation. The use of a 3 significant figure approximation in working does not ensure 3 significant figure accuracy in a final answer, candidates are advised to work to at least 4 significant figures throughout their intermediate working. In this case the use of 41.4 N for the tension in one string led to 58.5 N rather than 58.6 N for the other tension.

Question 3

(a) The most direct method in answering this question was to equate the initial potential energy minus 8 J to the final potential energy. Many candidates misread or misinterpreted the loss of kinetic energy on hitting the ground and equated 8 J to the final potential energy to obtain $h = 0.5$ m. A correct maximum height could still be obtained from $5 - 0.5 = 4.5$ m, but this complete method was rarely seen. Those who mistakenly increased the energy by 8 J on impact with the ground found an increased maximum height of 5.5 m. Less direct methods were also regularly seen, for example, using an energy equation to calculate the velocity after impact with the ground and a constant acceleration formula, usually $v^2 = u^2 + 2as$, with acceleration $-g$ m s^{-2} to find the height.

(b) Candidates who used $s = \frac{1}{2}gt^2$ to find the time taken to reach the ground and $s = -\frac{1}{2}(-g)t^2$ to find the time from the ground to the maximum height were generally more successful than those who involved the velocities at the ground. Whilst the time taken for the drop was frequently correct, the time for the rise was less straightforward. The initial velocity for the ball rising was sometimes used as 10 m s^{-1}, the velocity on impact with the ground rather than the changed velocity following the energy loss. It was also common to see the use of $v = \sqrt{10\,m\,s^{-1}}$ suggesting that the kinetic energy at the ground $\left(\frac{1}{2}\times1.6\times v^2\right)$ was 8 J rather than 80 J before and 72 J after impact.

Question 4

(a) (i) The work done against the resistance was usually calculated successfully as WD = $DF \times v \times t$. This was also seen correctly as $P \times t$. Occasional errors included the use of an incorrect formula such as $\text{WD} = DF \times \frac{v}{t}$ or $\text{WD} = \frac{P}{t}$.

(ii) The solution for this part was usually fully correct with only a few candidates answering in watts instead of the required kW.

(iii) Most candidates adjusted the driving force for the increased power as necessary, although a few decreased using $\frac{45\,000 - 12\,000}{36}$ instead of $\frac{45\,000 + 12\,000}{36}$. When forming the equation of motion, the resisting force 1250 N was sometimes missing e.g. $\frac{57\,000}{36} = 1400a$. An alternative method seen was to consider only the increased power causing the acceleration and to solve $\frac{12\,000}{36} = 1400a$. The final answer was sometimes seen as 0.24 m s^{-2} correct to 2 instead of 3 significant figures.

(b) The inclined plane required a component of weight down the plane whilst the constant speed implied no acceleration. When resolving the forces along the plane, errors seen included the use of $1400\sin\theta$ instead of $1400g\sin\theta$ leading to $\theta = 32.4$, or the omission of the resistance 1250 N leading to $\theta = 8.2$. A few candidates overlooked 'constant speed' and included '*ma*' in their equation using the acceleration from **part (a)(iii)**.

Question 5

(a) This question was well answered by the majority of candidates who understood that integration was needed and fully correct solutions were often seen. Only a few candidates attempted to use constant acceleration formulae, e.g. $s = ut + \frac{1}{2}at^2 = \frac{1}{2}kt^2\left(16 - t^2\right)$. Most integrated and then calculated k successfully following the substitution of $t = 4$ and $v = 8$. Those who included a constant of integration C without recognising that $C = 0$, were unable to evaluate k. Others integrated twice and then inappropriately used the given answer to find the value of k rather than finding k in order to show the answer given.

(b) Candidates were expected to solve $s(t) = 0$ in order to find the time taken and hence the speed on return to O. Some assumed that $a = 0$ rather than $s = 0$ on return. The most common error was to find the velocity correctly as $-29.4\ \text{m s}^{-1}$ but omit to state the speed $29.4\ \text{m s}^{-1}$, as was required by the question.

(c) Maximum displacement required the solution of $v(t) = 0$ to find the time taken. As in **part (b)** some candidates mistakenly used $a = 0$ leading to $t = 4$ and $s = 20$.

Question 6

Part (a) was generally well answered but **parts (b)** and **(c)** proved to be particularly challenging for many candidates.

(a) This was a straightforward question for most candidates who were able to apply $F = \mu R$ appropriately to obtain the given answer. When the answer is given in the question, sufficient detail is expected. Reference to the 40 N and 60 N weights was expected before reaching an equation such as $20 = 50\mu$. A few candidates considered mass instead of weight showing $6 - 4 = 5\mu$. Others incorrectly included a tension, e.g. $60 - T + T - 40 = 50\mu$.

(b) The solution was expected to involve three simultaneous equations for the three particles. The situation was frequently oversimplified by the assumption that the tensions in the two strings were equal. This usually resulted in the solution of $8g - T = 8a$ and $T - 4g = 4a$ with no consideration of the 5 kg mass and the friction between the table and this particle. Some erroneously included the frictional force in the equation of motion for the 4 kg or 8 kg mass, e.g. $T - 4g - 0.4 \times 5g = 4a$. The acceleration could be found directly from the equation of motion for the whole system $8g - 0.4 \times 5g - 4g = 17a$ with the two tensions then found from $T_1 - 4g = 4a$ and $8g - T_2 = 8a$.

(c) This solution depended on recognising that the acceleration after the 8 kg particle hit the ground needed to be calculated either from a system equation for the two particles in motion or from solving simultaneous equations of motion for the 4 kg and 5 kg particles. Many candidates assumed either that the acceleration was $-g\ \text{ms}^{-2}$ or the same as found in **part (a)**. Those who recalculated the acceleration, sometimes used a faulty equation, e.g. $-4g - 20 = 5a$ instead of $-4g - 20 = 9a$ or $-T - 20 = 5a$. Some calculated different accelerations for each of the two particles despite their connection and then found and combined two different times taken. It was common to see a calculation for the time taken for the 8 kg mass to reach the ground or for the velocity at this time with no further progress.

MATHEMATICS

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Paper 5 Probability & Statistics 1

Key messages

Candidates need to have an understanding of all the topics in the syllabus for this component.

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure final value. The only exception is if a value is stated within the question. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

General comments

Candidates appeared to have sufficient time to complete all the paper. Many candidates did not appear to have sufficient knowledge of the topics for this component, with some not attempting any questions.

The use of simple sketches and diagrams can help clarify both context and conditions. These were frequently present in good solutions.

Candidates would be well advised to enhance their interpretation of success criteria, such as ‘fewer than 2 days’, as this is essential when answering probability questions.

It is good practice to read the question again after completing a solution to ensure that all the requirements have been fulfilled. As this is an applied mathematics component, comments about data should be related to the context of the question.

Comments on specific questions

Question 1

Although many candidates did identify that the criteria for a geometric approximation was fulfilled, there were a significant number of solutions which used the binomial approximation. This was sometimes successful in **part (b)** when all the possible scenarios that did not fulfil the success criteria were identified.

Many candidates did not interpret the success criteria accurately, which is a fundamental expectation of this component. Candidates would be well advised to practice this process in preparation for the examination.

Many answers were stated to 2 significant figures or 3 decimal places, rather than 3 significant figures as required.

(a) The best solutions identified the possible outcomes for the two coins to find the probability of obtaining two tails. The geometric approximation was then used and evaluated accurately with the probability stated as a fraction. Weaker solutions misinterpreted the criteria and found the probability of obtaining two tails on the 8th throw. The weakest attempts used the binomial approximation and often found the probability of having 1 success in 7, or 8, throws.

A number of candidates stated that the probability of throwing two tails was $\frac{1}{2}$, or occasionally $\frac{1}{3}$, and used this value appropriately.

(b) Better solutions recognised that the question used a standard property of the geometric approximation and used this accurately to answer efficiently. Weaker solutions used a binomial approximation approach and calculated the probability of obtaining two tails for each number of throws up to 9 throws and subtracting from 1. Errors in interpreting the success criteria resulted in either the omission of the 9th throw, or the inclusion of the 10th throw.

Question 2

This question involving coding was found challenging by many candidates. The standard formulae were not seen in many of the solutions. Work was often presented without clear structure, and candidates did not always communicate their logic within their solution.

(a) The best solutions used the relationship 'the mean of coded values = the mean of the original values subtract the coding value', $\left(\frac{\sum(x-k)}{40} = \frac{\sum x}{40} - k\right)$, with the given values substituted followed by a clear algebraic solution. Many candidates were successful using a simple problem solving logic approach, although there was often little communication about the steps that were being undertaken or the reasoning behind them.

(b) A significant number of candidates made no attempt at this part. This part simply required the data provided to be substituted into the standard coded data variance formula. Good solutions did state the formula before substituting. A number of solutions either did not square the coded mean or simply used the original mean in their calculation.

Question 3

The majority of solutions identified that a conditional probability was being calculated and made some progress towards the expected value. Solutions with a tree diagram were often more successful, as this enabled the information provided to be clearly displayed within the required context. The best solutions stated the required conditional probability formula before substituting the unsimplified probability calculations. Weaker solutions often had difficulties in using the information 'when Suki has milk, she never has a biscuit' within the context, and it was often omitted from the expected denominator.

Question 4

Many good solutions to this probability question were noted. Candidates did show appropriate supporting work for the variance, often calculating E(X) initially before substituting in the variance calculation. Candidates should be aware that there is no expectation to convert probabilities stated accurately as fractions to inaccurate decimal values.

(a) Most successful solutions contained an outcome space to identify the possible outcomes for the two spinners. It was noted that working with negative numbers was an occasional problem, and some unexpected values were obtained as possible outcomes. Weaker solutions often identified the possible outcomes and then stated each had a probability of 0.2.

(b) Candidates who had produced a probability distribution table in **part (a)** were usually able to complete this part successfully. Good solutions included the unsimplified arithmetic expressions for both E(X) and Var(X), with all values left as fractions. Weaker solutions often did not square E(X), or square the probability rather than the outcome in the variance formula.

Question 5

Many candidates found this question on permutations and combinations challenging. The use of simple diagrams to help clarify the given scenario is often helpful and seen in more successful solutions. As the number of arrangements and selections is always exact, candidates should be aware that the final answer should not be rounded to 3 significant figures.

(a) Many good solutions to this question were seen. The best included a simple diagram to illustrate the scenario, which can aid understanding of the process needed. Almost all attempts included the unevaluated factorial term as well as stating the accurate value. A few solutions provided an answer rounded to 3 significant figures rather than the exact value that is expected. A common error was not excluding Raman from the group when calculating the arrangements. A small number of solutions calculated 8! + 1, presumably to account for the one way that Raman can appear in the arrangements. Some better responses considered the arrangements as being two groups of four, which is true, but needed to be treated as ${}^8P_4 \times {}^4P_4$ rather than ${}^4P_4 + {}^4P_4$.

(b) Some excellent solutions were presented for this question. These usually included a simple diagram to clarify the scenario. The most successful approach was to consider the total number of arrangements that the team could make and subtract the number of arrangements where Raman and Sanjay were together. A number of candidates provided explanations of what was being calculated being included in the solution. The alternative approach was often less successful as many did not allow Raman and Sanjay to swap places and used ${}^8C_2$ rather than ${}^8P_2$ when calculating how many ways they could be inserted into the line of other seven team members. Again, some candidates stated only a rounded final answer rather than the exact number of arrangements.

(c) Although many solutions identified that a group of five and a group of four was required, the calculation presented by many assumed that the groups were picked independently, and all team members were available for the second group that was being selected. The best solutions realised that once the first group was selected, all the remaining team members had to be in the second group – and so could be ignored in the calculation. A few solutions used permutations rather than combinations, so implying that the order of the team members was to be considered.

(d) Many candidates found this question challenging, and a significant proportion of candidates presented no attempt. The best solutions realised that **part (c)** had calculated the value for the denominator of the probability and used a similar approach to calculate the number of ways that Raman and Sanjay were in the same group. Common errors were to consider the number of ways they could be in just one of the groups, usually the group of five, not reducing the number of team members the group could be picked from if they had already been selected or finding the product rather than the sum of their answers. Many solutions simply calculated the number of ways the groups could be selected and made no attempt to use this information to find the probability.

Question 6

The overall quality of the statistical diagrams presented was good, although a number of candidates did not use a ruler when drawing the box-and-whisker plot, which does make interpreting the information presented more difficult.

(a) Many excellent back-to-back stem-and-leaf diagrams were seen. The best solutions used a ruler to align the stem initially, included units for both teams in the single key and ensured that the individual data values were inserted into the leaves equally space, so that the diagram was a good visual representation of the data spread and skew. Weaker solutions included errors where the corrections made the vertical alignment inaccurate. Candidates should appreciate that as a diagram, it is recommended to be completed in pencil, so that errors can be erased and replaced. If it is necessary to cross-through, then any replacement value should ideally be place above the error.

(b) Most candidates were able to find the median accurately. The most common error when finding the interquartile range was not finding the mid-values between the median and the maximum/minimum values, with the use of $\frac{n}{4}$ being seen regularly.

(c) Many good box-and-whisker plots were seen. A small number did have the whiskers incorrectly passing through the box. Some candidates had values from **part (b)** where the median was not between the quartiles, which they attempted to plot here. Candidates should be aware that a ruler should be used to construct these plots to ensure that the anticipated degree of accuracy can be achieved.

(d) Many candidates simply made comparisons of individual values, for example the greatest weights rather than interpreting the statistical information displayed in **parts (c)** and **(a)**. At AS and A level, comparisons are normally related to the range, central tendency or skew and must be made in context. Good solutions often identified that the players from Rebels tended to be heavier than the Sharks.

Question 7

This question was found challenging by a large number of candidates, and some parts were not attempted in a significant number of scripts.

Although the question identifies that the original data has a normal distribution, candidates should be aware that other approximations may be required because of the given context. A number of solutions attempted to use the normal distribution in **part (a)(ii)**, when the success criteria had evolved to discrete data and a binomial approximation was appropriate.

(a) (i) Almost all solutions started by using the normal standardisation formula to find a probability. The best solutions often had a sketch of the normal distribution curve and identified the probability area which fulfilled the given criteria. These solutions also identified that the original data for the times was continuous and so did not require a continuity correction. A probability to at least 4 significant figures was then used to calculate the expected number of days when Karli would spend more than 142 minutes on social media, with a final conclusion of an integer value of days, with no indication of rounding. Weaker solutions either showed a rounding process for the number of days, or simply omitted this process. A significant number of solutions misinterpreted the success criteria and found the probability of spending less than 142 minutes. Candidates are well advised to spend time in ensure that they are able to interpret this type of success criteria accurately.

(ii) To be successful, candidates needed to use a result that they had found in **part (a)(i)** in a context which required a binomial approximation. Although the actual mathematics required was quite standard, few solutions were fully correct. Common errors were not using the probability calculated in **part (a)(i)** or misinterpreting the success criteria by including two days in their solution. A significant number of candidates submitted no attempt at this question.

(b) Although a significant number of candidates made no attempt at this question, many good solutions were seen. The most successful often used a simple diagram of the normal distribution curve to help identify the required probability area. As 90 per cent is a critical value, candidates needed to use the stated critical z-value from the normal table rather than using the main tables or a calculator to find a z-value. Most solutions realised that the normal standardisation formula needed to be used to form an equation. Common errors were to equate to the probability of $z = 0.9$, or $z = 1.282$ which would find the time of 10 per cent of the days spending more than t minutes on social media. The majority of equations formed were solved appropriately with sufficient evidence to justify the stated answer. It was noted that some solutions had accuracy errors from either using the tables or rounding from the calculator inaccurately.

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Paper 5 Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. When errors are corrected, candidates would be well advised to cross through and replace the term, rather than overwrite it.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates work to at least 4 significant figures throughout to justify a 3 significant figure value. The only exception is if a value is stated within the question. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, candidates would be advised to use the response space in a logical and clear manner. Better solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that cumulative frequency graphs are constructed with a curve, and that this needs to be reasonably accurately drawn. It was noted that many of the statistical diagram were well labelled.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates did not complete the very last question. A few candidates found some topics and skills required particularly challenging, including being able to differentiate between the binomial and geometric approximation requirements. Many good solutions were seen for **Questions 3** and **5**. The context in **Questions 2**, **4** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

(a) Almost all candidates simply stated the required probability, although good solutions included the unsimplified calculation. The most common error was to truncate rather than round the decimal probability, which if not supported by an acceptable fraction could gain no credit.

(b) Most candidates recognised that the conditional probability was required. The most efficient method was to identify the values required in the data table and state the probability directly. The most common method was to use the standard formula $P(M|D) = \frac{P(M \cap D)}{P(D)}$, and find the required probabilities from the data table. A common error was to assume that M, male, and D, drums, were independent and use $P(M) \times P(D)$ to calculate $P(M \cap D)$.

(c) Some good solutions to this part were seen; these stated the appropriate test in context, clearly identified the required probabilities and made a numerical comparison before stating the conclusion. The majority of solutions omitted one or more of these elements, and so not fully justifying their conclusion. Almost all checked the identity $P(F \cap G) = P(F) \times P(G)$, but a small number of candidates used the alternative approach involving conditional probability.

Question 2

This combinations question was found challenging by many candidates. The best solutions often contained simple 'diagrams' which clarified the conditions and possible outcomes. A few solutions assumed that the condition applied in **part (a)** was also required in **part (b)**. Candidates should be aware that when the question is structured as here, then the conditions are only applied within the single part.

(a) Most candidates deduced that the group required five women to be chosen from the group of 11 and used ${}^{11}C_5$ as expected. A significant number of solutions stated this as the final answer, while better solutions multiplied by 4 with no justification. The best solutions included a statement to show that one man was being picked from the group of four men. A small number of solutions incorrectly added rather than multiplied these values.

(b) A variety of approaches were seen. The most successful was to calculate the total number of ways that a group of six people could be chosen from the full group and subtract the number of groups which contained Jane and Kate. Candidates who used the more common approach of considering the different scenarios which fulfilled the condition often omitted the scenario when neither Jane nor Kate were present. Better solutions identified the value that was being calculated at each stage. It is expected that the unevaluated combination calculation is stated within the working to support the solution.

Question 3

Many good solutions to this probability question were seen. Candidates should be reminded that there is no expectation for probabilities to be stated as decimal values. Although not required by the question, a tree diagram was often present in more successful solutions, as it enabled the success criteria to be identified easily and supported the necessary calculations.

(a) As this part required a statement to be shown to be true, candidates should be aware that full justification for any calculations used need to be provided. Where a tree diagram had been generated, simply identifying the required branches would be sufficient to show that there were three possible outcomes that fulfilled the success criteria. Better responses often used the combination method for calculating probabilities successfully. Weaker solutions simply had a calculation that provided the required value, with no justification and often not related to the context.

(b) A high number of candidates made no attempt at the probability distribution table. As one value had been provided in **part (a)**, it was anticipated that the initial table would include at least one correct probability. Solutions where a tree diagram had been used in **part (a)** often utilised this to support the work here. Poorer solutions did not seem to relate the work attempted in **part (a)** as guidance to complete the table. A common error was to omit the outcome 0 yellow marbles selected, and so the probabilities in the table did not sum to 1. A few solutions included 4 or 5 yellow marbles picked as outcomes, which did not apply the condition given in the initial question information.

At this level, individual probabilities greater than 1 should highlight to the candidate that there is an error in their work that should be checked.

(c) Where a probability distribution table had been completed, most were able to calculate the value. Candidates should be aware that answers need to be supported by an unevaluated calculation, which here enabled $E(X)$ found from incorrect probability distribution tables to gain credit.

Question 4

This question was found to be challenging by many. Good solutions often included simple diagrams which clarified the conditions that were being applied. Weaker solutions frequently had poor communication of the logic that was being applied to determine the number of arrangements that fulfilled the criteria.

(a) Almost all candidates were successful in determining the correct number of arrangements and providing sufficient evidence of the calculation performed. Weaker solutions often did not remove the effect of the repeated letter E in the word.

(b) The arrangement criteria appeared to be challenging for many. A large number of solutions had little indication why calculations were being attempted, and how they linked to the question. The most successful solutions had a simple diagram such as '^ ^ ^ ^ T ^ ^ C ^' initially and then stated that there were seven remaining letters when the T and C had been removed, that the T and C could be located in six different positions (sometimes these were illustrated) and that the order of T and C could be reversed. The calculation was then shown and evaluated. There were a number of different approaches based on this principle that were clearly communicated and successfully used, However, many solutions presented only a calculation with little justification, and credit would be awarded based on the approach it was most similar to. A common error was not realising that the T and C could be reversed and so not multiplying by 2. A number of solutions attempted to use a similar method to **Question 2(b)** but made little progress with identifying all the scenarios which needed to be removed.

Question 5

This question was successfully identified by most candidates as involving discrete random variables and so linked to the binomial and geometric approximations. However, there was some confusion as to the conditions required for the geometric approximation to be applied.

(a) Most solutions recognised that this was a fairly straightforward binomial approximation question. Good solutions clearly stated all the unsimplified terms required and then evaluated without including intermediate steps. Where candidates stated intermediate values, accuracy was often lost with premature approximation. The most common error was to misinterpret the success criteria and calculate the probability that three or more days will be wet. A less common, but still frequent, error was to include the probability that three days will be wet.

The interpretation of this type of success criteria is an important skill in this component, and candidates should be encouraged to focus on achieving a good understanding of the differences that occur.

(b) Most candidates identified that the geometric approximation was appropriate. There was some inconsistency with the interpretation of the context with a common error being having eight dry days and so actually finding the probability that the first wet day is 9 October. Less successful solutions found the eight term of a binomial approximation, normally out of 31 for the days in October. A significant number of solutions were not presented to 3 significant figures with the probability stated to 3 decimal places. At this level, candidates should be able to differentiate between decimal places and significant figures for accuracy.

(c) Many found this question very challenging. The best solutions realised that the probability from **part (b)** was required, recognised that a binomial approximation was appropriate and evaluated accurately. Weaker solutions seemed to continue with the geometric approximation and found the probability that year 4 was the first to fulfil the success criteria. Many solutions used the binomial approximation with the original probabilities, and so found the probability that 8 October was wet in exactly 1 of the 4 years.

Question 6

Most solutions recognised that normal approximation was appropriate throughout the question. Many answers were presented in a clear, logical order with good communication of the processes and mathematical techniques used. The context was generally identified as continuous, with few attempts to use the continuity correction. The most successful solutions often included a sketch of the normal distribution curve to help identify the required probability area.

(a) The majority of candidates successfully constructed an appropriate standardisation calculation and found the anticipated *z*-value accurately. Many were able to use normal tables effectively to find the probability, but many did not interpret the success criteria accurately and found the complement of the required answer. The use of a simple sketch of the normal distribution curve was seen in many successful solutions.

(b) The best solutions initially found the anticipated *z*-value and formed an equation using the standardisation formula with the mean and standard deviation values substituted. There was a clear algebraic process presented to find the value of *t*. Weaker solutions used the *z*-value for the incorrect probability area. It was noted that candidates who did not use the tables often truncated rather than round their answers, resulting in accuracy errors. A number of solutions simply equated the standardisation formula to a probability which was not sufficient.

(c) This question was found challenging by many and was omitted by some candidates. The most efficient solutions correctly interpreted the 15 minutes as the numerator of the standardisation formula, had a simple sketch which identified that both tails of the normal distribution were excluded and found the required probability area using $2\Phi(1\ 5625) - 1$. Less efficient approaches involved determining the times that were 15.0 minutes from the mean and substituting into the normal standardisation formula with the mean. Many of these solutions were incomplete and only found the probability that the task was completed in less than 47 2 minutes. The weakest solutions misinterpreted the criteria and substituted 15 as the time into the normal standardisation formula. Again, simple diagrams were often seen in successful solutions.

Question 7

(a) Most graphs used scales which enabled points to be plotted accurately. The labelling of axes was also much improved, with units being included on the distance axis and the cumulative frequency axes also labelled. Almost all candidates correctly interpreted the data as being continuous and did not use a continuity correction when plotting points. Not starting the graph at (0, 0) was the main error. Candidates should be aware that a curve is required for a cumulative frequency graph, as line segments creates a cumulative frequency polygon. Plotting the graph at mid-points was uncommon.

(b) Where there was an appropriate graph in **part (a)**, most candidates provided an attempt at the interquartile range. As the question required the use of the graph, there needed to be a clear indication that values had been obtained from **part (a)**, with the best solutions having lines drawn from the quartiles to the curve.

(c) Although some excellent solutions were seen for this question, many candidates appear to have found the task challenging. The most efficient approach used was to create a data table which included the class mid-values (m) and frequencies (f) which was then used to generate mf and mf^2. These were then used to calculate the mean and variance, and then the standard deviation. Most solutions simply substituted values into the mean and variance formula, which was both repetitive and time-consuming. Common errors were the use of the upper boundary or class size rather than the mid value and using the cumulative frequency instead of the frequency. A small number of these solutions then summed these cumulative frequency values to find the total number of children, even though the question stated that there were 140 children in the data.

Candidates should be aware that class details can be presented in a variety of ways, and would be advised to practice interpreting the different formats accurately.

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Key messages

- Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. Where formulae are used, as in **Questions 3b**, **6c** and **7c**, the unsimplified formula with the correct substitutions should be shown before the final answer is presented. In **Question 5b** where different scenarios were being considered, the scenarios need to be clearly identified.
- Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates work to at least 4 significant figures throughout to justify a 3 significant figure value. Candidates are reminded that if an answer is exact, as in **Questions 1** and **6b**, they should not round the answer to 3 significant figures.
- Candidates are expected to be able to use the normal distribution tables and they are expected to know and use the critical values quoted in the formula booklet where appropriate, as in **Question 4c**.

General comments

Better responses within this paper demonstrated that many candidates were careful to read the questions carefully, made sure that they showed all their working, labelled their diagrams and ensured that their answers make sense within the context of the question. It is important that the final answer fits the context of the question, for example in **Question 3b** it was not sensible to give an answer of $\frac{761}{48}$ when the context is the time taken to travel to work. Similarly, those who did not draw a diagram in **Question 4c** and worked with a positive *z*-value often did not query the answer that put 95% of the data above a time greater than the mean.

Comments on specific questions

Question 1

This question was answered well by the majority of candidates, most appreciating that a combination was required and that they needed to subtract 3 from both 26 and 20 to find $^{23}C_{17}$. Many candidates also identified that since the answer obtained was exact it should not be rounded.

Question 2

(a) Most candidates appeared to know what was required when drawing a back-to-back stem and leaf diagram and followed the request for Lakeview to be represented on the left-hand side. Better responses showed the schools labelled correctly and took care to line up the leaves in order from the centre with no commas in between. The stem needed to be a column of single digits starting with the lowest number, 1, at the top. With so little data, there was no need to split the leaves for any part of the stem.

Most candidates realised the importance of including a key for both schools with the units, metres, clearly stated. It is good practice to use three different digits in the key to avoid confusion. Many used 0|1|8 from the top row but those who used the second or third row digits, 2|2|0 or 0|3|0, introduced some ambiguity.

(b) This question was well answered with most candidates correctly identifying the positions of the quartiles as 3rd and 9th, naming the lower quartile as 19 and the upper quartile as 32 and subtracting. The most frequent error was to think that the lower quartile was the arithmetic mean of the 3rd and 4th items and, similarly, that the upper quartile was the mean of the 9th and 10th.

Question 3

(a) A significant minority did not appear to understand that in a histogram the area of the bars represents the frequency and plotted bars of the required widths with the frequencies for heights. However, most candidates knew that a histogram required them to calculate frequency densities, most of those correctly dividing the given frequencies by the correct class widths, 5, 5, 10, 10, 20. A few worked with incorrect class widths, usually either too small by 0.5 or too large by 1, and a few others incorrectly divided by the midpoint or the upper bound.

Better responses used a sensible scale and plotted the tops of the bars in the correct. The most common scales used on the vertical axis were 4 or 5 units to 2 cm. Candidates are encouraged to clearly write down their calculated frequency densities to ensure they are plotting the correct values. Candidates are advised to use a sharp pencil to ensure their lines are clearly defined; some candidates used a thick or blunt pencil which often obscured the height at which they wished to draw the top of the bar.

Most candidates appreciated that the bar ends should be at 0, 5, 10, 20, 30 and 50 but those who incorrectly tried to plot them at 4.5, 9.5 etc. frequently ended up with a horizontal axis that was not to scale from 0 to 4.5. Most also knew to label their vertical axis with ‘frequency density’ and ‘time in minutes’ on the horizontal axis. Some candidates omitted axes labels, which are essential to given meaning to their graph.

(b) Most candidates used the formula for an estimated mean appropriately and so long as they were using the correct class boundaries to find the mid-points were able to produce the correct answer. The mid-point attempts needed to be clearly substituted into the correct formula, with many candidates giving the final quotient only which was not sufficient. Presenting the final answer as $\frac{761}{48}$ was not appropriate in the context of the question, since it asked to find an estimate of the mean time taken by an employee to travel to work, with time conventionally presented as a decimal or a mixed number.

Question 4

(a) This question was answered well with most candidates correctly standardising 43.2 to find a *z*-value of $0.\dot{5}$. The most frequent error was to truncate the value to 0.555 rather than rounding, giving these candidates the final probability of 1 – 0.7105 rather than the required 1– 0.7108. Although very few drew a diagram to find the correct tail, most did find an appropriate area less than 0.5.

(b) Most candidates multiplied a probability by 365 and found the probability by subtracting their answer to **part (a)** from 1. However, many used a probability rounded to 3 significant figures, forgetting that to obtain an answer correct to 3 significant figures they need to use input numbers correct to 4 or more significant figures. Others either gave a decimal answer, used an approximation sign, or stated the answer correct to 3 significant figures and did not follow the instruction to find an estimate for the number of days, which required an integer as the final answer, which could have been either side of the decimal result.

(c) The majority of candidates used the inverse normal distribution to find thfe required *z*-value with only a small number using the tables the wrong way round. Candidates are required to be familiar with the tables provided and to use the critical values for the normal distribution where appropriate. Better responses, and especially those who used a diagram, found a negative *z*-value and usually obtained the correct final answer. Those who used a positive *z*-value did not seem to identify that their final answer, usually 47.1, was greater than the mean which should have flagged that an error had occurred.

Question 5

(a) Those who understood that the order of the letters and digits in the code mattered, knew to use permutations and usually obtained the correct answer. A significant number of candidates, however, used combinations and gave the answer ${}^5C_2 \times {}^7C_4 = 350$.

(b) Candidates found this to be one of the most challenging questions on the paper. The most efficient method was to subtract the number of codes that had neither A nor 5 from the total found in **part (a)**. This method was seen in only a minority of scripts.

The most frequently attempted method was to attempt identifying the scenarios and to add the three subtotals for the three identified scenarios, A and no 5; 5 and no A; A and 5. However, many candidates seemed to confuse the scenarios, and these were generally poorly described; most not being clear about whether 'With A' meant 5 to be included or not. This confusion meant that some obtained the correct totals for Method 3 in the mark scheme but added them all instead of subtracting the number with A and 5. Others did not appear to realise that the order mattered and hence all the sub-totals were incorrect with some candidates not describing the scenarios or explaining what they were trying to do, providing only values.

(c) Candidates also found this question to be particularly challenging. The most frequently seen method was to divide the number of successful codes by the total found in **part (a)**. However, many found it difficult to find the correct number of ways, often not realising that the hundreds digit could be 5, 6 or 7. Some found the correct number of successful codes but did not divide and make it into a probability.

Those who chose the probability method were often less successful. While a number of candidates realised that the probability of DE was $\frac{1}{20}$ they had difficulty finding the probability of a number between 4500 and 5 000, often forgetting the three possibilities for the hundreds digit again and multiplying $\frac{1}{7}$ by $\frac{1}{6}$ instead of $\frac{3}{6}$.

Question 6

(a) Almost all candidates were able to find the two required equations from summing the probabilities to 1 and correctly using the formula for E(X). A few candidates misunderstood how to calculate the E(X), sometimes dividing by 6, while others did not sum the probabilities to 1. Candidates should ensure they show their method when solving the two equations with the minimum requirement being to rearrange the equations into the same linear form, preferably with the unknowns on one side of the = sign and the number on the other side.

(b) This question was answered well with most candidates knowing how to use the variance formula with only a small number not subtracting the square of the mean.

However, some did not realise that their solution here was exact, the variance in this question being exactly equal to 0.6475 or $\frac{259}{400}$. Many candidates rounded their answer to 3 significant figures without showing this exact value at all; it is good practice to initially write an answer to more than 3 significant figures before rounding.

(c) This question was generally answered well. Most candidates recognised that it required use of the binomial distribution with the value of p from **part (a)** and $n = 12$ although a number omitted the ${}^nC_r$ element in the terms. Some candidates interpreted the wording of 'at least 3' to mean 'less than 3', 'more than 3' or 'exactly 3'. Others expressed the intention to subtract the P(fewer than 3) from 1 but then did not actually do this. Candidates should ensure they show the unsimplified binomial terms in their working.

(d) Candidates found this question to be particularly challenging. Of the candidates who recognised the need for the geometric distribution many often did not use the correct probability, choosing to use the value of p again rather than P(3) from the table in **part (a)**. Others did the correct calculation but rounded the initial answer to 3 decimal places instead of 3 significant figures.

Question 7

(a) Those who understood what was required in this question generally obtained the correct probabilities. Some ignored the requirement 'in terms of *x*' and filled in numerical answers after calculating them in **part (c)**. Others did not appear to have read all of the information, mainly omitting that the ball chosen from box *A* was 'placed in box *B*' and used the denominator $x + 9$ in all four of their probabilities.

(b) For those who had found the correct probabilities in **part (a)** this was relatively straightforward. Many of those who had not answered **part (a)** correctly tried to work backwards to find the P(blue in box *B*) that made the probability of both balls being blue correct. However, it was important that they showed a product that was consistent with their tree diagram.

(c) This question was generally answered well. Almost all candidates equated the answer to **part (c)** to $\frac{1}{6}$ and worked out that x equalled 14. Some stopped at this point, but most went on to attempt use of the conditional probability formula using $x = 14$. Some candidates showed little or no working in obtaining their final answer, it is important that clear method is shown throughout. The product used in the numerator and the sum of products in the denominator needed to be shown before the final answer. Candidates who had made errors in their tree diagram but who showed their working clearly were able to gain the relevant method and follow through marks.

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Paper 6 Probability & Statistics 2

Key messages

For questions that require several probability terms it is necessary to show all of the terms in full. In **Question 5(b)** it was necessary to write down all of the required binomial terms as well as the probability sum.

Final answers are generally required to be given to 3 significant figures, unless otherwise specified in a question. Candidates are reminded to ensure they round their final answer appropriately, ensuring that significant figure accuracy is used, rather than decimal places. To ensure the accuracy of the final answer it is important that candidates maintain a 4 significant accuracy throughout their working.

General comments

There were many clear and well explained answers to all questions throughout the question paper.

Comments on specific questions

Question 1

(a) The question required the distribution of the sample mean of the heights. This was a normal distribution with mean 12.5 and variance 0.4096.

(b) The requested probability was for a region symmetrically placed about the mean. Candidates who did not notice this could still find the probability by standardising twice, using the parameters found for the distribution of the sample mean. It was necessary to deal with the Φ values. A diagram could help candidates with this.

Question 2

A suitable approximating distribution to the given Poisson distribution Po(45.2) was the normal distribution N(45.2, 45.2). To find the probability that the customer service desk could not deal with all the enquiries in a day required the use of a continuity correction, 60.5, in the standardisation. Many candidates applied this correctly. Some candidates incorrectly used 60 or 59.5. Candidates then needed to subtract the corresponding *z*-value from 1 since the smaller tail probability was needed for the final answer.

Question 3

In answering this question candidates initially needed to find the *z*-value, 1.754, corresponding to the given confidence interval width by using the variance of proportions, $\frac{0.2\times0.8}{75}$. Secondly candidates needed to find the value of *α* corresponding to this z-value by using Φ – (1 –Φ) or by using 2Φ –1 or equivalent. In this question an answer given to 2 significant figures was requested. Some candidates found only the *z*-value and the Φ value, 0.9603, and incorrectly gave 96 as their answer for *α*. Other candidates incorrectly included *p* in their calculation with the given width.

Question 4

(a) To find the probability it *was* necessary to integrate f(x) between the limits 0 and 1.2. It was essential to clearly write down the integration result. Many candidates did all of this correctly. A few candidates used limits 1.2 to 3 but omitted to subtract their result from 1, which would have obtained the correct answer.

(b) To find E(X) it was necessary to integrate xf(x) between the limits 0 and 3. It was essential to clearly write down the integration result. Some candidates omitted the x or incorrectly multiplied by x^2. Other candidates integrated incorrectly.

(c) It was necessary to integrate f(x) between the limits 0 and m and to equate the result to 0.5. It was essential to clearly write down the integration result and then to show sufficient steps in reaching the given cubic equation. Many candidates did all of this correctly.

Question 5

(a) (i) The suitable approximating distribution for the given binomial distribution was the Poisson distribution with parameter 0.025 which could be expressed as Po(0.025). The justification involved the value of the sample size, n = 2500, and the value of np, 0.025. These two values had to be related to the critical values 50 and 5. Many candidates did not complete this justification fully. Some candidates incorrectly gave their answer as the binomial distribution. Other candidates incorrectly suggested a normal distribution.

(ii) Many candidates answered this question correctly by finding 1 – P(0) using the Poisson distribution. The answer required 3 significant figures and not just 3 decimal places which was often seen.

(b) The significance test required the hypotheses to be stated, the relevant probability to be found, the comparison to be shown and the conclusion to be stated. A one-tailed test was appropriate and the tail required the probability P($x \leqslant 4$) to be found using the binomial distribution B(28, 0.3). It was necessary to write down the binomial terms as well as the sum. Many candidates found this sum of probabilities correctly as 0.0474. Some candidates instead found P($x \leqslant 3$), though some follow through marks were available. A few candidates found only P(x = 4) which was not a valid test. It was necessary to show the comparison of the probability with 0.02 for the 2% significance level. Finally, the conclusion needed to be stated in the context of the question in a non-definite form with no contradictions. From the wording of the question the context could refer to the researcher's suspicion or to the percentage of the people having the medical condition being less than 30%. Many candidates answered this question very efficiently and accurately.

Question 6

(a) Most candidates found the unbiased estimates correctly, showing the substitution of the given sample values into the formulae accurately.

(b) It was necessary to create a new variable such as $S – T$ and to compare this with 0.1 seconds. For this variable the expectation was 0.2 and the variance was 0.5564. After standardising the given value 0.1 the required probability could be found. A diagram could be helpful in deciding which was the relevant area for the probability. Other new variables such as $S – T$ – 0.1 could be used. Some candidates made errors in finding the variance, such as subtracting Var(S) and Var(T) or including the 0.1 in the variance calculation. Other candidates did not attempt to find a new variable and so could make little relevant progress. Some candidates incorrectly gave the smaller area as their probability.

Question 7

(a) To carry out the significance test it was necessary to use the normal distribution of means of samples $\mathrm{N}\left(64.6, \frac{5.2^2}{100}\right)$. The mean mass, 63.5, of the sample of the 100 apples could then be standardised, the comparison with the significance level carried out and the conclusion made. The hypotheses needed to be for a one-tailed test. For the 2.5% significance level the critical z-value was 1.96 and the comparison using z-values was $-2.115 < -1.96$. Using the corresponding probabilities gave $0.0172 < 0.025$. These values indicated a significant result. Many candidates followed these steps accurately and produced the correct conclusion. The conclusion needed to be stated in the context of the question in a non-definite form with no contradictions. Some candidates lost some accuracy in their final answer by rounding prematurely, particularly when changing to the probability approach. Other candidates used an incorrect critical z-value. Alternatively, some candidates used the critical value method leading to $63.5 < 63.58$. This work could then be used in **part (b)** as well.

(b) To find the probability of a Type II error it was necessary to find the acceptance region for H_0 using the normal distribution $N\left(64.6, \frac{5.2^2}{100}\right)$. This value could then be used in the normal distribution $N\left(62.7, \frac{5.2^2}{100}\right)$. A few candidates completed this work successfully. Many candidates did not find the acceptance region and attempted only the second part of the method, for which some credit could be gained. Some candidates tried to follow the steps for a significance test in their attempt here; this was not appropriate. Two diagrams or a combined diagram could be helpful when answering this question.

MATHEMATICS

Paper 9709/62
Paper 6 Probability & Statistics 2

Key messages

- Candidates are advised to always read the question carefully.
- All answers must be fully supported with the relevant working.
- Candidates should round, rather than truncate, figures during working. Candidates should maintain a 4 significant figure accuracy throughout their working, and only round the final answer to 3 significant figures, or the degree of accuracy specified in the question.
- When writing conclusions to hypotheses tests the answer must be given in context and there must be a level of uncertainty in the language used.
- When carrying out a significance test the comparison between the test value and critical value (or equivalent comparison) must be clearly shown in order to justify the conclusion drawn.

General comments

Many candidates found some questions on this paper particularly challenging. This was specifically true for **Questions 3b**, **4a**, **7b** and much of **Question 6**. Questions that candidates found to be more straightforward and generally answered well were **Questions 1a**, **2a**, **5a** and **7a**.

This paper, therefore, generated a complete spread of marks; there were some good marks awarded, but also there were many cases where candidates appeared unprepared for the demands of the paper.

In general, timing did not appear to be a problem for candidates.

The comments below indicate common errors and misconceptions, however, there were also some full and correct solutions presented too.

Comments on specific questions

Question 1

(a) Many candidates successfully found the mean, though attempts at the unbiased estimate of the variance were varied; the most common error was the confusion between the two alternative formulae for the unbiased estimate of the variance. Calculation of the biased estimate was occasionally seen, and also the answer of 0.0056859 was often truncated to 0.00568, rather than correctly rounded to 0.00569.

(b) Many candidates successfully found the mean but attempts at the variance were not done as well. Errors included calculating 11 × their variance from **part (a)** and sometimes adding 0.5, or correctly using 11^2 × their variance but then incorrectly adding 0.5.

Question 2

(a) This question was well answered by the majority of candidates. Many candidates made a good attempt to explain why the given method to choose the sample was unsatisfactory. Many correctly identified that either the location or time led to the exclusion of many candidates, or some correctly identified that the 6 might have similar preferences due to the restrictive method. Some candidates made irrelevant comments about the size of the sample and some merely suggested it was ‘unrepresentative’ without explaining why.

(b) This part was not as well answered as **part (a)**. Some candidates did not appear to know what was required and, of those who did, many forgot to discard any repeated candidate numbers (for example 204) or to discard candidate numbers greater than 256. There were many partially correct rather than fully correct attempts.

Question 3

(a) This was a reasonably well attempted question. The confidence interval was correctly found by many candidates. Errors included use of an incorrect *z* value and attempting to centre the interval on 35 rather than $\frac{35}{140}$.

(b) It is important that candidates, as well as being able to find a confidence interval, also understand what a confidence interval is. Few recognised the fact that, for example, a 95 per cent confidence interval has a 0.95 chance of containing the true value of *p*. Thus, the probabilities required were 0.9, 0.95 and 0.99 which then needed to be combined correctly so that 2 out of the 3 intervals contained the true value of *p*. Many candidates used a probability of 0.25 or attempted to use *z* values or did irrelevant calculations finding further confidence intervals.

Question 4

(a) It was important for candidates to read the question carefully here to fully understand the scenario given. Many candidates gave answers relating to cost or time. Many did not identify that testing destroys the fireworks, and therefore it is necessary to take a sample rather than using all the fireworks, as testing the whole population would leave no fireworks left.

(b) This part was reasonably well attempted. Hypotheses were usually given, though not always fully correctly, and standardising was done well. It is important that the comparison is clearly shown and a conclusion is drawn that is in context and not definite. Errors here included use of an incorrect *z* value and giving a conclusion that did not have the correct level of uncertainty in the language used; for example, 'the mean time lasted is not less than 30 seconds' would not be acceptable as it is a definite statement.

(c) Few candidates explained the need for the Central Limit Theorem (CLT) fully correctly. Many did not make it clear that it was the population distribution that was unknown, stating that the distribution (or 'it') was unknown (or not known to be normal) was not enough. Some candidates thought the CLT was not needed and others gave statements relating to the size of the sample.

Question 5

(a) In general, this part was well answered. Many candidates correctly used a Poisson distribution here, though not all used the correct value of lambda; the value of 0.2 rather than 2 was often used. Another error seen was to calculate less than or equal to 3, or just equal to 3, rather than fewer than 3 as requested. Some candidates used an incorrect binomial distribution.

(b) Not all candidates used the correct approximating distribution N(40,40). Other normal distributions were used, and some credit could be gained in this instance for standardising. Many did not use, or used an incorrect, continuity correction.

(c) Many candidates used a Poisson distribution but with a mean of 6 rather than 10 (i.e. only considering the formatting errors rather than the formatting and typing errors as requested); it is important for candidates to read the question carefully in order to correctly interpret what is required. Other errors included calculating just 9 and 10, omitting to note the requirement 8 to 11 'inclusive', and other distributions e.g. normal and binomial) were incorrectly used. It was important that all supporting working was clearly shown.

Question 6

Candidates found this question, in general, to be particularly challenging and generally was not well answered. It is important that candidates know how to find a rejection region and understand the meaning of Type I and Type II errors.

(a) Many candidates correctly defined their null and alternative hypotheses in answering this part.

(b) To find the rejection region P(0) and P(0,1) using B(30,0.1) needed to be calculated. Candidates should have noted that P(0) was less than 0.1 and P(0,1) was greater than 0.1 thus concluding that the rejection region was 0. Some candidates used B(30,0.1) but calculated P(0) and only P(1), others calculated different point probabilities rather than cumulative probabilities, and some did not show any comparison with 0.1. A significant number of candidates gave unsupported answers and a common error was to give the answer (a rejection region) as a probability. Many candidates used incorrect distributions.

(c) A common incorrect answer of 0.1 was often seen here, though a few candidates, mainly those who had been successful in **part (b)**, stated the correct answer.

(d) This part was generally not well attempted. The Type II error was found by calculating 1 – P(0) using the distribution B(30, $\frac{1}{40}$). A large number of candidates used an incorrect binomial distribution, often B(40, $\frac{1}{40}$) or Bin(40, $\frac{1}{10}$), and others incorrectly used a normal or Poisson distribution.

(e) It was important that this was answered in the context of the question, many candidates did not do this. Generic definitions of a Type II error were not acceptable, and those that gave statements such as 'saying the probability is less than $\frac{1}{10}$ when it is not' were not fully correct, the contextual requirement was to say '… when it is actually less than $\frac{1}{10}$'.

Question 7

(a) (i) This was well attempted with the majority of candidates giving a full and credible solution. It is important in a 'show that' question that steps in the working are all fully shown.

(ii) This part was also well attempted with most candidates attempting to integrate xf(x) using the correct limits. Errors included omitting 'k' and calculation errors were made when substituting the limits.

(b) Candidates generally found this part to be very challenging. Although there were candidates who attempted to draw a diagram of a suitable probability density function, few candidates labelled their diagrams fully. Some candidates attempted to find a value for a (a = 1 being common), others worked with $\phi(0.2)$ trying incorrectly to fit a normal distribution to the information given, and some attempted to integrate various functions, for example y = x or the function given in **part (a)**. Many did not seem sure of what method to use, not realising that use of a diagram, as directed, and symmetry led to the solution.

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Paper 6 Probability & Statistics 2

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