Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME											
CENTRE NUMBER							CANDIDATE NUMBER				

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FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2017**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **24** printed pages.

JC17 06_9231_13/RP

CAMBRIDGE International Examinations

[Turn over

1 The roots of the cubic equation $x^3 + 2x^2 - 3 = 0$ are α, β and γ.

(i) By using the substitution $y = \dfrac{1}{x^2}$, find the cubic equation with roots $\dfrac{1}{\alpha^2}$, $\dfrac{1}{\beta^2}$ and $\dfrac{1}{\gamma^2}$. [3]

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(ii) Hence find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2}$. [1]

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(iii) Find also the value of $\dfrac{1}{\alpha^2\beta^2} + \dfrac{1}{\beta^2\gamma^2} + \dfrac{1}{\gamma^2\alpha^2}$. [1]

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2 **(i)** Verify that $\dfrac{2r+1}{r(r+1)(r+2)} = \dfrac{1}{2}\left\{\dfrac{(2r+1)(2r+3)}{(r+1)(r+2)} - \dfrac{(2r-1)(2r+1)}{r(r+1)}\right\}$. [2]

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(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2r+1}{r(r+1)(r+2)} = \frac{1}{2}\left\{\frac{(2n+1)(2n+3)}{(n+1)(n+2)} - \frac{3}{2}\right\}$. [2]

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(iii) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{2r+1}{r(r+1)(r+2)}$. [2]

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3 Prove, by mathematical induction, that $\sum_{r=1}^{n} r \ln\left(\frac{r+1}{r}\right) = \ln\left(\frac{(n+1)^n}{n!}\right)$ for all positive integers n. [6]

4 A curve C has equation $x^3 - 3xy + y^2 = 4$. Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 2)$ of C. [7]

5 A curve C has parametric equations

$$x = \tfrac{2}{5}t^{\frac{5}{2}} - 2t^{\frac{1}{2}}, \quad y = \tfrac{4}{3}t^{\frac{3}{2}}, \quad \text{for } 1 \leqslant t \leqslant 4.$$

(i) Find the exact value of the arc length of C. [5]

(ii) Find also the exact value of the surface area generated when C is rotated through 2π radians about the x-axis. [3]

6 Let I_n denote $\int_0^2 (4+x^2)^{-n}\,dx$.

(i) Find $\frac{d}{dx}\left(x(4+x^2)^{-n}\right)$ and hence show that

$$8nI_{n+1} = (2n-1)I_n + 2\times 8^{-n}.$$ [5]

(ii) Use the result for integrating $\dfrac{1}{x^2 + a^2}$ with respect to x, in the List of Formulae (MF10), to find the value of I_1 and deduce that

$$I_3 = \tfrac{3}{1024}\pi + \tfrac{1}{128}.$$ [5]

7 **(i)** Use de Moivre's theorem to prove that

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}.$$ [5]

(ii) Hence find the solutions of the equation

$$t^4 - 4t^3 - 6t^2 + 4t + 1 = 0,$$

giving your answers in the form $\tan k\pi$, where k is a rational number. [5]

8 Find the solution of the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 18t^2 + 6t + 1,$$

given that, when $t = 0$, $x = 3$ and $\frac{dx}{dt} = 0$. [10]

9 The plane Π_1 passes through the points $(1, 2, 1)$ and $(5, -2, 9)$ and is parallel to the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(i) Find the cartesian equation of Π_1. [4]

The plane Π_2 contains the lines

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}).$$

(ii) Find the cartesian equation of Π_2. [4]

(iii) Find the acute angle between Π_1 and Π_2. [3]

10 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -8 & 7 \\ 7 & -9 & 7 \\ 6 & -6 & 5 \end{pmatrix}.$$

(i) Given that $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector of **A**, find the corresponding eigenvalue. [2]

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(ii) Given also that -1 is an eigenvalue of **A**, find a corresponding eigenvector. [2]

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(iii) It is given that the determinant of **A** is equal to the product of the eigenvalues of **A**. Use this result to find the third eigenvalue of **A**, and find also a corresponding eigenvector. [3]

(iv) Write down matrices $\mathbf{P}$ and $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix, and hence find the matrix $\mathbf{A}^n$ in terms of n, where n is a positive integer. [6]

11 Answer only **one** of the following two alternatives.

EITHER

A curve C has polar equation $r = 2a\cos\left(2\theta + \frac{1}{2}\pi\right)$ for $0 \leqslant \theta < 2\pi$, where a is a positive constant.

(i) Show that $r = -2a\sin 2\theta$ and sketch C. [4]

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(ii) Deduce that the cartesian equation of C is

$$(x^2 + y^2)^{\frac{3}{2}} = -4axy. \qquad [2]$$

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(iii) Find the area of one loop of C. [5]

(iv) Show that, at the points (other than the pole) at which a tangent to C is parallel to the initial line,

$$2\tan\theta = -\tan 2\theta.$$ [3]

OR

The matrix $\mathbf{A}$, given by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 2 \\ 3 & -1 & 4 & 0 \\ 5 & -8 & -6 & 19 \\ -2 & 3 & 2 & -7 \end{pmatrix},$$

represents a transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$.

(i) Find the rank of $\mathbf{A}$ and show that $\left\{ \begin{pmatrix} 2 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for the null space of the transformation. [6]

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(ii) Show that if

$$\mathbf{Ax} = p\begin{pmatrix} 1 \\ 3 \\ 5 \\ -2 \end{pmatrix} + q\begin{pmatrix} -1 \\ -1 \\ -8 \\ 3 \end{pmatrix},$$

where p and q are given real numbers, then

$$\mathbf{x} = \begin{pmatrix} p + 2\lambda + \mu \\ q + 2\lambda + 3\mu \\ -\lambda \\ \mu \end{pmatrix},$$

where λ and μ are real numbers. [2]

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(iii) Find the values of p and q such that

$$p\begin{pmatrix}1\\3\\5\\-2\end{pmatrix}+q\begin{pmatrix}-1\\-1\\-8\\3\end{pmatrix}=\begin{pmatrix}3\\7\\18\\-7\end{pmatrix}.$$ [3]

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(iv) Find the solution of the equation $\mathbf{Ax}=\begin{pmatrix}3\\7\\18\\-7\end{pmatrix}$ of the form $\mathbf{x}=\begin{pmatrix}4\\9\\\alpha\\\beta\end{pmatrix}$, where α and β are positive integers to be found. [3]

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