FURTHER MATHEMATICS

Paper 9231/11
Paper 11

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth. They should take note of where exact answers are required.
- Candidates should ensure that any sketch graphs are fully labelled and carefully drawn to show behaviour at limits.
- Candidates should use the standard names for transformations and know what is required to describe each one.
- When dividing by a factor, candidates should consider the possibility that the factor is zero.

General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of transformations. There was no evidence that candidates were short of time.

Comments on specific questions

Question 1

(a) Most candidates realised that the first transformation to be applied is the second to be written. They recognised $\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ represented a one-way stretch but the shear represented by $\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$ was less familiar.

(b) Most candidates knew the method for writing down the inverse of a 2×2 matrix. Some candidates did not realise that it was the inverse matrix they were being asked to find.

(c) Many candidates realised that the determinant of M is the effect on area and could therefore write down $a = 2$. Being given an invariant line was less familiar than being asked to find it, but many were able to write down the appropriate transformation of a point on the line and use the fact that it gave a point of the same line.

Question 2

(a) This part was done to a very high standard with very few errors.

(b) This was done to a high standard, with correct partial fractions found. Most candidates wrote enough complete terms to justify cancellation.

(c) This part almost always correctly followed from **(b).**

Question 3

(a) Two main methods were used here. One was to first use the substitution $x = y^{\frac{1}{3}}$ and then to eliminate the cube root from the equation. The other was to manipulate the equation first to ensure that all terms involved x^3 and its powers and then replace x^3 by y. They were equally successful, with the necessary cubing of a binomial expression being accurately done.

(b) The strongest candidates realised that the answer was the sum of the squares of the roots of the new equation. Problems occurred when candidates tried to return to the original equation or used confusing notation.

(c) The evaluation of the determinant was usually accurate and 'singular' was understood. Some common sign errors eliminated the $\alpha^3\beta^3\gamma^3$ term and with it one of the connections with **(a)** leading to an incorrect value for c.

Question 4

Candidates were clearly familiar with the methods required and there were few errors apart from numerical ones. Candidates should be advised to check carefully that vectors calculated at the start of a question are correct, to avoid errors multiplying as the question progresses.

(a) Almost all candidates used the vector product to find the normal to the plane.

(b) The method of dividing the constant term of the plane by the modulus of the normal vector was well known and accurately applied.

(c) This was usually done well, with any errors being the result of earlier mistakes in arithmetic.

Question 5

Good solutions showed that the base case was correct by examining the formula and by also differentiating the expression. Candidates then expressed the assumption clearly and wrote down the expression under consideration. This helped to make it clear that two stages of differentiation would be required, and stronger candidates showed whether they were working on the first or second of these steps. There were many clear expressions of the conclusion.

Question 6

(a) The vertical asymptote was easily written down and most candidates knew how to find the oblique one. It was rare to see this not written as an equation.

(b) The majority of candidates used the method of using the discriminant of a quadratic to find the values of y where there are no real values of x to correspond. The strongest candidates explained clearly what they were doing and made it clear whether the inequality represented the region for which x does exist or the region for which it does not.

A small number of candidates attempted to use calculus methods. Finding the local maximum and local minimum is not sufficient to solve the problem. Proper reference to the shape of the graph and to the relative values of the maximum and minimum would be needed to produce a complete argument.

(c) Many graphs were carefully drawn, with a ruler used for lines and with all necessary labels. The basic shape was usually correct. The coordinates of the intersections with the axes were usually correct, although a few candidates omitted this part.

(d) There were some very good graphs, showing both branches and the correct behaviour towards infinity. It was rare to see an attempt at a smooth curve rather than the required sharp change of direction when the curve met the x axis.

Question 7

(a) The relationships between polar and Cartesian coordinates were well known and applied correctly. Some candidates made the solution more complicated by multiplying out brackets and having to recombine terms. The strongest candidates recognised directly the expressions for sin2θ and cos2θ and could then combine them to give the required result.

(b) The general shape was usually correct. Cartesian axes were frequently added to the sketch, rather than just the initial line, and rays marked with angles as a guide for drawing. The question asked for the equation of the line of symmetry and this was not always given.

(c) The majority of candidates were able to recall and apply the formula for finding the area of a polar curve and use the double angle formula correctly.

(d) Some candidates attempted to find the maximum distance from the initial line, rather than the line $\theta = \frac{\pi}{2}$

Using the given identity produced an expression in both sinθ and cosθ and some candidates spent considerable effort to express it in terms of one of these, possibly introducing errors. The strongest candidates differentiated using the product rule. The resulting equation in both sinθ and cosθ was most effectively solved by converting it to a quadratic in $\tan^2\theta$, although the possibility that cosθ = 0 should have been considered. There were a few very good, accurate solutions.

FURTHER MATHEMATICS

Paper 9231/12
Paper 12

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth. They should take note of where exact answers are required.
- Candidates should ensure that any sketch graphs are fully labelled and carefully drawn to show behaviour at limits.
- Candidates should use the standard names for transformations and know what is required to describe each one.
- When dividing by a factor, candidates should consider the possibility that the factor is zero.

General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of transformations. There was no evidence that candidates were short of time.

Comments on specific questions

Question 1

(a) Very few candidates made a sign error in writing down d.

(b) The most efficient solutions came from using the substitution $x = y - 1$. Some candidates returned to the original equation and used its root relationships to find the required coefficients for the new equation. This could become very complicated and risked introducing errors. In either case, the most common error was to forget to use $d = 1$ from **(a)**

(c) Candidates were asked to prove a given result and had to ensure that all necessary steps were clearly shown. Those candidates who wrote down the three relationships between the coefficients in **(b)** and the roots were far more likely to have correct signs and to make the structure clear. Then using the given relationship $\gamma + 1 = -\alpha - 1$ would simplify the expressions and make the required result quickly obtainable. Those candidates who tried to multiply brackets, or to substitute directly, found it difficult not to lose terms or signs.

Question 2

Most candidates showed good knowledge of the structure of an induction proof, though some did not communicate all the steps clearly. Sometimes the proposition was assumed for every integer rather than for $n = k$.

The inductive step was proved either by manipulating $7^{2(k+1)}$ or by considering the difference $7^{2(k+1)} - 7^{2k}$ and showing that it was divisible by 12. In this case candidates often neglected to move forward by using the assumption to show that $7^{2(k+1)}$ must then be divisible by 12.

Question 3

(a) The structure of the given expression as the difference of two squares was widely recognised and the required result usually justified.

(b) Most candidates used the result in **(a)** to express u_n as the difference of two non-fractional expressions, although a few candidates took the opposite approach and worked throughout with the reciprocals. Care needed to be taken to write sufficient complete terms to see and justify the cancellations.

(c) Only the strongest candidates recognised that for convergence to be possible, $|x| < 1$. Where the sum to infinity was attempted, it often omitted the limit of the term involving N.

Question 4

(a) Reflection was usually identified, although the equation of the axis was not always given.

(b) The rotation was not always described fully.

(c) The usual method was to calculate the matrix **AB** and to use the determinant to show that the area was unchanged. Only a few candidates used an argument considering the separate transformations, or the determinants of the individual matrices.

(d) Many candidates calculated **AB** rather than its inverse.

(e) The method for finding invariant lines was well understood and accurately applied. Problems arose for candidates whose notation did not distinguish between the point and its image properly, and also for those who tried to work with $y = mx+c$ without realising that $c = 0$ because the line passes through the origin.

Question 5

(a) There were some accurate sketches, although the behaviour as the curve approaches the pole was not always correct. Candidates should ensure that they state polar coordinates in the right order.

(b) Limits caused many problems in this question. When using a substitution, the limits should also be changed if later problems are to be avoided. When integrating $(\ln u)^2$ two main methods were used. Either using $\ln u \times \ln u$ or $1\times (\ln u)^2$ will require two integrations by parts. Many candidates completed both stages correctly. As the answer was given, all steps needed to be completed accurately, including the limits, to justify the result.

(c) Some candidates attempted to find the maximum distance from the line $\theta = \frac{\pi}{2}$, rather than the initial line.

When the product $\ln(1+\pi-\theta)\sin\theta$ is differentiated it needs to be divided by $\cos\theta$ to give the equation in the question. The possibility that $\cos\theta = 0$ was rarely considered. There were a few very good, accurate solutions.

Locating the root was successfully attempted by almost all candidates.

Question 6

(a) The asymptotes and the basic shape of the graph were usually correct.

(b) (i) There was a variety of methods for differentiating: quotient rule, product rule or separating into fractions before differentiating. Few candidates used the form of C_2 as given but expanded the equation first, involving extra work and losing sight of the factor $(x – 2a)$ which should have been rejected.

(ii) Many candidates correctly found both points of intersection. Some did not use both possible signs for $\frac{x-a}{x-2a}$ and others tried to square and expand both sides. This led to great complication.

A particularly elegant method was to replace $\frac{x-a}{x-2a}$ by z, and so $z^2 = |z|$ which has solutions 0, +1, –1 and from which the intersections follow easily.

(c) This graph was rarely drawn with sufficient attention to detail. C_2 and C_3 had two branches each and two points of intersection, so labelling had to apply to each section. The shape of each graph at the intersection with the x axis needed to be carefully considered and often the approach to the asymptotes was poorly handled.

The strongest candidates managed to write down the solution for the inequality correctly.

Question 7

Candidates were clearly familiar with the methods required and there were few errors apart from numerical ones. Candidates should be reminded to carefully check that vectors calculated at the start of a question are correct, to prevent errors multiplying as the question progresses.

(a) Almost all candidates used the vector product to find the normal to the plane.

(b) The only problems here were errors in calculating the vectors lying in the planes, and some wrong signs in cross products.

(c) The formula for shortest distance was well known and correctly used. An inaccurate final answer was usually the result of earlier errors in arithmetic.

FURTHER MATHEMATICS

Paper 9231/13
Paper 13

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth. They should take note of where exact answers are required.
- Candidates should ensure that any sketch graphs are fully labelled and carefully drawn to show behaviour at limits.
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General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of transformations. There was no evidence that candidates were short of time.

Comments on specific questions

Question 1

(a) Most candidates realised that the first transformation to be applied is the second to be written. They recognised $\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ represented a one-way stretch but the shear represented by $\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$ was less familiar.

(b) Most candidates knew the method for writing down the inverse of a 2×2 matrix. Some candidates did not realise that it was the inverse matrix they were being asked to find.

(c) Many candidates realised that the determinant of M is the effect on area and could therefore write down $a = 2$. Being given an invariant line was less familiar than being asked to find it, but many were able to write down the appropriate transformation of a point on the line and use the fact that it gave a point of the same line.

Question 2

(a) This part was done to a very high standard with very few errors.

(b) This was done to a high standard, with correct partial fractions found. Most candidates wrote enough complete terms to justify cancellation.

(c) This part almost always correctly followed from **(b).**

Question 3

(a) Two main methods were used here. One was to first use the substitution $x = y^{\frac{1}{3}}$ and then to eliminate the cube root from the equation. The other was to manipulate the equation first to ensure that all terms involved x^3 and its powers and then replace x^3 by y. They were equally successful, with the necessary cubing of a binomial expression being accurately done.

(b) The strongest candidates realised that the answer was the sum of the squares of the roots of the new equation. Problems occurred when candidates tried to return to the original equation or used confusing notation.

(c) The evaluation of the determinant was usually accurate and 'singular' was understood. Some common sign errors eliminated the $\alpha^3\beta^3\gamma^3$ term and with it one of the connections with **(a)** leading to an incorrect value for c.

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(b) The method of dividing the constant term of the plane by the modulus of the normal vector was well known and accurately applied.

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Question 5

Good solutions showed that the base case was correct by examining the formula and by also differentiating the expression. Candidates then expressed the assumption clearly and wrote down the expression under consideration. This helped to make it clear that two stages of differentiation would be required, and stronger candidates showed whether they were working on the first or second of these steps. There were many clear expressions of the conclusion.

Question 6

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(b) The majority of candidates used the method of using the discriminant of a quadratic to find the values of y where there are no real values of x to correspond. The strongest candidates explained clearly what they were doing and made it clear whether the inequality represented the region for which x does exist or the region for which it does not.

A small number of candidates attempted to use calculus methods. Finding the local maximum and local minimum is not sufficient to solve the problem. Proper reference to the shape of the graph and to the relative values of the maximum and minimum would be needed to produce a complete argument.

(c) Many graphs were carefully drawn, with a ruler used for lines and with all necessary labels. The basic shape was usually correct. The coordinates of the intersections with the axes were usually correct, although a few candidates omitted this part.

(d) There were some very good graphs, showing both branches and the correct behaviour towards infinity. It was rare to see an attempt at a smooth curve rather than the required sharp change of direction when the curve met the x axis.

Question 7

(a) The relationships between polar and Cartesian coordinates were well known and applied correctly. Some candidates made the solution more complicated by multiplying out brackets and having to recombine terms. The strongest candidates recognised directly the expressions for $\sin 2\theta$ and $\cos 2\theta$ and could then combine them to give the required result.

(b) The general shape was usually correct. Cartesian axes were frequently added to the sketch, rather than just the initial line, and rays marked with angles as a guide for drawing. The question asked for the equation of the line of symmetry and this was not always given.

(c) The majority of candidates were able to recall and apply the formula for finding the area of a polar curve and use the double angle formula correctly.

(d) Some candidates attempted to find the maximum distance from the initial line, rather than the line $\theta = \frac{\pi}{2}$

Using the given identity produced an expression in both $\sin\theta$ and $\cos\theta$ and some candidates spent considerable effort to express it in terms of one of these, possibly introducing errors. The strongest candidates differentiated using the product rule. The resulting equation in both $\sin\theta$ and $\cos\theta$ was most effectively solved by converting it to a quadratic in $\tan^2\theta$, although the possibility that $\cos\theta = 0$ should have been considered. There were a few very good, accurate solutions.

FURTHER MATHEMATICS

Paper 9231/21
Paper 21

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth, particularly when a geometric interpretation is required.
- Candidates should make use of results derived or given in earlier parts of a question.

General comments

The majority of candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. There was no evidence that candidates were short of time. Sometimes candidates did not fully support their answers and reached a conclusion without justification, particularly where answers were given within the question. There were many scripts of a high standard.

Comments on specific questions

Question 1

(a) This part was very well answered with most candidates accurately finding the Maclaurin's series, though a few accepted a zero value of $f''(0)$ without checking their work.

(b) Almost all candidates substituted the correct expansion from **(a)** which led to the required approximation.

Question 2

(a) Almost all candidates knew how to approach this question and completed it to a high standard. There was some inaccuracy when solving linear equations to find the particular integral and some problems with notation. A few candidates gave expressions instead of equations as their answer.

(b) Most candidates successfully stated an approximate solution using their answer to **(a)**.

Question 3

(a) The majority of candidates calculated the determinant accurately to show that the system of equations does not have a unique solution. A few candidates accepted a non-zero determinant without checking their work.

(b) Most candidates were able to show that the system is consistent by manipulating the equations. The strongest candidates also gave a complete and correct geometric description.

(c) – (d) Most candidates were able to derive a contradiction by manipulating equations. The strongest candidates also gave a complete and correct geometric description.

Question 4

(a) Strong candidates formed a correct expression for the sum of the areas of the rectangles and applied the standard result for the sum of cubes to accurately derive the given result.

(b) Strong candidates adapted their solution to **(a)** and derived a suitable lower bound.

Question 5

(a) Most candidates differentiated $\cos^{-1} t$ correctly using implicit differentiation.

(b) The majority of candidates found the first derivative correctly using parametric differentiation. The attempts to find the second derivative varied in length, with stronger candidates showing the required level of algebraic fluency and remembering to divide by $\frac{\mathrm{d}x}{\mathrm{d}t}$ after differentiating with respect to *t*.

Question 6

(a) After expanding $\left(z - z^{-1}\right)^4$ using the binomial expansion**,** stronger candidates grouped together terms clearly before applying the identity $z^n + z^{-n} = 2\cos n\theta$ to justify fully the given result. Alternatively, a few candidates applied $\cos^2\theta + \sin^2\theta = 1$, and were usually successful too.

(b) Most candidates found the integrating factor correctly and, after multiplying both sides of the equation by $\sin\theta$, were able to apply the result given in **(a)**. Stronger candidates maintained accuracy throughout, particularly when substituting in the initial conditions.

Question 7

(a) Most candidates found the first part of this question straightforward. A few candidates spent time finding $\det\left(\mathbf{P} - \lambda\mathbf{I}\right)$ instead of reading directly from the diagonal of the matrix.

(b) Stronger candidates were able to maintain accuracy throughout their solution, both when manipulating the characteristic equation and when substituting in for $\mathbf{P}$ and $\mathbf{P}^2$.

(c) Most candidates applied $\mathbf{A} = \mathbf{PDP}^{-1}$ using their answer to **(b)**. A few candidates took the lengthy approach of finding $\mathbf{P}^{-1}$ again without using the characteristic equation.

Question 8

(a) Most sketches were accurate, showing asymptotes $x = 0$ and $y = 1$. A few candidates unnecessarily sketched the curve for $x < 0$.

(b) This part of the question was answered well with the majority of candidates accurately substituting cot*h* and cosec*h* in terms of exponentials and combining using a common denominator.

(c) Stronger candidates clearly applied the chain rule and required double angle formula to justify the given derivative.

(d) Most candidates accurately recalled the formula for arc length and applied the results given in **(b)** and **(c)** to reach the given answer. Finding the logarithmic form of $\cosh^{-1} 2$ was not problematic for most candidates.

FURTHER MATHEMATICS

Paper 9231/22
Paper 22

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth. They should take note of where exact answers are required.
- Candidates should make use of results derived or given in earlier parts of a question.

General comments

The majority of candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. There was no evidence that candidates were short of time. Sometimes candidates did not fully justify their answers and reached a conclusions without justification, particularly where answers were given within the question. There were many scripts of a high standard.

Comments on specific questions

Question 1

This question was very well answered with most candidates accurately finding the Maclaurin's series, but a few did not divide $f''(0)$ by 2.

Question 2

Most candidates accurately recalled the formula for surface area with correct limits. Stronger candidates fully simplified $\sqrt{\dot{x}^2 + \dot{y}^2}$ before substituting into the formula, which caused fewer errors and showed a clear path to the answer.

Question 3

Almost all candidates wrote down the sixth roots of unity and most candidates adjusted these accurately to give all six solutions.

Question 4

This was answered well with almost all candidates dividing through by x and then finding the correct integrating factor. Stronger candidates multiplied their integrating factor by $x^{-1}e^x$, integrated the right-hand side by parts and accurately used the initial conditions. A few candidates multiplied the original equation by x, spotting that the left-hand side becomes x^2y differentiated.

Question 5

(a) The first part of this question was answered well by most candidates.

(b) Candidates who used the chain rule were usually successful. Those who chose to multiply out the expression were at risk of errors. A small number of candidates made y' the subject before differentiating using the quotient rule and were usually successful too.

Question 6

Almost all candidates knew how to approach this question and completed it to a high standard. There was some inaccuracy when solving linear equations to find the constants and some problems with notation. A few candidates used *x* as both a dependent and independent variable, and some gave expressions instead of equations as their answer.

Question 7

(a) The majority of candidates used the formula for the sum of a geometric progression to fully justify the given answer.

(b) Almost all candidates knew that de Moivre's theorem related the series to the geometric progression in **(a)**. Stronger candidates accurately took the real part, after simplifying the numerator and denominator, which led to the given answer.

Question 8

Stronger candidates showed consideration of the sum of the areas of the rectangles to justify the left-hand side of the inequality. When dealing with the right-hand side, the most common approach was to complete the square and use the logarithmic form of the inverse of sin*h* to justify the given answer. A few candidates worked instead with tan and sec via a suitable substitution and were usually successful too.

Question 9

(a) While it was common to see the determinant or solution found correctly, only stronger candidates gave a full geometric interpretation by emphasising that the three planes intersect at a single point.

(b) This was answered well with most candidates justifying the eigenvalues by setting $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ before writing down the three linear factors.

(c) This part of the question was also answered well, but a few candidates accepted zero eigenvectors without checking for errors in their working.

(d) The strongest candidates were able to maintain accuracy throughout their whole solution, both when manipulating the characteristic equation and when substituting in for **A** and $\mathbf{A}^2$.

FURTHER MATHEMATICS

Paper 9231/23
Paper 23

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth, particularly when a geometric interpretation is required.
- Candidates should make use of results derived or given in earlier parts of a question.

General comments

The majority of candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. There was no evidence that candidates were short of time. Sometimes candidates did not fully support their answers and reached a conclusion without justification, particularly where answers were given within the question. There were many scripts of a high standard.

Comments on specific questions

Question 1

(a) This part was very well answered with most candidates accurately finding the Maclaurin's series, though a few accepted a zero value of $f''(0)$ without checking their work.

(b) Almost all candidates substituted the correct expansion from **(a)** which led to the required approximation.

Question 2

(a) Almost all candidates knew how to approach this question and completed it to a high standard. There was some inaccuracy when solving linear equations to find the particular integral and some problems with notation. A few candidates gave expressions instead of equations as their answer.

(b) Most candidates successfully stated an approximate solution using their answer to **(a)**.

Question 3

(a) The majority of candidates calculated the determinant accurately to show that the system of equations does not have a unique solution. A few candidates accepted a non-zero determinant without checking their work.

(b) Most candidates were able to show that the system is consistent by manipulating the equations. The strongest candidates also gave a complete and correct geometric description.

(c) – **(d)** Most candidates were able to derive a contradiction by manipulating equations. The strongest candidates also gave a complete and correct geometric description.

Question 4

(a) Strong candidates formed a correct expression for the sum of the areas of the rectangles and applied the standard result for the sum of cubes to accurately derive the given result.

(b) Strong candidates adapted their solution to **(a)** and derived a suitable lower bound.

Question 5

(a) Most candidates differentiated $\cos^{-1} t$ correctly using implicit differentiation.

(b) The majority of candidates found the first derivative correctly using parametric differentiation. The attempts to find the second derivative varied in length, with stronger candidates showing the required level of algebraic fluency and remembering to divide by $\frac{\mathrm{d}x}{\mathrm{d}t}$ after differentiating with respect to *t*.

Question 6

(a) After expanding $\left(z - z^{-1}\right)^4$ using the binomial expansion**,** stronger candidates grouped together terms clearly before applying the identity $z^n + z^{-n} = 2\cos n\theta$ to justify fully the given result. Alternatively, a few candidates applied $\cos^2\theta + \sin^2\theta = 1$, and were usually successful too.

(b) Most candidates found the integrating factor correctly and, after multiplying both sides of the equation by $\sin\theta$, were able to apply the result given in **(a)**. Stronger candidates maintained accuracy throughout, particularly when substituting in the initial conditions.

Question 7

(a) Most candidates found the first part of this question straightforward. A few candidates spent time finding $\det(\mathbf{P} - \lambda\mathbf{I})$ instead of reading directly from the diagonal of the matrix.

(b) Stronger candidates were able to maintain accuracy throughout their solution, both when manipulating the characteristic equation and when substituting in for $\mathbf{P}$ and $\mathbf{P}^2$.

(c) Most candidates applied $\mathbf{A} = \mathbf{PDP}^{-1}$ using their answer to **(b)**. A few candidates took the lengthy approach of finding $\mathbf{P}^{-1}$ again without using the characteristic equation.

Question 8

(a) Most sketches were accurate, showing asymptotes $x = 0$ and $y = 1$. A few candidates unnecessarily sketched the curve for $x < 0$.

(b) This part of the question was answered well with the majority of candidates accurately substituting cot*h* and cosec*h* in terms of exponentials and combining using a common denominator.

(c) Stronger candidates clearly applied the chain rule and required double angle formula to justify the given derivative.

(d) Most candidates accurately recalled the formula for arc length and applied the results given in **(b)** and **(c)** to reach the given answer. Finding the logarithmic form of $\cosh^{-1} 2$ was not problematic for most candidates.

FURTHER MATHEMATICS

Paper 9231/31
Paper 31

Key messages

A diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although candidates can always choose to draw their own diagram as well.

When a result is given in a question, candidates must take care to give sufficient detail in their working, so that there is no doubt that the offered solution is clear and complete. However, in all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

General comments

Most candidates showed a good understanding of the use of restitution and momentum equations, even though a significant proportion of them did not include the mass in the momentum equation.

There were many candidates who did not know when to use the equilibrium of forces, as opposed to the conservation of energy.

The majority of candidates showed good algebraic manipulative skills in deriving the equation of the trajectory of a projectile from first principles.

Comments on specific questions

Question 1

This question proved challenging for most candidates and there were a number of candidates who did not offer a response. The most common error was to use an approach based on the equilibrium of forces rather than on the conservation of energy.

Question 2

Most candidates realised that this question required the use of the conservation of energy. Some candidates stated that at the lowest point of the circle the velocity of the particle was $\sqrt{5ag}$, but in most cases they did not give a justification. They then used this result to answer the question, with varied success. A common mistake was to think that at the highest point of the circle the velocity of the particle, rather than the tension on the string, was equal to zero. Another common error was in the terms representing the Gravitational Potential Energy.

Question 3

(a) Many candidates scored full marks in this part. Other candidates were still able to apply Hooke's law successfully. A common error was to miss the term $\sin\theta$ in the application of Newton's second law horizontally.

(b) Most of the candidates who realised they had to apply Newton's second law vertically did so correctly and were able to score at least one mark.

Question 4

(a) Even though most candidates wrote a dimensionally correct moment equation, only a few managed to achieve full marks in this part. The correct calculation of the distance of the centre of mass of the cones from AB proved particularly problematic. In some cases, candidates used the expression for the diameter instead of that of the radius when calculating the volume of the cones.

(b) Those candidates who scored full marks in **(a)**, usually scored full marks in this part too. A common mistake among those candidates who did not, was to calculate the reciprocal ratio (cotangent)**.**

Question 5

(a) This part asked candidates to derive the equation of the trajectory of *P* from first principles and was answered correctly by most candidates. However, some candidates used an overly complex approach.

(b) Different approaches were used to answer this part, including differentiation, halving the range, and using the formula for the maximum height $\frac{u^2(\sin\theta)^2}{2g}$. These were usually successful.

(c) Only a few candidates managed to answer this part correctly. Many candidates did not recognise that, even though point Q was not the highest point of the projectile's trajectory anymore, its *y*-coordinate could still be determined using the result from **(b)**. In most cases they ended up substituting an incorrect expression for the *y*-coordinate, leading to an incorrect answer. Candidates who substituted the correct expression $\left(\frac{u^2}{4g}\right)$, usually scored full marks.

Question 6

(a) This part was answered correctly by many candidates. However, some candidates did not include the masses in the momentum equations.

(b) Unlike **(a)**, this part was answered correctly only by a minority of candidates. The most common difficulty was in correctly calculating the components of the velocity of sphere B after the collision. Some candidates failed to include the masses in the equation involving the two kinetic energies.

Question 7

(a) Many candidates scored the method mark for using the initial condition, but only a few candidates scored the last mark. Those who realised that when working out the expression for the velocity, they had to take into account both signs and disregard the one that did not apply, given the direction of the velocity at $t = 0$. Many candidates did not express the acceleration as $v\frac{\mathrm{d}v}{\mathrm{d}x}$ and then tried to integrate the acceleration with respect to t or x .

(b) Most candidates who correctly integrated the equation in **(a)** correctly integrated again in **(b)**, often scoring full marks. Some candidates did not use the modulus function in the integration of $\frac{1}{1-2x}$ and, when using the initial condition, ended up with the natural logarithm of a negative value, which they then used in the subsequent steps. Only a few candidates managed to successfully determine the asymptotical behaviour of *x* when *t* tends to infinity and scored the last mark.

FURTHER MATHEMATICS

Paper 9231/32
Paper 32

Key messages

In Mechanics, a diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient for candidates to annotate that diagram, although they are always free to draw their own diagrams as well.

When a result is given in a question, candidates must take care to give sufficient detail in their working, so that there is in no doubt that the offered solution is clear and complete. However, in all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

General comments

Most candidates attempted all the questions on the paper and many fully correct solutions were seen.

Candidates had a good understanding of the use of momentum and restitution and its application in the oblique impact context of **Question 2**.

There was a mixed performance on **Question 3**, with most candidates scoring full marks in **(a)** but only a minority of candidates offering a meaningful solution to **(b)**.

Most candidates made good attempts at **Question 5** on projectiles and **Question 7** on motion under a variable force.

Attempts at **Question 6** on the motion of a particle on the end of an elastic string were very variable in quality. Most candidates appeared to know the method that they should be using but many were unable to apply it with accuracy or completeness to the specific situation under consideration.

Comments on specific questions

Question 1

Most candidates realised that this question required the use of the conservation of energy and many wrote down a correct equation representing this. The second requirement was the use of Newton’s second law of motion at the point *B*. A common error was to state that when the particle loses contact with the sphere its velocity will be zero. This is not the case and instead this loss of contact results in the reaction between the particle and the sphere being zero.

Question 2

This question was answered correctly by many candidates. The majority of candidates wrote down correct momentum and restitution equations for motion along the line of centres, but a few candidates used the velocity of sphere *A* instead of its component along the line of centres.

The common error in finding the speed of *A* after the collision was to omit the component perpendicular to the line of centres. This latter remains unchanged as a result of the collision but must be included.

Question 3

(a) Most candidates scored full marks in this part. A minority of candidates, who used the method of taking moments successfully, calculated the distance of the centre of mass of the object from the vertex of the cone instead of from the base of cylinder. Only a few of these candidates remembered to subtract their answer from the combined height of the object. It is usually advisable (and often simpler) to find the distance required in the question directly.

(b) This part of this question was the most challenging on this paper. Many candidates did not realise what they needed to do in order to show that the object could rest in equilibrium with the curved surface of the cone in contact with a horizontal surface. In many of these cases, it is possible that a diagram might have helped. This was a case where a sketch of the situation may have provided inspiration and signalled a simple trigonometrical approach. There were several different lines of approach, considering either lengths or angles.

Question 4

(a) A significant number of candidates scored only the first mark in this part, for a correct resolution in the vertical direction. A very common error in the horizontal resolution was to omit essential steps in the working. When an answer is given in the question, a full derivation must be provided by the candidate.

(b) The majority of candidates did not think of applying the result in **(a)** to a similar case with a different height and angular speed. This would have led them to two expressions from which all but r and x could be eliminated. A significant number of attempts at this part were abandoned with little real progress made.

Question 5

(a) This part asked candidates to start from the equation of the trajectory given in the list of formulae (MF19) and write it in a slightly different form. A significant minority of candidates ignored this instruction and instead derived the equation of the trajectory from first principles. The mark was still awarded if this was done correctly, but this approach would have been more time consuming.

(b) Most candidates answered this part well and proceeded with confidence in their method. However, some candidates made arithmetical errors.

Question 6

The first part of the request in this question was to show $k = 4\,mg$. The simplest way to do this was by use of Hooke's law and Newton's second law of motion. Those candidates who realised this were almost always successful. Other candidates immediately started on an energy approach and often ignored the given acceleration.

This first request was the precursor to consideration of the subsequent motion of the particle. Almost all candidates realised that this required energy considerations. Some found the initial and final energies and equated them, others found and equated the loss and gain in energy. Either approach had the potential for success so long as kinetic energy, elastic potential energy and gravitational potential energy were all considered at both ends of the motion. Candidates could also choose their points of reference for measuring heights/lengths and gravitational potential energies. All approaches should have led to a quadratic equation involving an unknown length and the natural length of the string, but there were many different forms of this. Only one of these is given on the mark scheme, but all were rewarded at equivalent steps in the working.

Although most candidates set out on an appropriate path, there were various errors introduced to many solutions. Some were algebraic or arithmetic, but more significantly, others involved the omission of one of the elastic potential energy terms or one of the gravitational potential energy terms.

Question 7

(a) Most candidates wrote down a correct expression from applying Newton's second law of motion and realised that it would be helpful to use the expression $v\dfrac{dv}{dx}$ for the acceleration. They were then able to integrate and apply the initial condition to find v in terms of x. The most common error was the omission of a negative sign in the initial expression, and candidates overlooked the fact that the force was resistive. Some candidates took a longer route to solving the question by first finding v in terms of t and then integrating again to find x in terms of t. This is a valid method, but was unnecessarily long.

(b) Many candidates answered this part well. There were two common errors seen. These were a sign error when integrating to obtain the log term and use of an incorrect initial condition. Most candidates used the initial condition found in **(a)**, and so found the total distance travelled in one step. An alternative to this was to find the extra distance travelled in this second part of the motion and add it on to the distance from the first part to give the total distance. This was an equally acceptable approach, but a significant minority of candidates found only the extra distance and not the total distance.

FURTHER MATHEMATICS

Paper 9231/33
Paper 33

Key messages

A diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although candidates can always choose to draw their own diagram as well.

When a result is given in a question, candidates must take care to give sufficient detail in their working, so that there is no doubt that the offered solution is clear and complete. However, in all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

General comments

Most candidates showed a good understanding of the use of restitution and momentum equations, even though a significant proportion of them did not include the mass in the momentum equation.

There were many candidates who did not know when to use the equilibrium of forces, as opposed to the conservation of energy.

The majority of candidates showed good algebraic manipulative skills in deriving the equation of the trajectory of a projectile from first principles.

Comments on specific questions

Question 1

This question proved challenging for most candidates and there were a number of candidates who did not offer a response. The most common error was to use an approach based on the equilibrium of forces rather than on the conservation of energy.

Question 2

Most candidates realised that this question required the use of the conservation of energy. Some candidates stated that at the lowest point of the circle the velocity of the particle was $\sqrt{5ag}$, but in most cases they did not give a justification. They then used this result to answer the question, with varied success. A common mistake was to think that at the highest point of the circle the velocity of the particle, rather than the tension on the string, was equal to zero. Another common error was in the terms representing the Gravitational Potential Energy.

Question 3

(a) Many candidates scored full marks in this part. Other candidates were still able to apply Hooke's law successfully. A common error was to miss the term $\sin\theta$ in the application of Newton's second law horizontally.

(b) Most of the candidates who realised they had to apply Newton's second law vertically did so correctly and were able to score at least one mark.

Question 4

(a) Even though most candidates wrote a dimensionally correct moment equation, only a few managed to achieve full marks in this part. The correct calculation of the distance of the centre of mass of the cones from AB proved particularly problematic. In some cases, candidates used the expression for the diameter instead of that of the radius when calculating the volume of the cones.

(b) Those candidates who scored full marks in **(a)**, usually scored full marks in this part too. A common mistake among those candidates who did not, was to calculate the reciprocal ratio (cotangent)**.**

Question 5

(a) This part asked candidates to derive the equation of the trajectory of *P* from first principles and was answered correctly by most candidates. However, some candidates used an overly complex approach.

(b) Different approaches were used to answer this part, including differentiation, halving the range, and using the formula for the maximum height $\frac{u^2(\sin\theta)^2}{2g}$. These were usually successful.

(c) Only a few candidates managed to answer this part correctly. Many candidates did not recognise that, even though point Q was not the highest point of the projectile's trajectory anymore, its *y*-coordinate could still be determined using the result from **(b)**. In most cases they ended up substituting an incorrect expression for the *y*-coordinate, leading to an incorrect answer.

Candidates who substituted the correct expression $\left(\frac{u^2}{4g}\right)$, usually scored full marks.

Question 6

(a) This part was answered correctly by many candidates. However, some candidates did not include the masses in the momentum equations.

(b) Unlike **(a)**, this part was answered correctly only by a minority of candidates. The most common difficulty was in correctly calculating the components of the velocity of sphere B after the collision. Some candidates failed to include the masses in the equation involving the two kinetic energies.

Question 7

(a) Many candidates scored the method mark for using the initial condition, but only a few candidates scored the last mark. Those who realised that when working out the expression for the velocity, they had to take into account both signs and disregard the one that did not apply, given the direction of the velocity at $t = 0$. Many candidates did not express the acceleration as $v\frac{\mathrm{d}v}{\mathrm{d}x}$ and then tried to integrate the acceleration with respect to t or x.

(b) Most candidates who correctly integrated the equation in **(a)** correctly integrated again in **(b)**, often scoring full marks. Some candidates did not use the modulus function in the integration of $\frac{1}{1-2x}$ and, when using the initial condition, ended up with the natural logarithm of a negative value, which they then used in the subsequent steps. Only a few candidates managed to successfully determine the asymptotical behaviour of *x* when *t* tends to infinity and scored the last mark.

FURTHER MATHEMATICS

Paper 9231/41
Paper 41

Key messages

In all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

Care must be taken with the language used when interpreting the result of any test. In general, a hypothesis test is not a proof and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, "there is insufficient evidence to support the claim that…." rather than "the test proves that….".

General comments

Almost all candidates attempted all the questions. The standard was generally high, with many candidates presenting clear and accurate solutions to many questions.

The instructions for this paper specifies that non-exact numerical answers be given to 3 significant figures. Candidates are therefore advised to work to a greater degree of accuracy while working towards the final answer. Premature rounding or working to only 3 significant figures may result in an error in the third figure in the final answer. This is particularly the case in statistics problems where several different values are calculated, each depending on the previous one. Such rounding errors were seen in **Question 3** and **Question 4**.

Comments on specific questions

Question 1

Most candidates answered this question well. Some candidates used the pooled estimate for the population variance, but there was no information given to suggest that the population variance could be assumed to be the same for both samples. A few candidates attempted to use a *t*-value, even though the samples are large.

Question 2

(a) Most candidates knew the basic method involved in carrying out a Wilcoxon signed-rank test. They were able to calculate the differences from the median 200 and the signed ranks and then find the sum of each of the negative and positive ranks. It is worth noting that a tabular approach to this is a helpful aid to accuracy. Solutions were less successful in other details of the test. The hypotheses were not often stated correctly in terms of population medians. One common error was to refer to means rather than medians, or simply use the notation μ.

The final mark was awarded for an appropriate statement of the correct conclusion from the result of the test. There were several common errors which led to this mark not being awarded. Some candidates did not state a conclusion in words, simply saying "accept H_0". Some candidates gave a definitive statement such as "the scientist's belief is true". In interpreting results, there needs to be a level of uncertainty in the language used, so for example, "there is insufficient evidence to support the scientist's belief" or "there is sufficient evidence to suggest that the scientist's belief is not true".

(b) Most candidates were able to give a relevant reason.

Question 3

Most candidates recognised that a goodness of fit test with a binomial distribution with mean 0.35 was required and calculated the expected frequencies accurately. In calculating the value of the test statistic a few candidates did not combine the last two columns in the table. This was necessary because the last observed frequency was less than 5. A few candidates incorrectly assumed that a Poisson distribution was appropriate.

A common error in carrying out the test was to use an incorrect tabular value. The final mark was awarded for a conclusion in context, allowing a follow-through on the candidate’s test statistic value.

Question 4

The majority of candidates correctly realised that this question required a paired *t*-test and they were able to execute the test accurately. There were a few reasons for some marks not being awarded: the hypotheses were not stated correctly, often with 0 instead of 0.3 or with ≠ instead of >, the test statistic had the incorrect numerator of 0.411 instead of 0.411 – 0.3 and most often, the conclusion did not contain any level of uncertainty.

Question 5

Many candidates answered this question well.

(a) This part was almost always answered correctly.

(b) Most candidates calculated the probabilities correctly and used them in a cubic expression for the probability generating function. A minority of candidates made errors in the probabilities but were still able to score the mark for their cubic expression.

(c) Almost all candidates knew that they needed to multiply together their expressions from the first two parts of the question and did so with accuracy to obtain a quintic expression.

(d) The method in this part was recognised by most candidates and applied with accuracy.

(e) Candidates were asked to use their probability generating function for Z to find the most probable value of Z. Many candidates understood that this was the power of the term with the largest coefficient and stated it correctly as 2. Some candidates identified the correct term but gave the answer as the coefficient rather than the power.

Question 6

(a) Most candidates answered this part well, usually using the given cumulative distribution function. A few candidates took the longer route of finding the probability density function first and using that to find the upper and lower quartiles. A very few candidates found the two quartiles but did not subtract to deduce the interquartile range.

(b) This part was almost always answered correctly.

(c) Most candidates answered this part correctly. Sources of error were incorrect substitution of $y = \sqrt{x}$ into the given expression for the cumulative distribution function $\mathrm{F}(x)$ and incorrect differentiation to find the probability density function of Y. Sometimes the range of y was given incorrectly.

FURTHER MATHEMATICS

Paper 9231/42
Paper 42

Key messages

When a result is given in a question, candidates must take care to give sufficient detail in their working, so that there is in no doubt that the offered solution is clear and complete. However, in all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

Care must be taken with the language used when interpreting the result of any test. In general, a hypothesis test is not a proof and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, "there is insufficient evidence to support the claim that…." rather than "the test proves that….".

General comments

Almost all candidates attempted all the questions. The standard was generally high, with many candidates presenting clear and accurate solutions to many questions.

The instructions for this paper specifies that non-exact numerical answers be given to 3 significant figures. Candidates are advised to work to a greater degree of accuracy while working towards the final answer. Premature rounding or working to only 3 significant figures may result in an error in the third figure in the final answer. This is particularly the case in statistics problems where several different values are calculated, each depending on the previous one. Such rounding errors were often seen in **Question 4** and **Question 6**.

Comments on specific questions

Question 1

Almost all candidates knew how to proceed with this question and scored at least four of the five marks. The final mark was awarded for an appropriate statement of the correct conclusion from the result of the hypothesis test. There were several common errors which led to this mark not being awarded. Some candidates did not state a conclusion in words, simply saying "accept H_0". Some candidates gave a definitive statement such as "the mean height is 172.5 cm". In interpreting results, there needs to be a level of uncertainty in the language used, so for example, "there is sufficient evidence to suggest that the mean height is 172.5 cm" or "there is insufficient evidence to suggest that the mean height is not 172.5 cm".

Question 2

(a) Most candidates knew the basic method involved in carrying out a Wilcoxon matched-pairs signed-rank test. They were able to calculate the differences and the signed rankings and then find the sum of each of the negative and positive ranks. It is worth noting that a tabular approach to this is a helpful aid to accuracy. Solutions were less successful in other details of the test. The hypotheses were not often stated correctly, in terms of the difference between population medians. One common error was to refer to the difference of means rather than medians, or simply to use the notation μ. Another common error was to use hypotheses such as "Judge 2 gives higher marks than Judge 1". As in all tests, a level of uncertainty is expected in the concluding statement.

(b) Most candidates made a good attempt at the explanation required in this part and scored both marks. A successful explanation needed some direct reference to the increase of the test statistic from 22 to 24.

Question 3

Both parts of this question were answered well by most candidates.

(a) The most common error in this part was a lack of detail in showing that p is equal to 10.19. When a result is given candidates must be careful to give sufficient detail to show that they both know what to do and that they are working accurately.

(b) Almost all candidates found the correct test statistic from the data. The most common error was in using an incorrect tabular value when applying the test.

Question 4

(a) Almost all candidates answered this part correctly.

(b) Most candidates differentiated the given cumulative distribution function to find the probability density function and then used this in an appropriate integral to find the required expectation value.

(c) The most common error in this part occurred through a lack of accuracy in carrying through the result of **(b)**. This usually gave a final answer of 0.124 instead of 0.134.

(d) Most candidates answered this part well, including those who had made errors of understanding in the previous two parts.

Question 5

Both parts of this question required candidates to show given answers and a common error was a lack of detail in the responses.

(a) Candidates were directed to the approach in this part by being asked to write down a general binomial coefficient. It was then expected that they would differentiate each term of the binomial expansion, collect like terms and obtain the given expression. A common error was to omit essential working, in some cases by giving only a two-line answer. A convincing solution requires appropriate detail.

(b) Again, this part required a convincing solution to obtain full marks. Answers were generally better than those in **(a)**.

Question 6

(a) Most candidates answered this part correctly. A minority of candidates used a z-value in their calculations rather than a t-value. This is not appropriate when the sample size, here 10, is small. Other candidates did not score the final mark because they gave an unbiased estimate for the population standard deviation and not the population variance.

(b) The majority of candidates answered this question well. A minority of candidates did not use or understand the information given in the penultimate sentence; that Nassa assumed that the two samples came from normal distributions with the same variance. This should have led to the use of a pooled estimate for the population variance. As in previous questions, some candidates did not express their conclusion with any level of uncertainty in their language or in context and others lost accuracy along the way.

FURTHER MATHEMATICS

Paper 9231/43
Paper 43

Key messages

In all questions candidates are advised to show all their working, as credit is given for method as well as accuracy.

Care must be taken with the language used when interpreting the result of any test. In general, a hypothesis test is not a proof and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, "there is insufficient evidence to support the claim that…." rather than "the test proves that….".

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(a) Most candidates knew the basic method involved in carrying out a Wilcoxon signed-rank test. They were able to calculate the differences from the median 200 and the signed ranks and then find the sum of each of the negative and positive ranks. It is worth noting that a tabular approach to this is a helpful aid to accuracy. Solutions were less successful in other details of the test. The hypotheses were not often stated correctly in terms of population medians. One common error was to refer to means rather than medians, or simply use the notation μ.

The final mark was awarded for an appropriate statement of the correct conclusion from the result of the test. There were several common errors which led to this mark not being awarded. Some candidates did not state a conclusion in words, simply saying "accept H_0". Some candidates gave a definitive statement such as "the scientist's belief is true". In interpreting results, there needs to be a level of uncertainty in the language used, so for example, "there is insufficient evidence to support the scientist's belief" or "there is sufficient evidence to suggest that the scientist's belief is not true".

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A common error in carrying out the test was to use an incorrect tabular value. The final mark was awarded for a conclusion in context, allowing a follow-through on the candidate's test statistic value.

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The majority of candidates correctly realised that this question required a paired *t*-test and they were able to execute the test accurately. There were a few reasons for some marks not being awarded: the hypotheses were not stated correctly, often with 0 instead of 0.3 or with ≠ instead of >, the test statistic had the incorrect numerator of 0.411 instead of 0.411 – 0.3 and most often, the conclusion did not contain any level of uncertainty.

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Many candidates answered this question well.

(a) This part was almost always answered correctly.

(b) Most candidates calculated the probabilities correctly and used them in a cubic expression for the probability generating function. A minority of candidates made errors in the probabilities but were still able to score the mark for their cubic expression.

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(d) The method in this part was recognised by most candidates and applied with accuracy.

(e) Candidates were asked to use their probability generating function for Z to find the most probable value of Z. Many candidates understood that this was the power of the term with the largest coefficient and stated it correctly as 2. Some candidates identified the correct term but gave the answer as the coefficient rather than the power.

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(a) Most candidates answered this part well, usually using the given cumulative distribution function. A few candidates took the longer route of finding the probability density function first and using that to find the upper and lower quartiles. A very few candidates found the two quartiles but did not subtract to deduce the interquartile range.

(b) This part was almost always answered correctly.

(c) Most candidates answered this part correctly. Sources of error were incorrect substitution of $y = \sqrt{x}$ into the given expression for the cumulative distribution function $F(x)$ and incorrect differentiation to find the probability density function of Y. Sometimes the range of y was given incorrectly.