Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

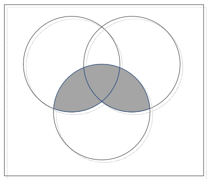
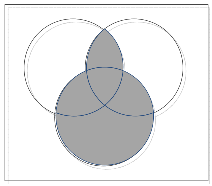
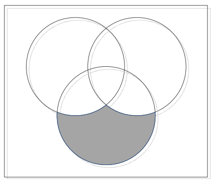
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	 $(A \cup B) \cap C$  $(A \cap B) \cup C$  $(A' \cap B') \cap C$	**B3**	**B1** for each
2	attempt at differentiating a quotient, must have minus sign and $(x+1)^2$ in the denominator	**M1**	
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}-\left(5x^2+4\right)^{\frac{1}{2}}}{(x+1)^2}$	**A1**	all else correct
	When $x=3$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{11}{112}$	**A1**	must be exact
	Alternative $y=\left(5x^2+4\right)^{\frac{1}{2}}(x+1)^{-1}$	**M1**	attempt to differentiate a product
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}10x\left(5x^2+4\right)^{-\frac{1}{2}}(x+1)^{-1}+\left(5x^2+4\right)^{\frac{1}{2}}\left(-(x+1)^{-2}\right)$	**A1**	all else correct
	When $x=3$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{11}{112}$	**A1**	A1 must be exact

Question	Answer	Marks	Partial Marks
3(a)	$\mathbf{v} = 3\sqrt{5} \times \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$	M1	attempt to find the magnitude of $(\mathbf{i} - 2\mathbf{j})$ and use
	$= 3\mathbf{i} - 6\mathbf{j}$	A1	for $3\mathbf{i} - 6\mathbf{j}$ only
3(b)	$\mathbf{w} = 2\cos 30^\circ \mathbf{i} + 2\sin 30^\circ \mathbf{j}$	M1	attempt to use trigonometry correctly to obtain components
	$= \sqrt{3}\mathbf{i} + \mathbf{j}$	A1	
4	$3^n - n3^{n-1}\left(\frac{x}{6}\right) + n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$ $3^n = 81$ **,** so $n = 4$	B1	
	$4 \times 3^3 \times -\frac{1}{6} = a$	M1	for $-n3^{n-1}\left(\frac{x}{6}\right)$**,** ${}^nC_1 3^{n-1}\left(-\frac{x}{6}\right)$ or $\binom{n}{1}3^{n-1}\left(-\frac{x}{6}\right)$**,** with/without *their n*
	$a = -18$	A1	using *their n* and equating to a to obtain $a = -18$
	$\frac{4 \times 3}{2} \times 3^2 \times \frac{1}{36} = b$	M1	for $n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$**,** ${}^nC_2 3^{n-2}\left(\frac{x}{6}\right)^2$ or $\binom{n}{2}3^{n-2}\left(\frac{x}{6}\right)^2$**,** with/without *their n*
	$b = \frac{3}{2}$	A1	using *their n* and equating to b to obtain $b = \frac{3}{2}$
5(i)	$v = -12\sin 3t$	B1	
5(ii)	12	B1	**FT** on *their* (i) of the form $k\sin 3t$, must be $\lvert k\rvert$
5(iii)	$a = -36\cos 3t$	B1	allow unsimplified
	$3t = \frac{\pi}{2}$, 1.57 or better	B1	
	$t = \frac{\pi}{6}$ or 0.524	B1	
5(iv)	4 cao	B1	may be obtained from knowledge of cosine curve

Question	Answer	Marks	Partial Marks
6(i)	$\frac{1}{\sin\theta}\times\frac{1}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$	M1	for $\cot\theta=\frac{\cos\theta}{\sin\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
	dealing with the fractions correctly	M1	
	$\frac{1}{\sin\theta}\times\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$	M1	use of identity
	$=\cos\theta$	A1	correct simplification, with all correct
	Alternative 1 $\frac{\operatorname{cosec}\theta}{\frac{1}{\tan\theta}\left(1+\tan^2\theta\right)}$	M1	dealing with fractions
	$=\frac{\tan\theta\operatorname{cosec}\theta}{\sec^2\theta}$	M1	use of appropriate identity
	$=\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}\times\cos^2\theta$	M1	for $\cot\theta=\frac{1}{\tan\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sec\theta=\frac{1}{\cos\theta}$, $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
	$=\cos\theta$	A1	correct simplification, with all correct
	Alternative 2 $\frac{\operatorname{cosec}\theta}{\frac{1}{\cot\theta}\left(\cot^2\theta+1\right)}$	M1	dealing with fractions
	$=\frac{\cot\theta\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta}$	M1	use of appropriate identity
	$=\frac{\cot\theta}{\operatorname{cosec}\theta}$ $=\frac{\cos\theta}{\sin\theta}\times\sin\theta$	M1	for $\cot\theta=\frac{\cos\theta}{\sin\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
	$=\cos\theta$	A1	correct simplification, with all correct

Question	Answer	Marks	Partial Marks
6(ii)	$\int_0^a \cos 2\theta \, d\theta = \left[\frac{1}{2}\sin 2\theta\right]_0^a$	**B1**	
	$\frac{1}{2}\sin 2a = \frac{\sqrt{3}}{4}$	**M1**	use of $\left[k\sin 2\theta\right]_0^a = \frac{\sqrt{3}}{4}$ to obtain $k\sin 2a = \frac{\sqrt{3}}{4}$
	$2a = \frac{\pi}{3}$	**DM1**	attempt to solve equation of the form $k\sin 2a = \frac{\sqrt{3}}{4}$, with $-1 \leqslant \frac{\sqrt{3}}{4k} \leqslant 1$, must have a correct order of operations dealing with the double angle
	$a = \frac{\pi}{6}$, 0.167π or better	**A1**	
7(i)	$\lg y = \lg A + bx$	**B1**	straight line form, may be implied by correct values of both A and b later
	Gradient = b,	**M1**	equating gradient to b
	$b = 3$	**A1**	
	Use of substitution into one of the following $2.2 = \lg A + 0.5b$ $3.7 = \lg A + b$ $158.489 = A \times 10^{0.5b}$ $5011.872 = A \times 10^{b}$ or equivalent valid method leads to $\lg A = 0.7$	**M1**	
	$A = 5,\ 5.01$ or $10^{0.7}$	**A1**	
	Alternative 1 $\lg y = \lg A + bx$	**B1**	straight line form, may be implied by correct work later
	$2.2 = \lg A + 0.5b$	**M1**	one correct equation
	$3.7 = \lg A + b$	**A1**	both equations correct
	attempt to solve 2 correct equations	**M1**	
	leading to $b = 3$ **and** $A = 5,\ 5.01$ or $10^{0.7}$	**A1**	

Question	Answer	Marks	Partial Marks
7(i)	**Alternative 2** $y = A\left(10^{bx}\right)$ $158.489 = A \times 10^{0.5b}$	**M1**	one correct equation
	$5011.872 = A \times 10^{b}$	**A1**	both correct
	$\dfrac{5011.872}{158.489} = 10^{0.5b}$	**M1**	attempt to solve 2 correct equations
	leading to $b = 3$	**A1**	correct *b*
	Use of substitution leads to $A = 5,\ 5.01$ or $10^{0.7}$	**A1**	correct *A*
7(ii)	Substitute *A* and *b* correctly into either $y = A\left(10^{0.6b}\right)$, $\lg y = \lg A + 0.6b$ or $\lg y = \lg A + 0.6\lg 10^{b}$ or using $\lg y = 1.8 + 0.7$	**M1**	correct statement using *their A* and *b* **correctly** in either equation or using $\lg y = 3x + 0.7$
	$y = 316$, 315 or $10^{2.5}$	**A1**	
7(iii)	Substitute *A* and *b* correctly into either $600 = A\left(10^{bx}\right)$, $\lg 600 = \lg A + bx$ or $\lg 600 = \lg A + x\lg 10^{b}$ or using $\lg 600 = 3x + 0.7$	**M1**	correct statement using *their A* and *b* **correctly** in either equation or using $\lg y = 3x + 0.7$
	$x = 0.693$	**A1**	
8(a)(i)	2520	**B1**	
8(a)(ii)	360	**B1**	
8(a)(iii)	1080	**B1**	
8(a)(iv)	6 or 8 to start with No of ways = $2 \times 5 \times 4 \times 3 \times 2$ = 240	**B1**	
	9 to start with No of ways = $1 \times 5 \times 4 \times 3 \times 3$ = 180	**B1**	
	Total number of ways = 420	**DB1**	Dependent on both previous B marks

Question	Answer	Marks	Partial Marks
8(a)(iv)	**Alternative 1** All numbers > 6000 – all odd numbers > 6000	**B1**	plan and attempt to use, must be using 1080
	$1080 - 180 - 480$	**B1**	for 180 and 480
	Total number of ways = 420	**DB1**	Dependent on both previous B marks
	Alternative 2 Even numbers > 60000 : Odd numbers > 60000 7 : 11	**B1**	correct ratio
	Total number of ways = $\frac{7}{18} \times 1080$	**B1**	
	= 420	**DB1**	Dependent on both previous B marks
8(b)(i)	480700	**B1**	
8(b)(ii)	26460	**B1**	
8(b)(iii)	With brother and sister ${}^{23}C_5 = 33649$	**B1**	for ${}^{23}C_5$ or ${}^{23}C_5 \times {}^{k}C_k$
	Without brother and sister ${}^{23}C_7 = 245157$	**B1**	for ${}^{23}C_7$ or ${}^{23}C_7 \times {}^{k}C_k$
	Total number of ways = 278806	**B1**	for ${}^{23}C_5 + {}^{23}C_7$ and evaluation
9(a)(i)	3×2	**B1**	
9(a)(ii)	correct attempt to multiply the 2 matrices	**M1**	
	$\mathbf{C} = \begin{pmatrix} 6 & -6 \\ 5 & 2 \\ 19 & -8 \end{pmatrix}$	**A2**	–1 for each incorrect element
9(b)(i)	$\mathbf{X}^{-1} = \frac{1}{13}\begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}$	**B2**	**B1** for correct use of determinant **B1** for correct matrix
9(b)(ii)	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13}\begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 26 \\ 52 \end{pmatrix}$	**B1**	
	attempt to evaluate using inverse from (i) together with pre-multiplication to obtain a 2×1 matrix	**M1**	
	$x = 34,\ y = 12$	**A2**	**A1** for each
10(i)	0.5	**B1**	for 0.5 from correct work only

Question	Answer	Marks	Partial Marks
10(ii)	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads	**M1**	use of cosine rule (or equivalent) to obtain angle *AOB*.
	$DOC = AOB - 2(\textit{their } AOD)$	**M1**	use of angle *AOD* and symmetry
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations
	Alternative 1 $15 = 2 \times 8 \times \sin\left(\frac{1 + DOC}{2}\right)$	**M1**	use of basic trigonometry
	use of $\frac{1 + 0.5DOC}{2}$	**M1**	may be implied
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 2		
	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads $\angle AOB \times 8 =$ arc AB	**M1**	use of cosine rule (or equivalent) to obtain angle AOB.
	$\frac{\text{arc}AB - 8}{8} = \angle DOC$	**M1**	attempt at *DOC,* must be a complete method with *AOB* found
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 3 Equating 2 different forms for the area of triangle *AOB* $\frac{15\sqrt{31}}{4} = \frac{1}{2} \times 8^2 \sin AOB$, $AOB = 2.43075$ rads	**M1**	using both different forms of the area of triangle *AOB*
	$DOC = AOB - 2(\text{their } AOD)$	**M1**	use of angle *AOD* and symmetry
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations

Question	Answer	Marks	Partial Marks
10(iii)	$\sin\left(\frac{1.43}{2}\right)=\frac{\frac{DC}{2}}{8}$ or $DC^2=8^2+8^2-(2\times 8\times 8\times\cos 1.43)$	**M1**	use of cosine rule or basic trigomoetry to obtain DC
	$DC=10.49$	**A1**	awrt 10.5, may be implied
	Perimeter = $10.49+4+4+15$ = 33.5	**A1**	awrt 33.5
10(iv)	$\frac{1}{2}\times 8^2(2.43-\sin 2.43)-\frac{1}{2}\times 8^2(1.431-\sin 1.431)$	**B1**	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	area of one appropriate triangle, allow unsimplified	**B1**	
	an appropriate segment, allow unsimplified	**B1**	
	= 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 1 Area of a trapezium + 2 small segments	**B1**	one appropriate small sector, allow unsimpified (could be doubled)
	Each small segment = $\frac{1}{2}\times 8^2(0.5-\sin 0.5)$	**B1**	an appropriate triangle, allow unsimplfied (could be doubled)
	Area of trapezium = $\frac{1}{2}(15+10.5)\times(6.041-2.784)$	**B1**	attempt at trapezium, must have a correct attempt at finding the distance between the parallel sides – allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 2 Area of 2 small sectors + area of triangle ODC – the area of triangle OAB Area of a small sector = $\frac{1}{2}\times 8^2\times\frac{1}{2}$	**B1**	area of small sector, allow unsimplified, (could be doubled)
	Area of triangle $ODC=\frac{1}{2}\times 8^2\times\sin 1.43$	**B1**	area of triangle ODC, allow unsimplified
	Area of triangle $OAB=\frac{1}{2}\times 8^2\times\sin 2.43$	**B1**	area of triangle OAB, allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer

Question	Answer	Marks	Partial Marks
10(iv)	**Alternative 3** Area of rectangle + 2 small triangles + 2 small segments Each small segment = $\frac{1}{2}\times 8^2\left(0.5-\sin 0.5\right)$	**B1**	area of a small segment, allow unsimplified, could be doubled
	$\frac{1}{2}\times\frac{\left(15-10.49\right)}{2}\left(6.041-2.784\right)$	**B1**	area of a small triangle, allow unsimplified, could be doubled
	Area of rectangle = $10.49\times\left(6.041-2.784\right)$	**B1**	allow unsimplified, could be doubled
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 4 Sector *AOB* – sector *AOD* – sector *COB* – triangle *DOC*	**B1**	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	$\left(\frac{1}{2}\times 8^2\times 2.43\right)-2\left(\frac{1}{2}\times 8^2\times 0.5\right)-\left(\frac{1}{2}\times 8^2\sin 1.43\right)$ Area = sector *AOB* – segment *DC* – triangle *AOB*	**B1**	area of one appropriate triangle, allow unsimplified
	$\left(\frac{1}{2}\times 8^2\times 2.43\right)-(\text{their segment})-\left(\frac{1}{2}\times 8^2\sin 2.43\right)$	**B1**	an appropriate segment, allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
11(i)	$m\mathrm{e}^{2x-1}$ where m is numeric constant	**M1**	
	$\mathrm{f}\left(x\right)=\frac{1}{2}\mathrm{e}^{2x-1}\quad\left(+c\right)$	**A1**	condone omission of +*c*
	$\frac{7}{2}=\frac{1}{2}+c$	**DM1**	correct attempt to find arbitrary constant
	$\mathrm{f}\left(x\right)=\frac{1}{2}\mathrm{e}^{2x-1}+3$	**A1**	must be an equation
11(ii)	$k\mathrm{e}^{2x-1}$ where *k* is a numeric constant	**M1**	
	$\mathrm{f}''\left(x\right)=2\mathrm{e}^{2x-1}$	**A1**	
	$2x-1=\ln\left(\frac{4}{k}\right)$	**DM1**	attempt to equate to 4 and use logarithms
	$x=\frac{1}{2}+\ln\sqrt{2}$	**A1**	