Cambridge O Level

ADDITIONAL MATHEMATICS **4037/14**

Paper 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

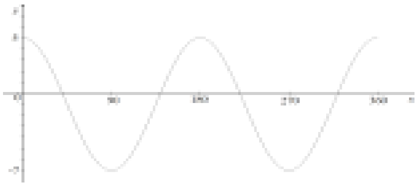
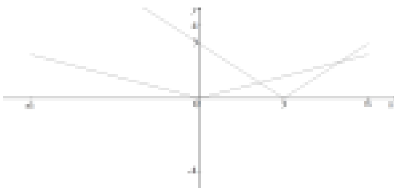
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1(a)		3	**B1** for shape, must have implied symmetry about $x = 180^{\circ}$, 1 complete maximum point above the x-axis, 2 minimum points below the x-axis, starting and finishing at the same positive y value. 5 and – 7 not necessary for this mark. **B1** for $-7 \leqslant y \leqslant 5$, may be implied by numbers in a table if not seen on the graph or by coordinates. Must have maximum point(s) at $y = 5$ only and minimum point(s) at $y = -7$ only. **B1** for a completely correct curve with maximum points implied at the end points.
1(b)	$a = 0$ $b = 4$ $c = 1$	2	**B1** for 2 correct.
	Alternative $a = 0$ $b = -4$ $c = -1$	**(2)**	**B1** for 2 correct.
2(a)		3	**B1** for the graph of either $y = \left\lvert \frac{2}{5}x \right\rvert$ or $y = \lvert x - 3 \rvert$, must be straight lines, not curves. **B1** for 2 correct graphs with 2 points of intersection only in the first quadrant soi. **B1** for $(0,\ 3)$ and $(3,\ 0)$, must have the correct graph of $y = \lvert x - 3 \rvert$.
2(b)	$\frac{2}{5}x = \pm(x-3)$ or $\left(\frac{2}{5}x\right)^2 = (x-3)^2$ $x = 5,\quad x = \frac{15}{7}$ or 2.14 or better	2	**B1** for each.
3(a)	$a^{10} - 30a^9x + 405a^8x^2$	3	**B1** for each term, must be simplified.

Question	Answer	Marks	Guidance
3(b)	$405a^8 = \frac{405}{256}$	**M1**	For the coefficient of *their* term in $x^2 = \frac{405}{256}$.
	$a = \frac{1}{2}$	**A1**	
	$p = \frac{1}{1024}$ $q = -\frac{30}{512}$ or $= -\frac{15}{256}$	**A1**	Need both. For *p*, allow for 0.000977 or better. For *q*, allow for -0.0586 or better.
4(a)	$5(2x+1)^{\frac{3}{2}}$	**2**	**M1** for $p(2x+1)^{\frac{3}{2}}$ where *p* is a constant, $p \neq 5$. **A1** allow with coefficients unsimplified.
4(b)	$\frac{1}{5}(2x+1)^{\frac{5}{2}} + c$	**2**	**M1** for $q(2x+1)^{\frac{5}{2}}$ where *q* is a constant. **A1FT** on *p* from (a), must have + *c*, must be simplified.
4(c)	$\int_0^a (2x+1)^{\frac{3}{2}}\,dx = 48.4$ $\left[\frac{1}{5}(2x+1)^{\frac{5}{2}}\right]_0^a = 48.4$ $\frac{1}{5}\left((2a+1)^{\frac{5}{2}} - 1\right) = 48.4$ $(2a+1)^{\frac{5}{2}} = 243$ $2a+1 = 9$ $a = 4$	**3**	**M1** for use of a correct integral or an integral of the form $\left[q(2x+1)^{\frac{5}{2}}\right]_0^a$ equated to 48.4. This may be implied by later work. **M1 dep** for correct use of limits and correct attempt to solve equation of the form $(2a+1)^{\frac{5}{2}} = p$ to obtain $2a+1 = p^{\frac{2}{5}}$ oe.
5(a)(i)	120	**B1**	
5(a)(ii)	$4 \times 3 \times 2 \times 1 \times 3$ =72 odd numbers 60%	**2**	**B1** for either: 72 or $4 \times 3 \times 2 \times 1 \times 3$ and no percentage, or $\frac{3}{5}$ and no percentage.
5(b)	495	**B1**	
	$495 \times 70 \quad (= 34650)$	**B1**	
	$\left(\frac{34650}{3!} = \right) \quad 5775$	**B1**	

Question	Answer	Marks	Guidance
6(a)	$x+y=1$	**B1**	
	$x+1=y^2$	**B1**	
	leading to $y^2+y-2=0$ or $x^2-3x=0$ $y=1,\ x=0$ only	2	**M1** for obtaining a 3-term quadratic equation in y or a 2-term quadratic equation in x. **A1** for the one pair only
6(b)	$2\log_p q \times \dfrac{3}{\log_p q}=6$ or $\dfrac{2}{\log_q p}\times 3\log_q p=6$	3	**M1** for use of power rule to obtain either $3\log_q p$ or $2\log_p q$. **M1** for change of base of logarithm for 1 term. **A1** for 6 nfww.
7	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\ -4\cos 2x\ \ (+c)$	2	**M1** for attempt to integrate to obtain $p\cos 2x,\ p\neq\pm 8,\ p\neq\pm 16$ **A1** all correct, condone absence of *c,* allow unsimplified.
	When $x=\dfrac{\pi}{2},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=6$ leading to $c=2$ $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\ -4\cos 2x+2$ soi	2	**M1** for attempt to find *c,* must be using $p\cos 2x$ with $x=\dfrac{\pi}{2}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=6$. **A1** may be implied by $c=2$ Allow unsimplified coefficients.
	$(y=)\ 2x-2\sin 2x\ \ (+d)$	2	**M1** for attempt to integrate *their* $\dfrac{\mathrm{d}y}{\mathrm{d}x}=p\cos 2x\ \ (+2)$ must be of the form $q\sin 2x\ \ (+rx+d)$, where $q\neq\pm\ 2\times$*their p* or ± 8. **A1** for $(y=)\ 2x-2\sin 2x\ \ (+d)$ condone absence of + *d*. Allow unsimplified coefficients.
	When $x=\dfrac{\pi}{2},\ y=4\pi$ leading to $d=3\pi$	**M1**	For attempt to find *d,* must be using $q\sin 2x\ \ (+rx)$
	$y=2x-2\sin 2x+3\pi$ or $y=2x-2\sin 2x+9.42$	**A1**	Must be an equation with simplified coefficients.

Question	Answer	Marks	Guidance
8(a)	$\text{p}\left(-\frac{1}{2}\right): -\frac{a}{8}+\frac{b}{4}-\frac{7}{2}+1$	M1	For substitution of $x=-\frac{1}{2}$, at least once, with simplification of first 3 terms. Allow one sign error.
	$-\frac{a}{8}+\frac{b}{4}-\frac{5}{2}=0$ oe $(2b-a=20)$	A1	May be implied by later work. Non algebraic terms need to be simplified correctly.
	$27a+9b=153$ oe	B1	Non algebraic terms must be simplified.
	$a=2,\ b=11$	2	**M1** for solution of *their* simultaneous equations **A1** for both
8(b)	$\text{p}'(x)=3ax^2+2bx+7$	M1	For attempt to differentiate *their* $\text{p}(x)$, allow in terms of *a* and *b*
	$\text{p}'(x)=6x^2+22x+7$	M1	**Dep** for use of $x=1$ in *their* $\text{p}'(x)$
	$\text{p}'(1)=35$	A1	
9(a)	$\frac{1}{2}\left(5\sqrt{3}-3+5\sqrt{3}-1\right)\left(\sqrt{3}+2\right)$	B1	For the area of complete trapezium
	$=\left(5\sqrt{3}-2\right)\left(\sqrt{3}+2\right)$ oe $=15-2\sqrt{3}+10\sqrt{3}-4$ oe $=11+8\sqrt{3}$	2	**M1** for attempt to simplify showing expansion of the brackets with at least 3 correct terms
	Alternative $\frac{1}{2}\times2\times\left(\sqrt{3}+2\right)+\left(5\sqrt{3}-3\right)\left(\sqrt{3}+2\right)$	(B1)	For the area of a triangle and a rectangle
	$=\sqrt{3}+2+\left(15-3\sqrt{3}+10\sqrt{3}-6\right)$ $=11+8\sqrt{3}$	(2)	**M1** for attempt to simplify showing expansion of the brackets with at least 3 correct terms.
9(b)	$\cot\theta=\frac{2}{2+\sqrt{3}}$	B1	Allow $\frac{1}{\tan\theta}$
	$\frac{2}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$	M1	For attempt to rationalise *their* $\cot\theta$.
	$4-2\sqrt{3}$	A1	

Question	Answer	Marks	Guidance
9(c)	$\text{cosec}^2\theta = \cot^2\theta + 1$ $= \left(4 - 2\sqrt{3}\right)^2 + 1$ $= 29 - 16\sqrt{3}$	2	**M1** for use of identity with *their* result from **(b)**,
10(a)	$\frac{1}{2}x^2\theta = 2$	M1	**M1** for use of sector area
	$\theta = \frac{4}{x^2}$	A1	
	Arc length = $\frac{4}{x}$	M1	**Dep** for use of arc length using *their* θ in terms of x.
	$P = \frac{10x}{3} + x\theta$	M1	For attempt to obtain P with evidence of adding sides and an arc length, allow $r\theta$.
	$= \frac{10x}{3} + \frac{4}{x}$	A1	nfww.
10(b)	$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{10}{3} - \frac{4}{x^2}$	B1	
	When $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$, $x^2 = \frac{6}{5}$	M1	For attempt to equate *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and attempt to obtain $x^2 =$
	$x = \sqrt{\frac{6}{5}}$ oe	A1	
	$P = \frac{4}{3}\sqrt{30}$	A1	
10(c)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{8}{x^3}$ positive so minimum	B1	Allow equivalent valid methods. Any evaluation must be correct.

Question	Answer	Marks	Guidance
11	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6x\times 2x}{x^2+1}+6\ln\left(x^2+1\right)$	3	**B1** for $\frac{2x}{x^2+1}$ **M1** for differentiation of a product **A1** for everything else correct, allow unsimpified.
	When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=6+6\ln 2$ oe	B1	May be in the form $6+\ln 64$ Must see an exact form
	When $x=1$, $y=6\ln 2$	B1	May be in the form $\ln 64$. Must be an exact form
	Tangent $y-6\ln 2=(6+6\ln 2)(x-1)$ oe	M1	For a correct attempt at a tangent using *their* gradient and *their y* and $x=1$.
	When $x=0$, $y=-6$	M1	**Dep** on previous M1 for attempt to get y coordinate of P using *their* tangent equation.
	When $x=2$, $y=6+12\ln 2$ oe	M1	**Dep** on M1 for tangent equation for attempt to get y-coordinate of Q using *their* tangent equation. May be in the form $6+\ln 4096$
	Area $PQR=\frac{1}{2}\times 2\times\left(12+12\ln 2\right)$ or $\frac{1}{2}\left\|\begin{matrix}0 & 2 & 2 & 0\\ -6 & 6+12\ln 2 & -6 & -6\end{matrix}\right\|$ oe	M1	**Dep** for attempt to find area (dependent on 2 previous M marks).
	$=12+12\ln 2$ or $12+\ln 4096$	A1	Allow equivalent exact answers.
	Alternative $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{12x^2}{x^2+1}+6\ln\left(x^2+1\right)$	(3)	**B1** for $\frac{2x}{x^2+1}$ **M1** for differentiation of a product **A1** for everything else correct, allow unsimplified.
	When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=6+6\ln 2$	(B1)	May be in the form $6+\ln 64$
	Base of triangle = 2	(B2)	
	Height of triangle = $2\times$ *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$	(M1)	
	$12+12\ln 2$ oe	(A1)	May be in the form $12+\ln 4096$ oe.
	Area $PQR=\frac{1}{2}\times 2\times\left(12+12\ln 2\right)$	(M1)	**Dep** for attempt to find area, must have $\frac{1}{2}$. Allow decimal equivalent for M1
	$=12+12\ln 2$ or $12+\ln 4096$	(A1)	Allow equivalent exact answers.