Example Candidate Responses

Cambridge International AS & A Level

Cambridge International AS and A Level Mathematics

9709

Paper 1

Cambridge Advanced

CAMBRIDGE International Examinations

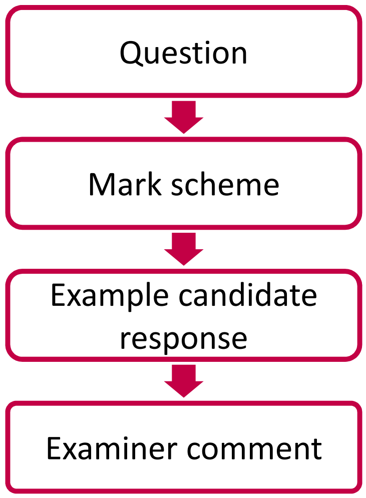
Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics (9709), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



Each question is followed by an extract of the mark scheme used by examiners. This, in turn, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at https://teachers.cie.org.uk

Assessment at a glance

The 7 units in the scheme cover the following subject areas:

- Pure Mathematics (units P1, P2 and P3);
- Mechanics (units M1 and M2);
- Probability and Statistics (units S1 and S2).

Centres and candidates may:

- take all four Advanced (A) Level components in the same examination session for the full A Level.
- follow a staged assessment route to the A Level by taking two Advanced Subsidiary (AS) papers (P1 & M1 **or** P1 & S1) in an earlier examination session;
- take the Advanced Subsidiary (AS) qualification only.

AS Level candidates take:

Paper 1: Pure Mathematics 1 (P1)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 60% of total

plus **one** of the following papers:

Paper 2: Pure Mathematics 2 (P2)	Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total

A Level candidates take:

Paper 1: Pure Mathematics 1 (P1)	Paper 3 Pure Mathematics 3 (P3)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 30% of total	**1¾ hours** About 10 shorter and longer questions 75 marks weighted at 30% of total

plus **one** of the following combinations of two papers:

Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics 1 (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 4: Mechanics 1 (M1)	Paper 5: Mechanics 2 (M2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 6: Probability and Statistics 1 (S1)	Paper 7: Probability and Statistics 2 (S2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

Teachers are reminded that the latest syllabus is available on our public website at **www.cie.org.uk** and Teacher Support at **https://teachers.cie.org.uk**

Paper 1

Question 1

1 In the expansion of $\left(x^2 - \frac{a}{x}\right)^7$, the coefficient of x^5 is -280. Find the value of the constant a. [3]

Mark scheme

1	$\left(x^2 - \frac{a}{x}\right)^7$ Term in x^5 is ${}_7C_3 \times (x^2)^4 \times (-a/x)^3$ This term isolated Equated to -280 $\rightarrow$ $a = 2$.	 B1 M1 A1 [3]	 Allow on own or in an expansion. Correct term in x^5 selected. Equated to -280

Example candidate response – 1

3 1 $\left(x^2 - \frac{a}{x}\right)^7 = (x^2)^7 + {}^7C_1 (x^2)^6 \left(\frac{-a}{x}\right)$
$+ {}^7C_2 (x^2)^5 \left(\frac{-a}{x}\right)^2 + {}^7C_3 (x^2)^4 \left(\frac{-a}{x}\right)^3$

B1

M1 $= x^{14} - x^{11}a + 21x^8a^2 - 35x^5a$

A0

2

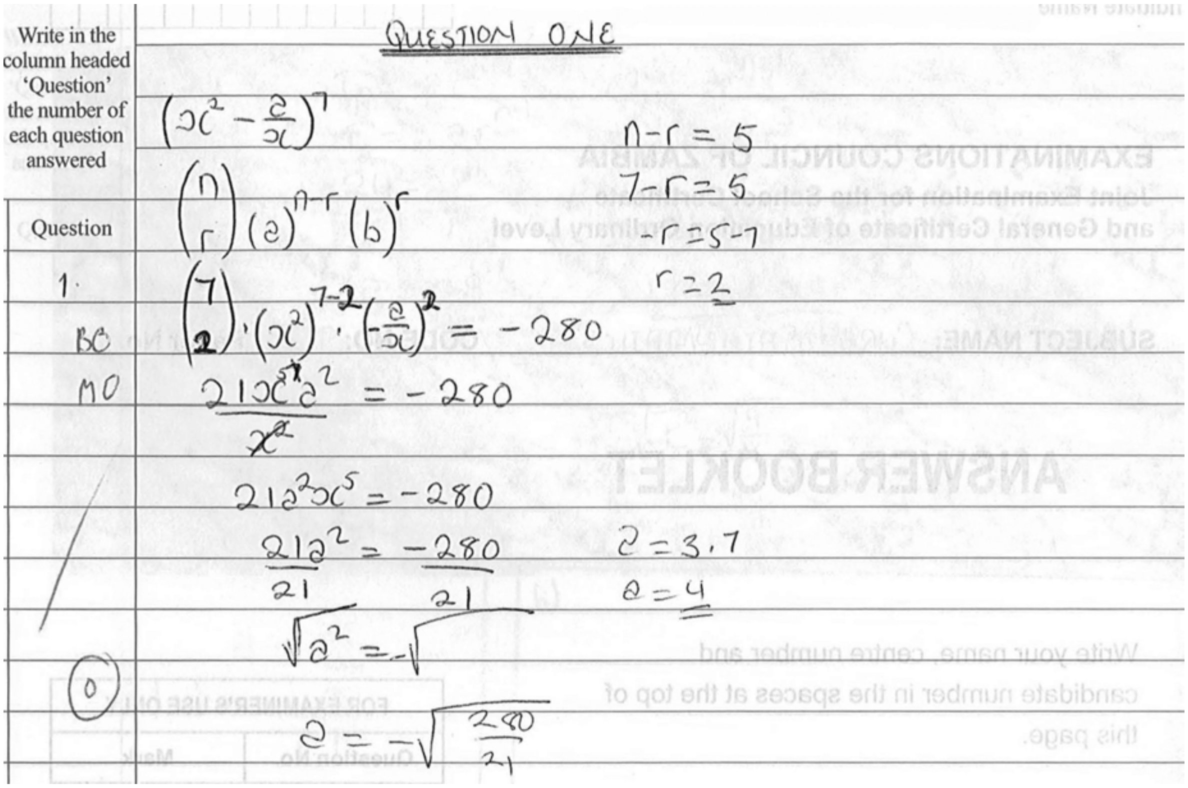
$\therefore x^5 = -280$

$-35a = -280$

$a = 8$

Total mark awarded = 2 out of 3

Example candidate response – 2


Write in the column headed 'Question' the number of each question answered

Question

1.

BC

MO

0

QUESTION ONE

$\left(x^2 - \frac{a}{x}\right)^7$ $\quad n - r = 5$

$\binom{n}{r}(a)^{n-r}(b)^r$ $\quad 7 - r = 5$

$-r = 5 - 7$

$r = 2$

$\binom{7}{2}\cdot(x^2)^{7-2}\cdot\left(-\frac{a}{x}\right)^2 = -280$

$\frac{21x^7a^2}{x^2} = -280$

$21a^2x^5 = -280$

$\frac{21a^2}{21} = \frac{-280}{21}$ $\quad a = 3.7$

$a = 4$

$\sqrt{a^2} = -\sqrt{}$

$a = -\sqrt{\frac{280}{21}}$

Total mark awarded = 0 out of 3

Examiner comment – 1 and 2

This question proved to be more successful for candidates who wrote down several terms of the expansion. In this particular case, candidate 2 only wrote down one term and made the common error of assuming that $(x^2)^5 = x^7$. This led to the incorrect term in x^5. Candidate 1 wrote down the first four terms in the expansion and correctly selected the term that would lead to the coefficient of x^5. Unfortunately, this candidate, although obtaining the correct value for the binomial coefficient ${}_7C_3$, made the common error of expanding $\frac{-a^3}{x}$ as $\frac{-a}{x^3}$. Similar errors, particular over the "–" sign, affected many scripts at both of these levels.

Question 2

2 A function f is such that $f(x) = \sqrt{\left(\frac{x+3}{2}\right)} + 1$, for $x \geqslant -3$. Find

(i) $f^{-1}(x)$ in the form $ax^2 + bx + c$, where a, b and c are constants, [3]

(ii) the domain of f^{-1}. [1]

Mark scheme

2	**(i)** $f(x) = \sqrt{\frac{x+3}{2}} + 1$, for $x \geq -3$ Make x the subject or interchanges x,y $\rightarrow\ 2(x-1)^2 - 3$ $\rightarrow\ 2x^2 - 4x - 1$	M1 M1 A1 [3]	Attempt at x as subject and removes +1 Squares both sides and deals with "+3" and "÷2". co
	(ii) domain of f^{-1} is ≥ 1.	B1 [1]	co. condone >1

Example candidate response – 1

2 (i) $f(x) = \sqrt{\frac{x+3}{2}} + 1$

Let $y = \sqrt{\frac{x+3}{2}} + 1$

$\sqrt{\frac{x+3}{2}} = y - 1$

$\frac{x+3}{2} = (y-1)^2$

$x+3 = 2(y-1)^2$

$x = 2(y-1)^2 - 3$

$x = 2[y^2 - 2y + 1] - 3$

M1 $x = 2y^2 - 4y + 2 - 3$

M1 $x = 2y^2 - 4y - 1$

A0

$f^{-1}(x) = 2y^2 - 4y - 1$

2(ii) $x \geq -3$

B0

2

Item marks awarded: (i) = 2/3; (ii) = 0/1

Total mark awarded = 2 out of 4

Example candidate response – 2

QUESTION TWO

2.

$f(x) = \sqrt{\left(\frac{x+3}{2}\right)} + 1$

let $y = f^{-1}(x)$

$\therefore a = 2$

$b = 0$

$c = -5$

$y = 2x^2 - 5$

$y = \sqrt{\left(\frac{x+3}{2}\right)} + 1$

$(x)^2 = \left(\sqrt{\left(\frac{y+3}{2}\right)}\right)^2 + (1)^2$

M0

M1

$x^2 = \frac{y+3}{2} + 1$

$y = 2x^2 - 5$

$2x^2 = y + 3 + 2$

$2x^2 = y + 5$

Write in the column headed 'Question' the number of each question answered

$f(x) = \sqrt{\frac{x+3}{2}}$

$= \sqrt{\frac{-3+3}{2}}$

$= \sqrt{0}$

domain $= \varnothing$

$f^{-1}(x) \geq 0$

X

Question

(i)

Item marks awarded: (i) = 1/3; (ii) = 0/1

Total mark awarded = 1 out of 4

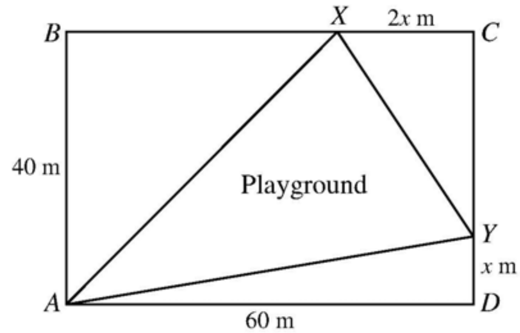
Examiner comment – 1 and 2

(i) Neither of these candidates made the common error of misreading the expression for $\mathrm{f}(x)$, i.e. $\sqrt{\dfrac{x+3}{2}}+1$ as either $\sqrt{\dfrac{x+3}{2}}$ or $\sqrt{\dfrac{x+3}{2}+1}$, but these scripts do illustrate two of the common errors which affected this question. Candidate 2 made a basic algebraic error in replacing $\left(\sqrt{\dfrac{x+3}{2}}+1\right)^2$ with $\dfrac{x+3}{2}+1$, although they did then proceed to make x the subject. Candidate 1 correctly manipulated the algebra to make x the subject, but then did not realise that the answer to $\mathrm{f}^{-1}(x)$ must be given in terms of x. The question illustrates the need to read the question carefully, to avoid the common misread and to ensure that answers are given in the form requested and in terms of x.

(ii) This part of the question was very badly answered by candidates at all levels. Candidates seemed to be unsure of the fact that the domain of f^{-1} was the same as the range of f and that substituting $x = -3$ into the expression for $\mathrm{f}(x)$ would lead to $x \geqslant 1$. Candidate 2 did at least attempt to substitute $x = -3$, but made the mistake of omitting the "$+1$". Candidate 1 assumed that the domain of f and f^{-1} were the same.

Question 3

3



The diagram shows a plan for a rectangular park $ABCD$, in which $AB = 40$ m and $AD = 60$ m. Points X and Y lie on BC and CD respectively and AX, XY and YA are paths that surround a triangular playground. The length of DY is x m and the length of XC is $2x$ m.

(i) Show that the area, $A\ \mathrm{m}^2$, of the playground is given by

$$A = x^2 - 30x + 1200. \qquad [2]$$

(ii) Given that x can vary, find the minimum area of the playground. [3]

Mark scheme

3	**(i)** $A = 2400 - 20(60 - 2x) - x(40 - x) - 30x$ $\rightarrow A = x^2 - 30x + 1200.$ (could be trapezium − triangle)	 M1 A1 [2]	 Needs attempts at all areas co answer given
	(ii) $\dfrac{dA}{dx} = 2x - 30$ or $(x - 15)^2 + 975$ $= 0$ when $x = 15$ or Min at $x = 15$ $\rightarrow A = 975$.	B1 M1 A1 [3]	co - either method okay Sets differential to 0 + solution. co co.

Example candidate response – 1

3i) Area of playground

Area of rectangle $= 40 \times 60 = 2400m$

Length of $XY = \sqrt{(40-x)^2 + (2x)^2}$

$\sqrt{40(40-x) - x(40-x) + 4x^2}$

$\sqrt{1600 - 40x - 40x + x^2 + 4x^2}$

$1600 - 80x + 5x^2$

$x = 8$

$CX = 2(8) = 16$

$\sqrt{8^2 + 60^2}$ $\quad YD = 8 \text{ cm}$ ✗

~~$XAY = 60,5$~~

Length of $AY = \sqrt{60^2 + 8^2}$

$\underline{\underline{60,5}}$ ✗

~~Area of playground~~ $=$ Length of $XY = \sqrt{(2x)^2 + (40-x)^2}$

Length

$\sqrt{(2(8))^2 + (40-8)^2}$

$\sqrt{128 + 1024}$

$XY = 33,94$ ✗

Midpoint of $XY = M \quad 16,97 \qquad AM = 58$

M0 i) $40 \times 60 - \frac{1}{2}(60 \times x) - \frac{1}{2}(2x \times 40 - x) - \frac{1}{2}(60 \times 2x)$

$2400 - \frac{60x}{2} - \frac{80x + 2x^2}{2} - 30\cancel{2x} = x^2 - 30x + 1200$ [shown]

ii) $A = x^2 - 30x + 1200$

B1 $\frac{dy}{dx} \quad 2x - 30 -$

$2x - 30 = 0$

M1 $\frac{2x}{2} = \frac{30}{2}$

A0 $\underline{x = 15}$ ✓

(2)

$A = 15^2 - 30(15) + 1200 =$

Item marks awarded: (i) = 0/2; (ii) = 2/3.

Total mark awarded = 2 out of 5

Example candidate response – 2

3. ~~Area of Rectangle – [Area of ΔBAX + Area of ΔXCY + Area of ΔYAD].~~

Area of playground =

Area of rectangle – [Area of BAX + Area of XCY + Area of YAD]

$= 40 \times 60 - \left[\left(\frac{1}{\cancel{2}}\overset{20}{\cancel{(40)}}(60-2x)\right) + \left(\frac{1}{\cancel{2}}(\cancel{2}x)(40-x)\right) + \left(\frac{1}{\cancel{2}}\overset{30}{\cancel{(60)}}(x)\right)\right]$

$= 800 - [(1200 - 40x) + (40x - x^2) + (30x)]$

$= 800 - [1200 \cancel{- 40x} \cancel{+ 40x} - x^2 + 30x]$

Area $= 800 - 1200 + x^2 - 30x$

$= 400 + x^2 - 30x$.

$= x^2 - 30x + 400$

Example candidate response – 2, continued

Question No.

no 3 Area of playground

M1 $= (40 \times 60) - \left[\left(\frac{1}{2}(\overset{20}{\cancel{40}})(60-2x)\right) + \left(\frac{1}{2}\right)(2x)(40-x) + \left(\frac{1}{2}(x)(\overset{30}{\cancel{60}})\right)\right]$

$= 800 - [1200 - 40x + 40x - x^2 + 30x]$

A0 $= 800 - 1200 + \cancel{40x} - \cancel{40x} + x^2 - 30x$

$= x^2 - 30x - 400$

let $f(x) =$

(ii) $x^2 - 30x + 1200$

$f(1) = 1^2 - 30(1) + 1200 = 1229 \cdot 4$

$f(0) = 0^2 - 30(0) + 1200 = 1200$

0 $f(-1) = -1^2 - 30(-1) + 1200 = 1231$

0 $\therefore$ minimum Area of playground $= 1229\ m^2$

Item marks awarded: (i) = 1/2; (ii) = 0/3

Total mark awarded = 1 out of 5

Examiner comment – 1 and 2

(i) Many candidates found this part difficult. Many did not realise that the required area could be obtained by subtracting the sum of the areas of the three triangles from the area of the large rectangle. Many candidates attempted to use Pythagoras's Theorem, as did candidate 1, before changing direction, and many others adjusted their answer to that given. Candidate 2 used a correct method, but made a careless error in attempting to obtain the required answer. Candidate 1 made an error with the area of one of the triangles.

(ii) It was pleasing that most candidates, even if unable to answer part **(i)**, proceeded to use the given answer to obtain the minimum value of A. Only a few candidates did not realise the need to use calculus, and candidate 2 was one of them. Most differentiated correctly and set the differential to 0, though many others thought the second differential was 0. A surprising proportion, at least a third, did exactly the same as candidate 1, obtaining a correct value for x but failing to read the question carefully to find the corresponding value of A. This again illustrates the need to read questions carefully.

Question 4

4 The line $y = \frac{x}{k} + k$, where k is a constant, is a tangent to the curve $4y = x^2$ at the point P. Find

(i) the value of k, [3]

(ii) the coordinates of P. [3]

Mark scheme

4	$y = \frac{x}{k} + k \quad 4y = x^2$		
	(i) $\frac{x^2}{4} = \frac{x}{k} + k \rightarrow kx^2 - 4x - 4k^2 = 0$	M1	Eliminates x or y completely.
	Uses $b^2 - 4ac \rightarrow k = -1$	M1 A1 [3]	Uses $b^2 - 4ac$ for a quadratic = 0 co nb a,b,c must not be f(x)
	(calculus $\frac{1}{k} = \frac{2x}{4}$ B1 $\rightarrow x = \frac{2}{k},\ y = \frac{1}{k^2}$ M1 $\rightarrow k = -1$ A1)		
	(ii) $y = -x - 1,\ 4y = x^2$ $\rightarrow x^2 + 4x + 4 = 0$ $\rightarrow P(-2, 1)$	M1 M1 A1 [3]	Elimination of x or y Soln of eqn. co.

Example candidate response – 1

(4i) $y = \frac{x}{k} + k$, $4y = x^2$

$y = \frac{x^2}{4}$

$\cancel{\frac{x}{k} + k} = \cancel{\frac{x^2}{4}}$

$4\left(\frac{x}{k} + k\right) = x^2$ ✓

$\frac{4x}{k} + 4k = x^2$

✓ $4x + 4k^2 = kx^2$ ✓

M1 $kx^2 - 4x - 4k^2 = 0$

M1✓ $a = k,\ b = -4,\ c = -4k$

A0 $\cancel{b^2 - 4ac \leq 0}$ $b^2 - 4ac = 0$

$\cancel{(-4)^2 - 4(k)(-4) \leq 0}$ $\cancel{b^2 - 4a}\ (-4)^2 - 4(k)(-4k) = 0$

$\cancel{16 + 16k \leq 0}$ $\cancel{16}\ 16 + 16k = 0$

$\cancel{16k \leq -16}$ $k = -1$

$\cancel{k \leq -1}$ not www

(ii) when $k = -1$ ... ①, $4y = x^2$ ... ②

$y = \frac{x}{-1} - 1$ replace in equation (2) M1✓

$4\left(\frac{x}{-1} - 1\right) = x^2$

$\frac{4x}{-1} - 4 = x^2$

$4x + 4 = -x^2$

Example candidate response – 1, continued

Question No.

Nothing is to be written in this margin

$x^2+4x+4=0$ $\quad$ $P=4$

$x^2+2x+2x+4=0$ $\quad$ $S=4$

$x(x+2)+2(x+2)=0$ $\quad$ $(2,2)$

$(x+2)\ (x+2)=0$

$x=-2$ or $x=-2$

$y=1$ $\quad$ $y=1$ $\quad$ M1 M1

coordinates of $P=(-2,1)$ $\quad$ not www $\quad$ A0

4

(ii) Gradient of BA $= \left(\dfrac{3-11}{\ }\right) = -8 \quad = 2$

Item marks awarded: (i) = 2/3; (ii) = 2/3

Total mark awarded = 4 out of 6

Example candidate response – 2

no. 4 line: $y = \frac{x}{k} + k$ Curve: $4y = x^2$

$y = \frac{x^2}{4}$

(i) Solving

M1

$\frac{x}{k} + k = \frac{x^2}{4}$

$\frac{x + k^2}{k} = \frac{x^2}{4}$.

$x + k^2 = \frac{kx^2}{4}$.

$k^2 - \frac{kx^2}{4} = -x$

$4k^2 - kx^2 = -4x$.

~~At y-axis, x = 0.~~

$\cancel{4k^2 - k(0)^2 = -4(0)}$

$\cancel{4k^2 = 0}$.

~~At x-axis, y = 0~~

$-kx^2 + 4x - 4k^2 = 0$

$kx^2 - 4x + 4k^2 = 0$.

Item marks awarded: (i) = 1/3; (ii) = 0/3

Total mark awarded = 1 out of 6

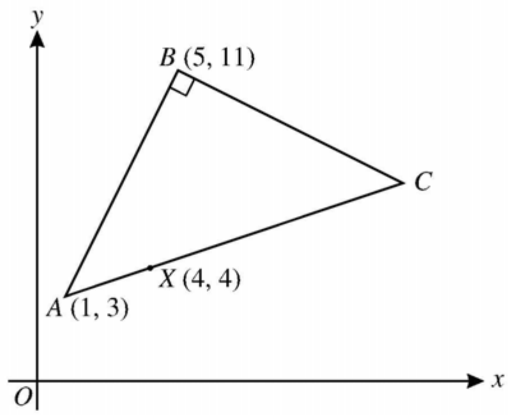
Examiner comment – 1 and 2

(i) Weaker candidates found this question difficult and algebraic errors were very common. Both of these candidates realised the need to eliminate y from the two given equations and to form a quadratic equation in x. Weaker candidates, such as candidate 2, did not recognise that the discriminant, "$b^2 - 4ac$", needed to be equated with 0. There were a lot of algebraic errors over signs and in expressing a, b and c correctly in terms of k, and the error made by candidate 1 in taking "c" as "−4" instead of "$-4k^2$" was very common across all levels of ability.

(ii) Several candidates were unable to proceed with this part of the question, but most realised the need to substitute their value of k into the earlier quadratic equation and to then solve for x. The last accuracy mark was not gained if the answer, as with candidate 1, had been fortuitously obtained in part **(i)**.

Question 5

5



The diagram shows a triangle ABC in which A has coordinates (1, 3), B has coordinates (5, 11) and angle ABC is 90°. The point X (4, 4) lies on AC. Find

(i) the equation of BC, [3]

(ii) the coordinates of C. [3]

Mark scheme

5	A (1, 3), B (5, 11), X (4, 4)		
	(i) Gradient of $AB = 2$		
	Gradient of $BC = -½$	B1	co
	$\rightarrow$ Eqn of BC is $y - 11 = -\frac{1}{2}(x-5)$	M1	For use of $m_1m_2 = -1$
		A1	co – unsimplified is fine
		[3]	
	(ii) gradient of AC (or AX) is ⅓		
	$\rightarrow$ eqn of AC is $y - 3 = \frac{1}{3}(x-1)$	B1	co
	or $y - 4 = \frac{1}{3}(x-4)$	M1	Correct form of line equation + sim eqns
	Sim equations $\rightarrow$ C (13,7)		co
		A1	answer only -0/3- assumed $AB = BC$.
		[3]	Uses graph or table and gets exactly (13,7) allow the 3 marks for **(ii)**.

Example candidate response – 1

5i Gradient $= \dfrac{y-y}{x-x}$. $\quad \dfrac{11-3}{5-1}$. $\quad \dfrac{8}{4}$.

$= 2$ ✓ $\quad (m_1 \times m_2) = -1$.

grad of normal h $= -\frac{1}{2}$ ✓.

$y - 11 = -\frac{1}{2}(x-5)$. ✓

3 ✓

$2y - 22 = -x + 5$.

$2y + x = 27$

ii Coords of C = .

$\cancel{y + 13 =}$

M0

$y - 3 = 5(x - 1)$

$y - 3 = 5x - 5$

$y + 2 = 5x$.

$y = 5x - 2$. (1).

$2y + 2(5x-2)$ X

3

Item marks awarded: (i) = 3/3; (ii) = 0/3

Total mark awarded = 3 out of 6

Example candidate response – 2

5. Gradient of AB.

$\overset{x_1\ y_1}{A(1,3)}$ $\quad$ $\overset{x_2\ y_2}{B(5,11)}$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{11-3}{5-1}$$

$$= \frac{8}{4}$$

$$= 2$$

$\therefore$ Gradient of BC $= -\frac{1}{2}$ ✓

Equation of BC $= \dfrac{y-11}{x-5} = -\frac{1}{2}$

B(5,11)

$2(y-11) = -1(x-5)$

$2y - 22 = -x + 5$

$2y = -x + 5 + 22$

$2y = -x + 27$.

3

To find coordinates of C.

Midpoint $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$A(\overset{x_1}{1}, \overset{y_1}{3})$

M0

$\dfrac{1+x_2}{2} = 4 \Rightarrow 1 + x_2 = 8$

$x_2 = 8 - 1$

$x_2 = 7$.

$\dfrac{3+y_2}{2} = 4 \Rightarrow 3 + y_2 = 8$

$y_2 = 8 - 3$

$y_2 = 5$

3

Coordinates of C $= (7, 5)$

Item marks awarded: (i) = 3/3; (ii) = 0/3

Total mark awarded = 3 out of 6

Examiner comment – 1 and 2

(i) This part of the question was correctly answered by nearly all candidates across the ability range and both of these candidates offered correct solutions. Other candidates made a few numerical errors in finding either the gradient of the line AB, the perpendicular gradient of BC or the equation of BC, but these were relatively infrequent.

(ii) As with part **(i)**, most candidates obtained full marks for this part of the question. Candidate 1 however made no attempt to find the gradient of AX, and hence AC, and there is no evidence for taking the gradient as 5. The solution offered by candidate 2 came from an assumption made by many candidates about properties of the diagram. Several assumed that the triangle ABC was isosceles, but without reason, and others assumed that AX was one quarter of AC, but again with no reasoning. Candidates should be aware that such assumptions cannot be given credit. In this particular case, candidate 2 assumed that X was the midpoint of AC and no credit could be given.

Question 6

6 **(i)** Show that the equation $2\cos x = 3\tan x$ can be written as a quadratic equation in $\sin x$. [3]

(ii) Solve the equation $2\cos 2y = 3\tan 2y$, for $0° \leqslant y \leqslant 180°$. [4]

Mark scheme

6	$2\cos x = 3\tan x$ **(i)** Replaces $\tan x$ by $\sin x \div \cos x$ $\rightarrow 2c^2 = 3s \rightarrow 2s^2 + 3s - 2 = 0$	 M1 M1 A1 [3]	 Uses $t = s \div c$ Uses $s^2 + c^2 = 1$. Correct eqn .
	(ii) Soln of quadratic $\rightarrow y = 15°$ $2y$ can also be $180 - 30$ $\rightarrow y = 75°$.	M1 A1 DM1 A1√ [4]	Method for quadratic = 0 and ÷2 co Works with $2y$ first before ÷ 2 for $90° - 1^{st}$ answer. (loses √ mark if extra soln in range)

Example candidate response – 1

Question No.

6(i) $2\cos x = 3\tan x$

$\cancel{2\cos x = 3\cos x}$

$2\cos x = \frac{3\sin x}{\cos x}$

$2\cos^2 x = 3\sin x$

$2(1-\sin^2 x) = 3\sin x$

$2 - 2\sin^2 x = 3\sin x$

$-2\sin^2 x - 3\sin x + 2 = 0$

$(\div -1)$ $2\sin^2 x + 3\sin x - 2 = 0$

$\cancel{(2\sin x - 1)(x}$

$2\sin^2 x + 3\sin x - 2 = 0$

3

(ii) $2\cos 2y = 3\tan 2y$

$(2\sin 2y - 1)(\sin 2y - 2) = 0$

$2\sin 2y - 1 = 0$ or $\sin 2y = 2$

(rejected since $-1 \le \sin y \le 1$)

$2\sin 2y = 1$

$\sin 2y = \frac{1}{2}$

S | A
T | C

B.a = $30°$

$y = 30°, 180° - 30°,$

$y = 30°, 150°$

M0

A0

DM0

A0

3

Item marks awarded: (a) = 3/3; (b) = 0/4

Total mark awarded = 3 out of 7

Example candidate response – 2

6(i) $2\cos x = 3\tan x$.

$2\cos x = 3\left(\dfrac{\sin x}{\cos x}\right)$ ✓

$2\cos^2 x = 3\sin x$. ✓

$\cancel{2(\sin^2 x)}$ $2(1 - \sin^2 x) = 3\sin x$.

$2 - 2\sin^2 x = 3\sin x$.

$-2\sin^2 x - 3\sin x + 2 = 0$. ← shown. ~~proved~~. ✓

3

ii) $2\cos 2y = 3\tan 2y$.

$2\cos 2y = 3\left(\dfrac{\sin 2y}{\cos 2y}\right)$

$2(\cos^2 x - \sin^2 x) = \dfrac{3(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$

$2 = 3(\sin x\cos x)$

0

3

Item marks awarded: (a) = 3/3; (b) = 0/4

Total mark awarded = 3 out of 7

Examiner comment – 1 and 2

(a) This part of the question was answered well by virtually all candidates. The basic formulae relating $\sin x$, $\cos x$ and $\tan x$ were accurately applied and both of these candidates had little difficulty in obtaining a correct solution.

(b) This part of the question proved to be more difficult. Many candidates, like candidate 2, failed completely to spot the link between the two parts. This particular candidate attempted to use the double angle formulae (not in fact a specific part of this syllabus) and was unable to make any progress. Candidate 1 recognised the link between the two parts, but having correctly obtained angles of 30º and 150º, did not gain the available method marks by not dividing by 2.

Question 7

7 The position vectors of the points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix},$$

where k is a constant.

(i) In the case where $k = 2$, calculate angle AOB. [4]

(ii) Find the values of k for which $\overrightarrow{AB}$ is a unit vector. [4]

Mark scheme

7	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix}$		
	(i) $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = 10$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$= \sqrt{5} \times \sqrt{24} \cos\theta$	M1	Product of 2 moduli
	$\rightarrow \theta = 24.1°$	M1 A1 [4]	All connected correctly. co
	(ii) $\overrightarrow{AB} = \begin{pmatrix} k-1 \\ -k \\ 2k-2 \end{pmatrix}$ allow each cpt ±	M1	Correct for either **AB** or **BA**.
	$(k-1)^2 + k^2 + (2k-2)^2$	M1	Sum of 3 squares (doesn't need =1)
	$\rightarrow 6k^2 - 10k + 4 = 0$	A1	Correct quadratic
	$\rightarrow k = 1$ or ⅔	A1 [4]	co

Example candidate response – 1

7. a

i) $\vec{OA} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ $\quad \vec{OB} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

$a \cdot b = 2 + 0 + 8 = 10$

$\cos\theta = \dfrac{a \cdot b}{|a||b|}$

$|a| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}$

$|b| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}$

$\cos\theta = \dfrac{10}{\sqrt{5}\sqrt{24}}$

$\theta = \cos^{-1} 0.913$

$\theta = 24.1$

Angle $AOB = 24.1°$

4

ii) $\vec{AB} = \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} k-1 \\ -k \\ 2k-2 \end{pmatrix}$

Example candidate response 1, continued

unit vector $= \dfrac{a+b+c}{\sqrt{a^2+b^2+c^2}}$

$= \dfrac{(k-1)+(-k)+(2k-2)}{\sqrt{(k-1)^2+(-k)^2+(2k-2)^2}}$

M2

$= \dfrac{k-1-k+2k-2}{\sqrt{(k-1)(k-1)+k^2+(2k-2)(2k-2)}}$

$= \dfrac{2k-3}{\sqrt{k^2-k-k+1+k^2+4k^2-4k-4k+4}}$

$= \dfrac{2k-3}{\sqrt{6k^2-10k+5}}$

$6k^2-10k+5 = 0$

30
1×30
2×15
3×10
5×6

$k>0$

6

Item marks awarded: (i) = 4/4; (ii) = 2/4

Total mark awarded = 6 out of 8

Example candidate response – 2

Question No.

7(i) Angle $A\hat{O}B = \cancel{+OA}\ \dfrac{\vec{OA}\cdot\vec{OB}}{|\vec{OA}|\cdot|\vec{OB}|}$

$\vec{OA} = \begin{pmatrix}1\\0\\2\end{pmatrix}$

$\vec{OB} = \begin{pmatrix}k\\-k\\2k\end{pmatrix}$

if $k=2$

$\vec{OB} = \begin{pmatrix}2\\-2\\4\end{pmatrix}$

Angle $A\hat{O}B = \dfrac{\begin{pmatrix}1\\0\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\4\end{pmatrix}}{\sqrt{1^2+0^2+2^2}\ \times\ \sqrt{2^2+(-2)^2+4^2}}$

$= \dfrac{\cancel{2}-\cancel{2}+8}{\sqrt{5}\times\sqrt{24}}$

M3

$\sqrt{3}\times\sqrt{3} = 3$

$= \dfrac{8}{120} = \cancel{\dfrac{1}{15}}$

$\cos A\hat{O}B = \dfrac{1}{15}$

$A\hat{O}B = \cos^{-1}\left(\dfrac{1}{15}\right)$

$= 86\cancel{2}^\circ$

Example candidate response – 2, continued

7 (ii) $\vec{AB} = \dfrac{\vec{AB}}{|\vec{AB}|}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} k-1 \\ -k \\ 2k-2 \end{pmatrix}$ ✓

$= \dfrac{k-1-k+2k-2}{\sqrt{(k-1)^2+(-k)^2+(2k-2)^2}}$ ✗ M1 M1

5

Item marks awarded: (i) = 3/4; (ii) = 2/4

Total mark awarded = 5 out of 8

Examiner comment – 1 and 2

(i) Candidates of all ability levels showed a very good understanding of the scalar product of two vectors and invariably the three available method marks were obtained. Both of these candidates were comfortable in their approach to the question. Unfortunately, candidate 2 made a common numerical error when the scalar product was evaluated as 8 instead of 10. This arose from assuming that "$-2 \times 0 = -2$".

(ii) This part of the question proved to be difficult and correct solutions from candidates at all levels were rare. Both of these candidates recognised that vector $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ and proceeded to obtain an expression in terms of k. Both candidates then introduced the modulus of vector $\overrightarrow{AB}$, but neither realised that this modulus, on its own, was equal to 1. Candidate 1 isolated the modulus, but set it to 0 instead of 1, whilst candidate 2 made no further progress.

Question 8

8 **(a)** In a geometric progression, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

(i) the first term, [3]

(ii) the sum to infinity of the progression. [2]

(b) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5°. Find the value of n. [4]

Mark scheme

8	**(a)** **(i)** $ar = 24$, $ar^3 = 13\frac{1}{2}$ Eliminates a (or r) $\rightarrow$ $r = \frac{3}{4}$ $\rightarrow$ $a = 32$	 B1 M1 A1 [3]	 Both needed Method of Solution. co
	(ii) sum to infinity $= 32 \div \frac{1}{4} = 128$	 M1A1√ [2]	 Correct formula used. √ on value of r
	(b) $a = 3$, $d = 2$ $\frac{n}{2}(6 + (n-1)2)$ $(= 360)$ $\rightarrow$ $2n^2 + 4n - 720 = 0$ $\rightarrow$ $n = 18$	B1 M1 A1 A1 [4]	Correct value for d Correct S_n used. no need for 360 here. Correct quadratic co

Example candidate response – 1

8a i) $ar = 24$ (i)

$ar^3 = 13{,}5$ (ii)

$$\frac{ar^3}{ar} = \frac{13{,}5}{24}$$

$$r^2 = 9/16$$

$$\sqrt{r^2} = \sqrt{9/16}$$

$$r = 3/4$$

Substituting r for $3/4$ in (i)

$$a(3/4) = 24$$

$$a = 24 \div 3/4$$

$$a = 32$$

3

ii) $$S_\infty = \frac{a}{1-r}$$

$$\frac{32}{1-3/4}$$

$$\frac{32}{1/4}$$

$$S_\infty = 128$$

2

b) $T_n = a + (n-1)d$

$5' = 3^{\circ} + (n-1)2$

Example candidate response – 1, continued

b) $a = 3°$

$a + d = 5°$

B1 $d = 2°$

$T_n = a + (n-1)d$

0 $5 = 3 + (n-1)2$

$5 = 3 + 2n - 2$

$5 - 3 = 2n - 2$

$2 = 2n - 2$

6 $\frac{4}{2} = \frac{2n}{2}$

$\underline{n = 2}$

Item marks awarded: (a)(i) = 3/3; (a)(ii) = 2/2; (b) = 1/4

Total mark awarded = 6 out of 9

Example candidate response – 2

8 a) 2^{nd} term $= 24$ $\qquad 24, \frac{24}{10.5}, 13\frac{1}{2}$

4^{th} term $= 13\frac{1}{2}$

$U_n = ar^{n-1}$

$24 = ar^1 \; - ③ \longrightarrow a = \frac{24}{r} \; - ①$ ✓

$13.5 = ar^3 - ②$ ✓ $\qquad$ B1

Replace ① in ② $\qquad 13.5 = \frac{24}{r} \times r^{3}{}^{r^2}$ ✓ $\qquad$ M1

$r^2 = 13.5 - 24$

$r^2 = 10.5 \qquad r = \sqrt{10.5}$

Replace in ③

$a = \frac{24}{10.5} = 2.28$

$a = \frac{24}{\sqrt{10.5}} = 7.4$

$a = 24 \times (\sqrt{10.5}) = 77.8$ $\qquad$ M1

ii). $S_\infty = \frac{77.8}{1-\sqrt{10.5}} = -34.7$

b) $1^{st} = 3^0$ $\qquad$ B1

$2^{nd} = 5^0 \qquad d = 2$ ✓ $\qquad S_n = \frac{1}{2}n\{2a + (n-1)d\}$ $\qquad$ M1

$8 = \frac{1}{2}n\{2(3) + (n-1)2\}$

$U_n = 3 + (n-1)2 \qquad 8 = \frac{1}{2}n(6 + 2n - 2)$

$n = 3 + 2n - 2 \qquad 8 = 3n + n^2 - 1$

$U_n = 2n - 1 \qquad n^2 + 3n - 9 = 0$

Example candidate response – 2 continued

$n^2 + 3n - 9 = 0$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-3 \pm \sqrt{9 - (-36)}}{2}$

5

$= \dfrac{-3 \pm \sqrt{45}}{2}$

$n = 1.85$ or $n = -4.85$

Item marks awarded: (a)(i) = 2/3; (a)(ii) = 1/2; (b) = 2/4

Total mark awarded = 5 out of 9

Examiner comment – 1 and 2

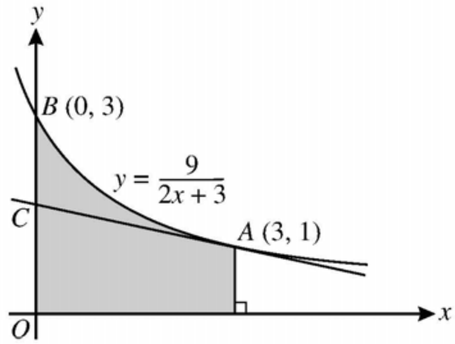
(a) (i) Most candidates were able to write down two correct equations relating *a* and *r*, i.e. "$ar = 24$" and "$ar = 13.5$". The algebra needed to eliminate either a or r often proved difficult and candidate 2 made the algebraic error of quoting $r^2 = 13.5 - 24$, instead $13.5 \div 24$. Candidate 1 correctly obtained the first term as 32.

(ii) Both candidates recognised that the sum to infinity was given by the formula $S_\infty = \dfrac{a}{1-r}$. Candidate 1 obtained a correct answer (128), but although there was a follow through accuracy mark available, this could not be awarded to candidate 2 since the value of $|r|$ was greater than 1. Candidates should be aware that the sum to infinity does not exist if $|r| > 1$ and that this implies that some error has been made in their earlier working.

(b) This question caused most candidates a lot of problems. Most realised that the common difference of the arithmetic series was 2° and the majority, including candidate 2, realised the need to use the sum, S_n, of n terms given by the formula $S_n = \frac{1}{2}n(2a + (n-1)d)$. Candidate 1 used the nth term instead of the sum of n terms, but neither candidate, along with almost half of the total intake, realised that the sum of the n terms was 360°.

Question 9

9



The diagram shows part of the curve $y = \dfrac{9}{2x+3}$, crossing the y-axis at the point $B\,(0, 3)$. The point A on the curve has coordinates $(3, 1)$ and the tangent to the curve at A crosses the y-axis at C.

(i) Find the equation of the tangent to the curve at A. [4]

(ii) Determine, showing all necessary working, whether C is nearer to B or to O. [1]

(iii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Mark scheme

9	$y = \dfrac{9}{2x+3}$ $A\,(3, 1)$ $B\,(0, 3)$		
	(i) $\dfrac{dy}{dx} = \dfrac{-9}{(2x+3)^2} \times 2$	B1 B1	Correct without the ×2. For × 2, independent of first part.
	$\rightarrow m = -\frac{2}{9}$	M1	Correct form of tan - numerical dy/dx
	$\rightarrow y - 1 = -\frac{2}{9}(x-3)$	A1√ [4]	For his m following use of dy/dx. (normal →max 2/4, no calculus 0/4)
	(ii) Meets the y-axis when $x = 0$, $y = 1\frac{2}{3}$. This is nearer to B than to O.	B1 [1]	Sets x to 0 in his tangent. The $1\frac{2}{3}$ and part **(i)** must be correct.
	(iii) Integral of $\dfrac{81}{(2x+3)^2} = \dfrac{-81}{2x+3} \div 2$	B1 B1	Correct without the ÷ 2. For ÷ 2,
	Uses limits 0 to 3 $\rightarrow \dfrac{-9}{2} - \dfrac{-81}{6} = 9\pi$	M1 A1 [4]	Use of limits with integral of y^2 only no π – max ¾. Use of area - 0/4,

Example candidate response – 1

(9) $y = \frac{9}{2x+3}$

(i)

$y = 9(2x+3)^{-1}$

$\frac{dy}{dx} = -9(2x+3)^{-2} \times 2.$

$= \frac{-18}{(2x+3)^2}$

At the pt $(3,1)$.

$\frac{dy}{dx} = \frac{-18}{[2(3)+3]^2}$

$= -\frac{2}{9}.$

Equation of tangent ; $m = \frac{-2}{9}$ $(3,1)$

$\frac{y-1}{x-3} = \frac{-2}{9}$

$9y - 9 = -2x + 6.$

$9y = -2x + 15$

$y = -\frac{2x}{9} + \frac{15}{9}$

4

(ii) $y = -\frac{2x}{9} + \frac{15}{9}$

At y-axis, $x = 0$

$y = -\frac{2(0)}{9} + \frac{15}{9}$

$y = 15/9$

$y = 1.667$

$\therefore$ C is nearer to B.

B1

Example candidate response – 1, continued

9(ii)

$$V = \pi \int_0^3 y^2 \, dx$$

$$V = \pi \int_0^3 \left(\frac{9}{2x+3}\right)^2 dx$$

$$V = \pi \int_0^3 \frac{81}{(2x+3)^2} dx$$

$$V = \pi \int_0^3 81(2x+3)^{-2} dx$$

B1 BO

$$V = \pi \left[\frac{81(2x+3)^{-1}}{-1}(\times 2)\right]_0^3$$

$$V = \pi \left[\frac{-162}{2x+3}\right]_0^3$$

M1

$$V = \pi \left[\frac{-162}{9} - \frac{-162}{3}\right]$$

A0

$$V = 36\pi$$

7

Item marks awarded: (i) = 4/4; (ii) = 1/1; (iii) = 2/4

Total mark awarded = 7 out of 9

Example candidate response – 2

9(i)

$$y = \frac{9}{2x+3}$$

$$\frac{dy}{dx} = 9(2x+3)^{-1}$$

$$= -9(2x+3)^{-2} \times 2$$

$$= \frac{-9}{(2x+3)^2}$$

B1

$x = 3$

$$= \frac{-9}{(2(3)+3)^2}$$

$$= \frac{-9}{(6+3)^2}$$

$$= \frac{-9}{9^2}$$

$$= \frac{-9}{81}$$

Gradient of tangent at A $= -\frac{1}{9}$

Equation of tangent

$\cancel{\text{tor}}$ $(3,1)$

$$\frac{y-1}{x-3} = -\frac{1}{9}$$

$$9(y-1) = -1(x-3)$$

$$9y-9 = -x+3$$

$$9y = -x+3+9$$

$$9y = -x+12$$

$$9y+x = 12$$

2

Example candidate response – 2, continued

(ii)

(iii) $V = \pi \int \left(\frac{9}{2x+3}\right)^2$

$= \pi \int_0^3 \frac{81}{(2x+3)^2}$

$= \pi \int_0^3 81(2x+3)^{-2}$

B1 $= \pi \int_0^3 81 . \frac{(2x+3)^{-3}}{-3\times 2}$

$= 81\pi \left[\frac{(2x+3)^{-3}}{-6}\right]_0^3$

$= \frac{81\pi}{-6} \left[(2x+3)^{-3}\right]_0^3$

M1 $= \frac{81\pi}{-6}\left[\frac{1}{(2x+3)^3}\right] = \frac{81\pi}{-6}\left[\frac{1}{(2(3)+3)^3}\right] - \left[\frac{1}{(2(0)+3)^3}\right]$

$= \frac{8\pi}{-6}\left[\frac{1}{(6+3)^3}\right] - \left[\frac{1}{(2+3)^3}\right]$

$= \frac{8\pi}{-6}\left[\frac{1}{9^3}\right] - \left[\frac{1}{6^3}\right] = \frac{-35}{432}$ units.

5

Item marks awarded: (i) = 3/4; (ii) = 0/1; (iii) = 2/4

Total mark awarded = 5 out of 9

Examiner comment – 1 and 2

(i) This question was a source of high marks for many candidates and both of these candidates attained reasonable marks. Candidate 1 had a completely correct response, whereas candidate 2 did not realise that the equation was composite and omitted to multiply by 2 in the differentiation. The final accuracy mark was follow-through and this was obtained by both candidates.

(ii) This part of the question, worth just 1 mark, required a correct answer to part **(i)**. Surprisingly, many candidates did not realise the need to check whether the value of *y* at which the tangent meets the *y*-axis was greater or less that $1\frac{1}{2}$ (half way between 0 and 3). Candidate 2 made no attempt at the question, whereas candidate 1 obtained a correct deduction.

(iii) Generally, this was a source of high marks for most candidates. These two scripts highlight two of the errors that occurred in a large number of scripts. Candidate 1 correctly realised that the integral of $(2x + 3)^{-2}$ required $\frac{(2x+3)^{-1}}{-1}$, but then multiplied by 2 instead of reversing the process in part **(i)** and dividing by 2. Candidate 2 made an error with the integral of $(2x + 3)^{-2}$ by expressing this as $\frac{(2x+3)^{-3}}{-3}$, but correctly divided by 2. Use of the limits 0 to 3 was correct, but the final answers were incorrect in both cases.

Question 10

10 A curve is defined for $x > 0$ and is such that $\frac{dy}{dx} = x + \frac{4}{x^2}$. The point $P\,(4, 8)$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Show that the gradient of the curve has a minimum value when $x = 2$ and state this minimum value. [4]

Mark scheme

10	$\frac{dy}{dx} = x + \frac{4}{x^2}$ and $P\,(4, 8)$		
	(i) $y = \frac{x^2}{2} - \frac{4}{x} + (c)$	B1 B1	co.co (ignore $+c$ at this stage)
	Uses (4, 8) $\rightarrow c = 1$	M1 A1 [4]	Uses the point after integration for c
	(ii) $\frac{d^2y}{dx^2} = 1 - \frac{8}{x^3}$	B1	Co
	$= 0$ when $x = 2$	B1	Sets to 0 + solution or verifies and states a conclusion (stationary or min)
	$\rightarrow$ gradient of 3	B1	Allow for $x = 2$ into dy/dx.
	$d/dx(1 - \frac{8}{x^3}) = \frac{24}{x^4} \rightarrow$ +ve $\rightarrow$ Min.	B1 [4]	Any valid method - 3rd differential +ve 2nd diff goes −0+, or 1st goes >3,3,>3

Example candidate response – 1

(10) $\frac{dy}{dx} = x + \frac{4}{x^2}$ $\quad P(4,8)$.

$\frac{dy}{dx} = x + 4x^{-2}$

(i) Equation of curve $= y = \int \frac{dy}{dx}\, dx$

$\int \frac{dy}{dx} dx = \int x + 4x^{-2}\, dx = \left[\frac{x^2}{2} + \frac{4x^{-1}}{-1} + c\right]$

$= \frac{x^2}{2} - \frac{4}{x} + c$.

Take $P(4,8)$

~~When $x = 4$, $\frac{dy}{}$~~

$\therefore y = mx + c$

$y = \frac{x^2}{2} - \frac{4}{x} + c$.

$8 = \frac{(4)^2}{2} - \frac{\cancel{4}}{\cancel{4}} + c$

$8 = 8 - 1 + c$.

4 $C = 1$

$\therefore$ Equation of curve is $y = \frac{x^2}{2} - \frac{4}{x} + 1$

(ii) $\frac{d^2y}{dx^2} = 1 + 4x - 2x^{-3}$

B1 $\frac{d^2y}{dx^2} = 1 + \frac{8}{x^3}$

B1 When $x = 2$, $\frac{d^2y}{dx^2} = 1 - \frac{8}{(2)^3} = 1 - \frac{8}{8} = 0$ $\therefore$ minimum

6 Since $\frac{d^2y}{dx^2}$ is positive, there is a minimum value

Item marks awarded: (i) = 4/4; (ii) = 2/4

Total mark awarded = 6 out of 8

Example candidate response – 2

10 $\frac{dy}{dx} = x + \frac{4}{x^2}$

(i) $y = \int x + \frac{4}{x^2}$.

$y = \frac{x^2}{2} - \frac{4}{x}$ ✓✓ $+c$ B2

when $x = 2$.

$\cancel{y = \frac{(2)^2}{2} - \frac{4}{2} = 2 - 2}$

$y = \frac{(2)^2}{2} - \frac{4}{2}$

$= 2 - 2$

$= 0$.

2

Item marks awarded: (i) = 2/4; (ii) = 0/4

Total mark awarded = 2 out of 8

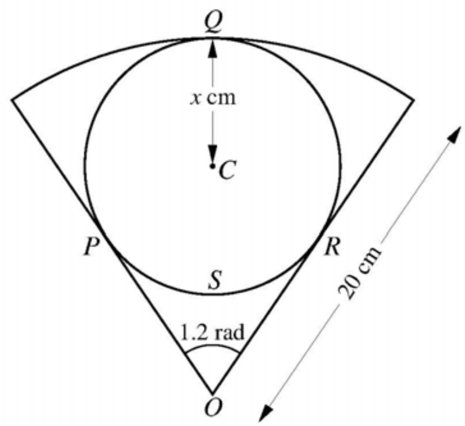
Examiner comment – 1 and 2

(i) This part proved to be one of the more straightforward questions on the paper and candidates from all ability levels scored well. Candidate 1 offered a perfectly correct solution. Candidate 2 integrated correctly, but made the mistake of omitting the constant of integration. Neither candidate made the error of thinking that the equation of the curve is the same as the equation of the tangent.

(ii) This proved to be a difficult question, with only a small minority obtaining full marks. Candidates reading the question carefully would have realised that the question did not ask for the maximum or minimum values of x, but requested the maximum or minimum value of the *gradient*. This meant setting $\frac{d^2y}{dx^2}$ to 0, instead of setting $\frac{dy}{dx}$ to 0, and looking at the sign of $\frac{d^3y}{dx^3}$ to determine whether the value of the gradient was positive or negative. If candidates had labelled the gradient $\left(\frac{dy}{dx}\right)$ as m and looked at $\frac{dm}{dx}$, then at $\frac{d^2m}{dx^2}$, they would have been more successful. Neither of the candidates obtained the easy mark obtained by substituting $x = 2$ into $\frac{dy}{dx}$ to obtain a value of 3. Candidate 2, like many candidates, made no attempt at the question. Candidate 1 obtained a correct expression for $\frac{d^2y}{dx^2}$ and deduced that this was 0 when $x = 2$.

Question 11

11



The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P, Q and R. Angle POR = 1.2 radians.

(i) Show that $x = 7.218$, correct to 3 decimal places. [4]

(ii) Find the total area of the three parts of the sector lying outside the circle with centre C. [2]

(iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR. [4]

Mark scheme

11	**(i)** $OQ = x + OC = 20$	B1	Used somewhere – needs "20".
	$\sin 0.6 = \dfrac{x}{OC} \rightarrow OC = \dfrac{x}{\sin 0.6}$	M1	Use of trig in 90° triangle
	$x + \dfrac{x}{\sin 0.6} = 20 \quad \rightarrow \quad x = 7.218$	M1 A1 [4]	Soln of linear equation. (answer given, ensure there is a correct method)
	(ii) Area = $\frac{1}{2}.\ 20^2 \times 1.2 - \pi \times 7.218^2$ $= 76.3$	M1 A1 [2]	Use of $\frac{1}{2}r^2\theta$ - needs r=20 and $\theta = 1.2$ co
	(iii) Angle $PCR = \pi - 1.2$ Arc $PR = 7.218 \times (\pi - 1.2) = (14.01)$	B1 M1	co Use of $s=r\theta$ with $r = 7.218$ -any θ -even $2\pi/3$
	$OP = OR = \dfrac{x}{\tan 0.6}$ $\rightarrow$ Perimeter of 35.1 cm	M1 A1 [4]	Correct use of trig or Pythagoras co

Example candidate response – 1

11 (i) Arc $PSR = 4.33$

$\theta = 0.6$

$\therefore x = \dfrac{4.33}{0.6}$

$= 7.21666$

$\approx 7.217 \leftarrow$ (proved)

0

(ii) Area of Sector $OPQR = \frac{1}{2} \times 20 \times 20 \times 1.2 = 240\ cm^2$

M

Area of circle $= 2\pi r = 2 \times \pi \times 7.218$

$= 45.352 \approx 45.4\ cm^2$

$\therefore$ Area of three parts of sector lying outside the circle $= 240 - 45.4 = 194.6\ cm^2 \leftarrow$

(iii) Perimeter of region $OPSR$ = Arc $PSR + OP + OR$.

Arc $PSR = 7.218 \times 0.6 = 4.3308$

≈ 4.33

M

$OP = OR = 20 - 7.218 = 12.782$

$\therefore$ Perimeter of region $OPSR = 4.33 + 12.782 + 12.782$

$= 28.894$

$\approx 29.9\ cm \leftarrow$

2

Item marks awarded: (i) = 0/4; (ii) = 1/2; (iii) = 1/4

Total mark awarded = 2 out of 10

Example candidate response – 2

11/.

ii) Area of Sector $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(20)^2$

$= 628.32$

Area of Circle $: 2\pi r^2$

$= 327.35$

Total area of the 3 parts outside $= 628.32 - 327.35$

$= 300.97$

iii) $BR = \frac{1}{2}\pi r^2$

$= \frac{22.68}{2}$

$= 11.34$

PO $\Rightarrow$ ~~tan~~ $0.6 = \frac{7.218}{x}$

PO $\Rightarrow$ $x = 10.55$

~~RO x 2 = 10.55~~

perimeter of OPSR $= 10.55 + 10.55 + 11.34$

$= 32.44$

A 0 0 M1 1

Item marks awarded: (i) = 0/4; (ii) = 0/2; (iii) = 1/4

Total mark awarded = 1 out of 10

Examiner comment – 1 and 2

(i) This proved to be a very difficult part, requiring the use of trigonometry in triangle OCR and realising that $CR = x$ and that $OC = 20 - x$. Only a small minority of candidates were successful. Neither of these two candidates was able to make a correct start.

(ii) This proved to be more successful and both these candidates were not concerned about their failure to cope with part **(i)**. Both candidates realised the need to subtract the area of a circle from the area of a sector. Unfortunately, candidate 2 quoted incorrect formulae for both areas. Candidate 1 obtained a correct answer for the area of the sector, but then used the formula for the circumference instead of the area of a circle.

(iii) Again, this proved to be a difficult part of the question. Two basic errors affected the solutions, the failure to realise that the angle PCR was $\pi - 1.2$ radians and that the length of OR or OP required the use of trigonometry in triangle OCR. Candidate 1 used the formula "$s = r\theta$" with the correct radius, but an incorrect angle (0.6 radians) and then assumed that $OP = OR = 20 - 7.218$. Candidate 2 correctly used trigonometry to find $OP = 10.55$, but then attempted to find the arc length using the formula $\frac{1}{2}\pi r^2$.

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