Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0123456789

FURTHER MATHEMATICS **9231/02**

Paper 2 Further Pure Mathematics 2 **For examination from 2020**

SPECIMEN PAPER **2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2. \qquad [6]$$

2 Find the exact value of $\int_0^1 \frac{1}{\sqrt{3+4x-4x^2}}\,dx$. [6]

3 Find the solution of the differential equation

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x}$$

for which $y = 0$ when $x = \frac{1}{2}\pi$. Give your answer in the form $y = f(x)$. [8]

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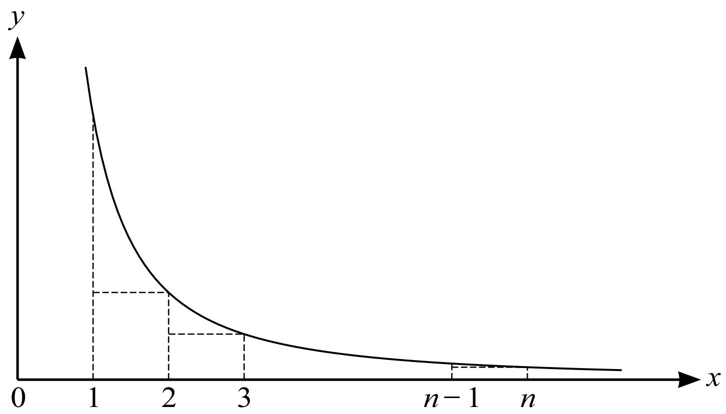

The diagram shows the curve with equation $y = \frac{1}{x^2}$ for $x > 0$, together with a set of $(n-1)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{r^2} < \frac{2n-1}{n}.$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound for $\sum_{r=1}^{n} \frac{1}{r^2}$. [3]

5 The curve C has parametric equations

$$x = e^t - 4t + 3, \qquad y = 8e^{\frac{1}{2}t}, \qquad \text{for } 0 \leqslant t \leqslant 2.$$

(a) Find, in terms of e, the length of C. [5]

(b) Find, in terms of π and e, the area of the surface generated when C is rotated through 2π radians about the x-axis. [5]

6 (a) Using de Moivre's theorem, show that

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$ [5]

(b) Hence show that the equation $x^2 - 10x + 5 = 0$ has roots $\tan^2\left(\frac{1}{5}\pi\right)$ and $\tan^2\left(\frac{2}{5}\pi\right)$. [5]

7 **(a)** Starting from the definition of tanh in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]

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(b) Given that $y = \tanh^{-1}\left(\frac{1-x}{2+x}\right)$, show that $(2x + 1)\frac{dy}{dx} + 1 = 0$. [4]

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(c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1-x}{2+x}\right)$ in the form

$$a \ln 3 + bx + cx^2,$$

where a, b and c are constants to be determined. [5]

8 **(a)** **(i)** Find the set of values of a for which the system of equations

$$\begin{aligned} x - 2y - \ \ 2z + \ \ 7 &= 0, \\ 2x + (a-9)y - 10z + 11 &= 0, \\ 3x - 6y + 2az + 29 &= 0, \end{aligned}$$

has a unique solution. [4]

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(ii) Given that $a = -3$, show that the system of equations in part **(i)** has no solution. Interpret this situation geometrically. [3]

(b) The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

(i) Find the eigenvalues of **A**. [4]

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(ii) Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [4]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.