ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 1

Key messages

It is important that candidates show each step in a method and read each question carefully. Candidates should think carefully before embarking on a substitution to solve simultaneous equations and choose the easier substitution where possible.

Final answers should be checked to see if they are in the form required and given to the accuracy requested. Candidates found trigonometric identities, exponential functions and integration as the reverse function of differentiation challenging.

General comments

There seemed to be sufficient time for candidates to complete the question paper. A few candidates did not attempt the later parts of **Question 12**, but this seemed to be due to finding the questions challenging rather than due to lack of time.

Candidates should recognise the meaning of the word 'hence' in later question parts.

The questions that most candidates produced strong answers for were **Questions 2**, **4,** and **9(a)**. Those that proved to be the most challenging were **Questions 5(b)**, **7**, **9(b)**, **10(c)** and **12(b).**

For questions involving drawing graphs, all intercepts must be clearly labelled. Putting a scale on an axis is insufficient.

Comments on specific questions

Question 1

(a) Most candidates understood that the curve was a cubic and were able to calculate both the x-axis and y-axis intercepts. Some candidates then drew a positive cubic. Many candidates did not gain the first mark for the general cubic shape because the local maximum was placed either on the y-axis or in the first quadrant.

(b) The strongest responses to this part linked the inequality with the graph, recognising that the regions to be identified were those where the graph was above the x-axis. Many candidates tried complicated algebra that was not necessary. Occasionally strict inequalities were used.

Question 2

(a) This part of question was well answered. Candidates who used either long division or equating coefficients were generally successful. Candidates who used the synthetic method did not gain any credit unless they recovered from $(x-\frac{5}{2})(6x^2-20x-16)+5$.

(b) The expression 'product of linear factors' was not understood by some candidates, and some restarted the question again in this part. The most common error was to include –5 in the final answer.

(c) **C**andidates with linear factors in **part (b)** often carried on correctly and gained credit in this part. However, many candidates did not understand what was required, instead equating each linear factor to 5 and solving the resulting equations.

Question 3

(a) Many candidates correctly identified 1 as log10 and were able to use the division or the product rule for logarithms. Some candidates, however, did not read the full question and did not factorise $(x^2 - 1)$ to simplify their answers.

(b) Changing the base was attempted by most and generally candidates were successful. Those who used substitution of log made the algebra easier and generally solved the resulting two term quadratic. A significant number used the power rule to convert $4\log(x+1)$ into $\log(x+1)^4$ but did not reach any solutions. Some candidates who solved the quadratic dealt with the logs correctly but either forgot the negative solution or rejected it. Some candidates did not gain the final mark because they did not rationalise surds to give their answers in the form required by the question.

Question 4

(a) This question was very well answered, with many correct answers seen. A few candidates slipped up with finding p because they incorrectly used $(px)^2$ or expanded incorrectly.

(b) This question was also well answered. The most common error in this part was not squaring the 3 in $\left(\frac{-1}{3y^2}\right)^2$ or keeping the negative sign after squaring $\left(\frac{-1}{3y^2}\right)$.

Question 5

(a) Many candidates found correct values for a and c with very little or no working. Working with the period of the graph proved more challenging. Only a few candidates drew a standard cosine graph and compared the graphs to identify the changes needed.

(b) This question proved very challenging, with few candidates recognising that the line $y = p$ must be horizonal. Many tried values of theta but seemingly at random. This question was omitted by a reasonably high number of candidates.

Question 6

Candidates who integrated both terms correctly tended to substitute the limits and gain full credit. Many candidates made a mistake in integrating the first term as $2\ln(2x-3)$ and a lot of candidates used $3\ln(2x-3)^2$ when they integrated the second term.

Question 7

Most candidates found this question very challenging. The most successful approach was to rearrange to make $\cot\theta$ the subject of the formula, square both sides without expanding the brackets and then use the identity $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$. Successful responses then used $y = \sin^2\theta$ which allowed them to rearrange. Many candidates gained partial credit for rearranging to get $y = \left(\frac{\cos\theta}{3x-2}\right)^2$. However, they often thought that this was the final answer, not realising that this is not y in terms of x only.

Question 8

Many candidates gained partial credit for finding at least one correct solution. Candidates who did not state $\pm\frac{1}{2}$ generally found only two solutions, using $+\frac{1}{2}$ .Two common errors were not dealing correctly with the order of the operation to solve $\sin\left(2\alpha - \frac{\pi}{3}\right) = \pm\frac{1}{2}$ and mixing degrees with radians.

Question 9

(a) This question was well answered. Many candidates correctly simplified the powers of e and used the knowledge that ln1 = 0 or e^0 =1 to reach $4x - y = 0$. Many were able to solve the simultaneous equations by substitution. Many candidates rearranged the second equation to $y = \frac{256}{x^2}$ and substituted into the first equation. Although some were successful this way, many made arithmetic errors. Weaker attempts often involved multiplying the powers rather than adding them.

(b) This question proved to be very challenging for candidates. It required correct use of the reciprocal and choosing a good substitution. Substituting $y = e^{(2x-1)}$ with the use of the reciprocal led to a quadratic to solve from which taking logs gave the required solution. A few candidates used other substitutions such as $y = e^{2x}$ or $y = e^x$ but these involved far more complicated algebra, and hence were less successful.

Question 10

(a) Although some correct responses were seen, most candidates obtained $t = 0$ only. The question asked for the first time the velocity is zero after *P* leaves *O,* clearly indicating that the required *t* was greater than zero.

(b) Many candidates had difficulty integrating $3\sin 2t$ correctly with multiplication, rather than division, by 2 being the most common error. Having integrated, few candidates included the arbitrary constant and attempted to find its value. Some candidates did not express their answer in a form of *s* = ... so did not gain full credit.

(c) This question proved very challenging for candidates, with many obtaining a distance travelled of 0. A few candidates found 0 initially, but realised this must be wrong and made the necessary changes to their limits. A few candidates applied correct limits to start with and obtained 6 m and some used a cosine graph sketch and recognised that the distance would be 4 × 1.5.

Question 11

This question was generally well answered. Candidates needed to produce a plan and to refer back to the question regularly. To find the equation of the tangent, most candidates knew to differentiate and substitute to find the correct gradient and credit was given to those that attempted the differentiation but made slips. Almost all correctly found the *y* coordinate and proceeded to find the tangent equation. The co-ordinates of *A* and *B* needed to be found next and most managed this although the *x*-axis co-ordinate being negative caused arithmetic errors.

Many candidates did not attempt to find the mid-point which was essential for finding the equation of the perpendicular bisector. Mostly the perpendicular gradients were found correctly, often immediately after the tangent gradient. Credit was awarded on a follow through basis for finding a perpendicular bisector from any incorrect mid-points.

Most candidates knew that the final step involved letting $x = y$ or preferably both equal a, and credit was awarded when this was attempted following a valid method to find the equation of the bisector.

Question 12

(a) Most candidates correctly applied either the quotient rule or the product rule. The most common errors were not differentiating $\ln 3x$ correctly, and putting the terms in the numerator of the quotient rule the wrong way round.

(b) Although this question started with 'Hence find …', many candidates did not attempt to use their work from **part (a)** in **part (b)**. Those who realised the connection between the integration as a reverse of differentiation gained at least partial credit, but some responses did not include $+c$ with their integral.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 1

Key messages

Candidates are required to show each step of their working in solutions, especially when calculator use is not permitted**.** Careful reading of the information given in a question is essential. Candidates are also advised to ensure that they have fully met the demands of the question and to recognise the meaning of the word 'hence' in a question part.

General comments

Many candidates made good attempts at most questions. There appeared to be no timing issues and most candidates had sufficient room in their examination booklets to answer the questions.

Comments on specific questions

Question 1

Most candidates were able to find the values of *a* and *c*. Fewer candidates found a correct value for *b*. It was intended that the graph should be read to determine the period of the function, in this case 540^{o}, and hence to calculate the value of *b*.

Question 2

Many fully correct responses were seen. It was essential that candidates changed the base of either the logarithmic term in *r* or the logarithmic term in *s*. The addition law of logarithms could then be applied to find either $\log_3 rs = 8$ or $\log_9 rs = 4$. The final answer could then be obtained from either of theses statements. Common errors included an incorrect change of base, which meant that little progress could be made, and incorrect manipulation of either $\ln_3 9$ or $\ln_9 3$.

Question 3

Candidates are expected to know the derivative of $\tan x$ and consequently $\tan mx$, where *m* is a multiple. Many candidates omitted $\frac{1}{2}$ from their derivatives but were able to continue to obtain a final answer of $\frac{4}{3}$, which gained partial credit. Some candidates, unable to recall the appropriate derivative, differentiated $\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$. This was a commendable approach, although the factor of $\frac{1}{2}$ again tended to be omitted. Most candidates who made any progress gave their final answer in exact from as required, and many completely correct solutions were seen.

Question 4

(a) Most candidates were able to write down the required answer of 6435 by considering the correct number of combinations.

(b) Most candidates were able to write down the required answer of 9 by considering the correct number of combinations.

(c) Many fully correct solutions were seen. Work was often clearly set out, stating the three possible cases that needed to be considered and subsequently the associated combinations.

Question 5

Candidates were instructed not to use a calculator in this question, but many responses contained insufficient working to evidence this. To obtain full credit, it was essential that each step in the calculation was shown to demonstrate understanding of how to manipulate surds.

(a) Candidates needed to find the length of the line *AD* using Pythagoras' theorem. Most candidates did just this and provided sufficient detail in their solution to reach the answer of $\sqrt{22}$. It was then a case of adding the lengths involved to obtain the perimeter.

(b) Most candidates chose to find the area of the complete trapezium, obtaining the expression $\frac{1}{2}\left(17\sqrt{7}-16\right)\left(\sqrt{7}+2\right)$. To gain full credit, evidence of multiplying the two brackets had to be shown in the form of at least three terms. Many candidates omitted this step. Splitting the shape into a triangle and a rectangle was seen less frequently, but was an equally successful method provided sufficient detail was provided in the solution.

(c) Many candidates were unable to write down $\cot DBC$ by using the diagram and the ratio $\frac{BC}{DC}$ to obtain $\cot DBC = \frac{\sqrt{7}+2}{9\sqrt{7}-9}$. Rationalisation of this fraction was then required. Many candidates first found $\tan DBC$, obtaining $27-9\sqrt{7}$ and then attempted to rationalise $\frac{1}{27-9\sqrt{7}}$. There is nothing incorrect about this approach, but it is less efficient and candidates using this method tended to make more arithmetic slips.

Question 6

(a) Most candidates were able to use the given arc length to show that $\theta = 0.75$.

(b) Many correct solutions were seen, with most candidates setting their solutions out so that they could be followed easily. Most responses used the cosine rule to find the lengths of the lines *AB* and *DC*. The arc lengths *AD* and *BC* were usually found correctly. Candidates should work to at least 4 significant figures in their solutions and give their final answer correct to 3 significant figures. Some candidates rounded their lengths to 1 decimal place, and subsequently obtained an inaccurate answer. Other candidates took the length of the line *DC* as 3.75. This is the length of the arc *DC*. Careful reading of the information given in a question is essential to avoid misunderstanding.

(c) Fewer correct solutions were seen to this part. There were several different ways of obtaining the required area. The most successful method involved finding the difference in area between the two segments involved. Candidates who attempted this approach usually obtained full credit. Where this approach was not used, solutions were often difficult to follow, with calculations written down but not related to anything specific. Candidates are advised to either draw another diagram and label it appropriately with reference to the calculations written down or make use of the original diagram to do the same.

Question 7

(a) Many completely correct solutions to this unstructured question were seen. Most candidates were able to formulate a correct plan of action and apply the appropriate techniques to obtain a correct final answer, gaining full credit.

Candidates that made arithmetic errors, often when obtaining a quadratic equation in one variable at the start of the question, were able to subsequently gain method marks for a correct plan of action.

Candidates should aim to use the most efficient method. Many responses found the gradient of the line AB using their coordinates of *A* and *B* rather than reading the gradient from the given equation of the line *AB* in the stem of the question. While not incorrect, this method does make errors more likely.

(b) Fewer correct solutions were seen for this question part. It was intended that candidates make use of displacement vectors. A diagram showing the points *A*, *B* and the midpoint of *AB*, together with a perpendicular line showing *D* and hence the position of *C* would have enabled candidates to visualise the situation. Including the coordinates for each point in the diagram would then have made the application of displacement vectors straightforward.

Question 8

(a) Most candidates gained credit for recognising the need to differentiate as a quotient. Incorrect differentiation of $\left(3x^2-5\right)^{\frac{1}{3}}$ meant that some candidates did not gain full credit, but many gained credit for a correct unsimplified derivative. Candidates who attempted to differentiate the expression as a product tended to be less successful. If an expression is written as a quotient, it is best to differentiate it as a quotient and not attempt to rearrange it as a product. Simplifying to the given form required was challenging for many candidates. If an error had been made with the differentiation of $\left(3x^2-5\right)^{\frac{1}{3}}$ then the final form could not be obtained. Candidates should then have gone back and checked their work for errors. Some candidates with a correct unsimplified derivative often found the subsequent necessary algebraic manipulation difficult.

(b) Candidates with a quadratic numerator in their answer to **part (a)** could gain credit for equating it to zero and attempting to solve the resulting equation. Most candidates gave their answers in exact form as required.

Question 9

(a) Many candidates had difficulty finding the required velocity vector. It was necessary to find the magnitude of the direction vector $\begin{pmatrix}-20\\21\end{pmatrix}$ and relate this to the given speed of *P*.

(b) Candidates were required to 'Write down' the position vector of *P* at time *t*, which implies that a candidate already has all the information needed. If candidates did not reach a correct final answer, credit was available if they obtained the form $\begin{pmatrix}3\\5\end{pmatrix}+$ their velocity vector from **part (a)**.

(c) A subtraction between the position vectors of *P* and *Q* was needed and many candidates attempted this. However, many did not realise that the distance between *P* and *Q* was being requested, not the displacement, and so left their answer in vector form. It was essential that the magnitude of the vector *PQ* be found to complete the question part.

(d) Very few fully correct answers to this part were seen. The key word was 'hence', which meant that the final answer from **part (c)** needed to be used. Many candidates had not obtained their final answer in non-vector form and so showed that when like vectors were equated inconsistent equations were obtained. However, candidates were expected to use the quadratic expression for *PQ* to show that this quadratic did not have any real solutions when equated to zero and so there would be no collision.

Question 10

(a) (i) Most candidates obtained the correct common difference and used a correct sum formula to obtain the given form of the answer. Sometimes algebraic errors meant that candidates did not obtain the correct answer.

(ii) Many candidates missed the crucial instruction in the question to find the 'exact' sum and so made use of their calculator to obtain a decimal answer instead of the exact answer required. Candidates are reminded of the importance of reading questions carefully. It was intended that candidates make use of their answer to **part (a)** and substitute in $x = \frac{2\pi}{3}$. Some candidate started again which was acceptable as the word 'hence' had not been used.

(b)(i) Few correct answers were seen. It was intended that candidates rewrite the geometric progression as $\ln 2y,\ \ln(2y)^2,\ \ln(2y)^4$ and then $\ln 2y,\ 2\ln 2y,\ 4\ln 2y$ in order to obtain a common ratio of 2 and hence a correct *n*th term. Many candidates did not deal with the logarithms correctly and ended up with a common ratio in terms of logarithms.

(ii) Provided candidates had a non-logarithmic common ratio, credit could be gained by making correct use of the sum formula. Full credit for a correct final answer required simplification of the correct numerical denominator.

(c) Most candidates realised that the common ratio was $2w - \frac{1}{4}$. To obtain further credit it was necessary to use the condition for the common ratio of a geometric progression which has a sum to infinity. Candidates needed to state $\left|2w - \frac{1}{4}\right| < 1$ or $-1 < 2w - \frac{1}{4} < 1$. A final answer in the form of one correct continuous inequality was expected. Some candidates misunderstood the question and attempted to find the sum to infinity, again highlighting the need to read questions carefully.

Question 11

(a) Many fully correct solutions were seen, with most candidates able to differentiate the given expression as a product.

(b) Most candidates attempted to make use of their answer to **part (a)** and find the given integral. Many candidates obtained $\frac{1}{2}x^2 \ln x$, often accompanied by a term in x^2. Many answers of $\frac{1}{2}x^2 \ln x - \frac{x^2}{4}$ were seen, but the inclusion of an arbitrary constant was required for full credit.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 1

Key messages

Candidates are required to show each step of their working in solutions, especially when calculator use is not permitted. Careful reading of the information given in questions is essential. Candidates are also advised to ensure that they have fully met the demands of the questions.

General comments

Many candidates found this examination very challenging. However, there appeared to be no timing issues and most candidates had sufficient room in their examination booklet to answer the questions.

Comments on specific questions

Question 1

Most candidates were able to find the values of *a* and *c*. Fewer candidates found a correct value for *b*. It was intended that the graph should be read to determine the period of the function, in this case 960°, and hence to calculate the value of *b*.

Question 2

Many candidates were able to identify the given equation as a disguised quadratic in 2^x, but were unable to simplify $3\left(2^{2x+1}\right)$ to $6\left(2^{2x}\right)$. Candidates with incorrect quadratic equations were able to gain credit for solving to obtain values for 2^x and a subsequent correct method to find *x*.

Question 3

(a) Most candidates either expanded the function and differentiated the result, or made use of the product rule. Those candidates who used the product rule obtained a form that was more easily factorised than those who chose expansion as a first step. The only errors appeared to be arithmetic slips in finding the values of *x* and/or *y.*

(b) Many correct sketches were seen, with candidates showing a correct cubic shape and identifying the intercepts with the axes.

(c) Many candidates found this part of the question challenging. The necessary work had already been done in **part (a)** by calculating the *y* coordinate of the minimum point and sketching the curve, but few candidates realised this. Candidates should be aware that information and results from preceding parts of a question may be connected to, or useful in, later parts.

Question 4

Many candidates approached this question thinking that a form of a ‘reverse chain rule’ was involved. Little progress was made by these candidates. It was expected that the integrand be expanded and then each term integrated separately. Those candidates who had a correct approach usually gave their answer in exact form as required.

Question 5

(a) This question elicited many fully correct solutions, but scripts were also seen where no valid progress was made. Where candidates understood the requirements of the question, most found the gradient of the line and then used the coordinates of one of the given points to find the equation of the line in terms of e^{2y} and x^3. Most were then able to then find *y* in terms of *x*.

(b) This question part required candidates to have obtained an equation of the correct form in **part (a)**. Where this was the case, most candidates gained at least partial credit for a correct method in this part.

Question 6

(a) Most candidates gained credit for recognising the need to differentiate as a quotient. Incorrect differentiation of $\ln\left(2x^2+1\right)$ meant that some candidates did not obtain a correct final answer, but many candidates obtained a correct unsimplified derivative and were awarded credit for this. Candidates who attempted to differentiate the expression as a product tended to be less successful. If an expression is written as a quotient, it is best to differentiate it as a quotient and not attempt to rearrange it as a product.

(b) Most candidates found this part challenging and did not realise that a substitution of $x=1$ into the derivative from **part (a)** was necessary to find the corresponding change in *y*.

(c) Some candidates did not attempt this question part as they had had problems solving **part (b)**. Again, a substitution of $x=1$ into the derivative from **part (a)** was necessary, but if the correct work had been done in **part (b)**, this value would already have been calculated. Few correct answers were seen.

Question 7

(a) The strongest responses to this question contained a clear plan of action. There were two scenarios that needed to be considered, namely a number ending in zero and a number ending in 5. Candidates with a clear plan were often able to obtain the correct final answer.

(b) Very few correct solutions were seen, with most candidates not able to achieve the required first step of writing down a correct equation in terms of factorials.

Question 8

Many completely correct solutions to this unstructured question were seen. Most candidates were able to formulate a correct plan of action and apply the appropriate techniques correctly to obtain a correct final answer, gaining full credit.

Candidates that made arithmetic errors were often able to gain partial credit for a correct plan of action.

Question 9

(a) Many candidates attempted to use an incorrect approach based on linear motion. Other candidates had difficulty with differentiating $-4\cos 2t$, so completely correct expressions for the velocity of *P* were rare.

(b) Candidates with a velocity in the form of a sine curve were usually able to gain partial credit for finding the correct intercept on the *v*-axis and attempting to find an intercept on the *t*-axis. However, few correct sketches were seen, with many candidates not finding the intercepts with the *t*-axis.

(c) Candidates with a correct form of $v=4+k\sin 2t$ in **part (a)** were usually able to obtain credit for correct differentiation of their expression for *v* to obtain an expression for the acceleration.

(d) Candidates who considered an equation of the form $k\cos 2t=0$ were often able to gain full credit for this part.

Question 10

(a) (i) Many candidates found this question part very challenging. It was expected that candidates write down the equation $(a)+(a+d)+(a+2d)=42$ from the information given about the sum of the first 3 terms of the arithmetic progression and the equation $a(a+d)(a+2d)=-6720$ from the information given about the product of the first 3 terms of the arithmetic progression. With the first equation simplifying to $(a+d)=14,$ substitution of 14 into the second equation was then required. However, many candidates tried to solve the equations and often expanded out the terms in the equation for the product of the first 3 terms.

(ii) Candidates also found this question part challenging. It was expected that candidates use the result from **part (a)** and solve simultaneously with the equation $(a+d)=14,$ also from **part (a)**. This question part could also be done by using the original information from the stem of the question to obtain a quadratic equation in either *a* or *d*. Candidates who found *d* first often chose the positive value for *d* which gave a negative value for *a*. The question states that *a* is positive. This highlights the need for candidates to read each question part carefully.

(b) Most candidates were able to write down the third and tenth term of the geometric progression, but few were able to eliminate *a* (the first term) in order to find the value of r^7, where *r* is the common ratio, and hence to find the values of the first term and the common ratio.

Question 11

There were few completely correct responses to this question. It was important that all terms were written as logarithms with a common base. It was expected that candidates would write the equations which each term in terms of $\log_3$. Once this was done, the equations could be solved as standard simultaneous equations. Another option was to write the equations in terms of indices after a change to $\log_3$. Candidates using this approach tended to have more success at obtaining the correct final answers.

Question 12

Provided an initial equation of $\cos\left(3\theta-\frac{\pi}{2}\right)=\frac{1}{2}$ was obtained, most candidates used the correct order of operations to obtain $\frac{5\pi}{18}$ but very few candidates were able to obtain any of the other solutions, which involved considering negative angles.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 2

Key messages

Candidates must show all key steps in their method. When values are incorrect and the method used to find them is not seen, credit cannot be awarded.

Candidates should not rely on calculators to solve equations or to work out the values of derivatives or integrals for particular values. Candidates should use calculators as checking tools in these cases. When such a check indicates that they have made an error, candidates should find the point in the solution where the error arises and ensure that they correct it fully.

General comments

Most candidates showed necessary working to gain any partial credit available and their work was usually well presented. Most candidates attempted most questions. The question most commonly omitted by candidates was **Question 12b**. This proved to be the most challenging question on the paper for candidates who did attempt it. As most candidates attempted **Question 12a**, the high omission rate on **Question 12b** seemed to be caused by the difficulty of the question rather than insufficient time.

The question paper challenged candidates across the attainment range, with the full range of marks seen. A minority of candidates found the paper extremely challenging and gained zero or very few marks.

Comments on specific questions

Question 1

(a) This question was very well answered, with very many fully correct solutions seen. Candidates must remember to label the axes as instructed.

(b) This question was also very well answered, with the vast majority of scripts containing fully correct solutions. Many candidates opted for the more difficult quadratic approach, solving $12x^2 - 48x + 36 = 0$, rather than the conventional linear equations approach.

Question 2

(a) This question was well answered by most candidates. However, some candidates found it challenging to deal with the negative coefficient of x^2.

(b) This question proved to be challenging for many candidates, although candidates who gained full credit on **part (a)** often also gained credit on this part.

Question 3

This was another question that was very well answered by most candidates. Many candidates knew what was required. However, some chose the more complicated quadratic formula method, which often resulted in an incorrect conclusion. Candidates who used the discriminant method usually produced a fully correct solution.

Question 4

(a) This question instructed candidates not to use a calculator. It was therefore crucial that candidates showed full working and worked with exact figures. Some candidates did not show sufficient working in their answers. However, most candidates equated y to zero and solved the quadratic equation correctly. Many candidates found working with surds challenging, using the distance formula instead of a simple subtraction of x values.

(b) Many candidates understood the method required for this question but found the algebra challenging. Some candidates attempted to make y the subject of the first curve and then equate the two curves. A significant number of candidates did not find the y coordinates and some candidates used calculators to do the rationalisation, so were unable to gain full credit.

Question 5

Correct solutions were seen to all of the parts. However, **part (b)(ii)** proved to be the most challenging for candidates, and **part (b)(i)** the best answered. It was rare to see partial credit being gained on any parts, with candidates either able to reach a fully correct solution or not being able to make any valid progress.

Question 6

Candidates usually recognised that this was a product rule question, but many were unable to differentiate $\cos x \sin^2 x$ correctly. Many candidates did not show the substitution of $x = 3$ into their derivative, instead using a calculator that was often set in degrees.

Question 7

Most candidates gained at least partial credit for this question, but fully correct solutions were rare. Most candidates were able to do the differentiation and substitution into the differential equation, but very few obtained the values of m and n.

Question 8

(a) Most candidates knew how to use the information about the first 30 terms and used the correct formula. However, many were unsure about how to use the information about the next 20 terms. S_{20} was a common error as was $S_{50} = -2210$. The simultaneous equations were usually solved well although candidates who used the substitution method fared less well than those using elimination.

(b) Candidates who used $4 + 4r + 4r^2 = 7$ were generally successful in producing a fully correct response. However, many candidates used the sum formula and were often unable to solve the cubic equation correctly. Many candidates found it difficult to determine which values of r were valid for the sum to infinity.

Question 9

Part (a) proved to be a very challenging question, with many candidates omitting this part or giving an incorrect reason. Standard phrases such as ‘not a 1:1 function’ were seen and did not gain credit. **Part (b)** was very well answered. **Part (c)** proved challenging for candidates who tried to use the change of subject approach and became stuck with the algebra. Those that used the quadratic equation approach were generally successful although sign errors were sometimes seen. Very few candidates justified the negative square root.

Question 10

(a) Many candidates who squared the given expression found the question challenging and did not complete the identity. Candidates who changed into $\sin x$ and $\cos x$ usually manged to complete the work correctly. There was some confusion with notation between $\sin^2 x$ and $\sin x^2$ and other candidates often left out the letter x completely. Candidates must use notation correctly so that their answers are clear.

(b) Many candidates gained at least partial credit on this question. Common errors were giving two answers instead of four, using $7\sin x = 5$ and not taking account of the 3θ or not using **part (a)**.

Question 11

This question proved very challenging for most candidates. Considerable confusion was seen between area and perimeter, with many candidates doing a large amount of working involving the shaded area. There were however a reasonable number of fully correct solutions seen. Some responses handled the combination of differentiation, surds and π very well so that some partial credit was obtained.

Question 12

This question was very challenging for many candidates, with **part (b)** more so than **part (a)**. Candidates would be supported by presenting work clearly and accurately, to help keep track of method, progress and expressions. Many candidates did not write down two expressions for **OP** as required by **part (a)** and many different vector expressions were seen. Many candidates omitted **part (b)** and very few correct solutions were seen.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 2

Key messages

Candidates must show all key steps in their method. When values are incorrect and the method used to find them is not seen, credit cannot be awarded.

Candidates should not rely on calculators to solve equations or to work out the values of derivatives or integrals for particular values. Candidates should use their calculators as efficient checking tools in these cases. When such a check indicates that they have made an error, candidates should find the point in the solution where the error arises and ensure that they correct it.

Candidates need to read each question carefully and identify any key words or phrases. When a question part includes the key word ‘hence’, it is expected that candidates use a previously found result to answer the current question part. When a question uses the key phrase ‘show that’, candidates need to show a clear, correct, complete method.

Candidates should take care to present their answers in correct mathematical form. For example, when the argument of a logarithm is a binomial term, brackets should be placed around it.

General comments

Most candidates showed that they were able to correctly combine varied techniques and knowledge when attempting problems requiring the use of several skills, for example in **Question 3(a)** and **Question 9**.

A good proportion of candidates offered well-presented solutions, making the logical progression of their work easy to follow. In some cases, solutions which were not well-presented led to errors such as candidates misreading their own work. Presentation was often poor in **Questions 6(b)**, **8(b)** and **11**, for example. Some candidates benefited from making good use of additional paper to rewrite or continue their solutions. In this case, it was helpful when candidates indicated that they had done this, and where the solution could be found.

Candidates seemed to have sufficient time to complete the examination paper.

Comments on specific questions

Question 1

(a) Most sketches contained the correct intercepts and shape. Some candidates did not have the correct shape due to not extending the ends of their curve sufficiently beyond the x-axis or sketching the minimum point on the y-axis. Some candidates did not label the intercept $x = 2.5$ on the graph, instead only marking the scale at 2 and 3, which was not sufficient. Similarly, some intercepts were not written in the correct order, and some negative signs were omitted. A minority of responses gave quadratic or linear functions rather than cubic.

(b)(i) Candidates who used their graph from **part (a)** were more often successful in solving the inequality. Some candidates gained partial credit for obtaining one inequality. A few candidates expanded their expression before writing inequalities, implying that the critical values had been found using a calculator. This approach often resulted in errors.

(ii) Many correct solutions were seen. Some sketches contained linear maxima rather than curved, and some cusps were drawn as rounded, which was not condoned. A few candidates drew a correct shape but had all three cusps on the positive x-axis.

Question 2

(a) A good number of fully correct solutions were seen. A few candidates gave inaccurate final answers despite using a correct method, due to the use of rounded decimals. Several candidates did not gain full credit as they had their calculator in degree mode. Some common mistakes were:

- altering the argument of the trigonometric function in some way
- differentiating the trigonometric function
- multiplying the trigonometric function by a function of x rather than a constant
- substituting the limits before integrating
- treating the integral as $\frac{\sin x}{4}$.

(b) A good number of fully correct solutions were seen. Once a correct solution had been seen, any subsequent working was ignored. A few candidates benefitted from this as it was evident that the simplification of the second term was sometimes an issue. The first term was often incorrectly integrated to give either:

- an incorrect multiple of $\ln(4x - 3)$ or $\frac{1}{4}\ln 4x - 3$, earning partial credit
- $\frac{(4x-3)^0}{0}$ which, usually then became 0, or $\frac{1}{4}\ln(x-3)$ following incorrect factorisation of the denominator, which did not gain credit.

The second term was often incorrectly integrated to give $\ln(x^3)$ or a multiple of x^{-4} . Some candidates attempted to combine the terms in some way and made no progress.

Question 3

(a) Most candidates started this question well by rearranging to form a three-term quadratic equation. Many of these went on to find the discriminant, which was the most efficient method. However, some candidates did not state clearly that the reason why the equation had no real roots was that the value of the discriminant was negative.

(b) A good number of candidates gave fully correct solutions. Some candidates incorrectly discarded the negative solution at some point. Many candidates used a substitution such as $y = \sqrt[3]{x}$. A few candidates stated and used $x = \sqrt[3]{x}$. This was acceptable but often resulted in errors. Some candidates found the correct values for $\sqrt[3]{x}$ and then took the cube root as the final step rather than cubing as required.

Question 4

(a) This part of the question was correctly answered by almost all candidates. The most popular solution was to find p(2), although some candidates did attempt algebraic long division or synthetic division.

(b) (i) As candidates were required to 'show that' $2x - 1$ was a factor, it was important that solutions contained no errors. This was often the case and full credit was awarded. Responses that used the factor theorem sometimes omitted brackets at some stage. Those using algebraic long division sometimes made miscopying errors.

(ii) Most candidates correctly used the factor $2x - 1$ to find the corresponding quadratic factor. A few candidates made no attempt to then factorise the quadratic factor. Candidates who used synthetic division often omitted to divide the coefficients of the quadratic factor by 2. The weakest responses attempted to use $x - 2$ as a factor, misinterpreting **part (a)**.

(iii) Complete success in this part of the question depended on success in the previous part. A good proportion of responses were fully correct. Occasionally, candidates found a solution from $3\sin\theta + 5$

= 0, which was not condoned for full credit. Common errors included solving $\sin^3\theta - 1 = 0$ or stating values for x only.

Question 5

This question was well answered. A few candidates used correct methods but did not state the equation of the tangent in the required form. The most common error in the initial stage of the solution was to state the derivative as $10e^{2x-1} + e$.

A few candidates stated the decimal value of $\left.\frac{dy}{dx}\right|_{x=1}$ without stating $10e^{2x-1}$, which could not be credited.

Question 6

(a) Many concise and accurate solutions were seen for this part. Most candidates made the correct substitutions for $\cot x$ and $\operatorname{cosec} x$. A small number of candidates wrote, for example, $\sin^3 x$ as $\sin x^3$. Some candidates wrote their trigonometric functions without arguments, which should be avoided.

(b) Candidates who fully factorised the left-hand side of the equation generally offered neat and accurate solutions. Of these candidates, some did not find all solutions in the required range, often because the candidate only considered positive square roots. Some candidates did not make it clear which angles were in their final set of solutions.

Question 7

(a) A fair number of fully correct solutions were seen. Candidates who attempted to find the number of cases where O and A were next to each other and subtract from all possible arrangements were more successful than those who attempted to list all the cases where they were not next to each other. Common errors were:

- not multiplying 8! by 2 before subtracting from 9!
- subtracting $2 \times 7!$ from 9!
- using combinations where permutations or simple factorials were appropriate.

(b) More responses gave the correct answer in this part of the question, with fewer gaining partial credit. Common errors were to evaluate:

- $5! \times 4!$ or $5! \times 5!$
- ${}^9C_5$ or ${}^9P_5$
- an incorrect multiple of 24.

Question 8

(a) Most candidates gained full credit. Some common errors were drawing lines that passed through the origin, using logarithms to base 10 or attempting to plot values of $\frac{y}{10}$, which enabled points to be plotted on the scaled axes.

(b) Responses to this part of the question were much more varied. Candidates who used two of the first, third and fifth points to find the gradient gave more accurate solutions than those who used the second point or the fourth point, which were not quite on the line. Some candidates read the value of the y-intercept from the graph. A small number of candidates stated a correct equation for the line. More commonly, candidates wrote 'y =' rather than '$\ln y$ =' or did not attempt to form the equation.

(c) Candidates who found $\ln y = 2.8$ and then used the graph usually offered an accurate solution. This was not the most common method used, however, and many candidates chose to use an equation. This was sometimes sufficiently accurate, although not as reliable.

Question 9

The question was well answered, with many fully correct solutions seen and others gaining partial credit. It was quite common for candidates to make errors which could have been avoided if they had marked their values on their diagram. For example, some candidates found the coordinates of A to be (6, 0). When finding the area, the most successful approach was to find the area between the curve and the x-axis from $x = 2$ to $x = 4$ and to subtract the area of triangle ACD.

Question 10

(a) This question proved challenging, with few correct explanations seen. Candidates who stated 'it' was a one-one function were not credited as it was not clear that they were indicating that f was one-one.

(b) A small number of candidates stated the correct inverse function. In many responses, an incorrect order of operations was used to find the inverse function, or expressions were in terms of y rather than x and so the method was incomplete. A few candidates used incorrect notation, either using x rather than f^{-1} or stating range < 0 or similar. Fewer candidates were able to state the correct domain. More commonly, completely incorrect domains were stated, such as $x > 0$ or $f^{-1} > e^3$.

(c) A very small number of candidates completed this correctly as success depended on having a correct inverse function in **part (b)**. It was common for candidates to try to find the expression for $g(x)$ by forming $y^2 + 3 = 2x$ and solving for y. Most candidates who used this approach were not successful.

Question 11

A reasonable number of fully correct solutions for this question were seen. Candidates who presented their work logically, writing the expansion they needed and forming three equations before combining them, were the most successful. It was more common for candidates to try to find an equation in terms of n than an equation in terms of a. Many candidates were unable to form all three equations. This was usually because they found it challenging to simplify either the indices or the factorial form of ${}^nC_2$. Some candidates found a non-integer or negative value for n, which is not permissible as in the binomial theorem for this syllabus n is a positive integer.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 2

Key messages

Candidates should read each question carefully and identify any key words or phrases. When a question includes the key word 'hence', candidates are expected to use a previously found result to answer the current part of the question.

Sufficient working needs to be shown for credit to be awarded. This is especially the case when the question indicates that a calculator must not be used.

When values are incorrect and the method used to find them is not shown, credit is not given. For example, the substitution of limits into an integral should be shown so that the method can be seen and assessed. A calculator should not be used instead of showing this necessary method step.

Candidates should check that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus or solving trigonometric equations. Candidates should also be aware of how to use their calculator to check their solutions.

General comments

Some candidates demonstrated good problem-solving skills. For example, in **Question 6(b)(i)**. These candidates interpreted the information given correctly then formed and carried out a plan which involved a combination of skills.

Solutions were generally well presented, making the logical progression of the work easy to follow. In some cases, solutions which were not well-presented led to errors, such as candidates misreading their own work. Presentation could have been improved in **Questions 6(b)**, **8(b)** and **11**. Some candidates benefited from making good use of additional paper to rewrite or continue their solutions. In this case, it was helpful when candidates indicated that they had done this, and where the solution could be found.

Candidates seemed to have sufficient time to complete the examination paper.

Comments on specific questions

Question 1

Responses to this question were varied. Many candidates drew a simple diagram to help them interpret the information in the question. Most candidates attempted to find the equation of the perpendicular bisector and use this to find the intercepts with the coordinate axes. Some candidates were able to find the coordinates of the midpoint and the gradient of the line *AB*.

A few candidates used the gradient of the perpendicular bisector to find a displacement vector and applied this to the coordinates of the midpoint in order to find the intercepts.

Question 2

Some fully correct and well-presented solutions were seen. Most candidates were able to form an expression for the discriminant. However, some candidates did not simplify their expression correctly. A small number of candidates stated a single value for *k*, not a 'set of possible values'.

Question 3

(a) Most candidates drew a pair of acceptable graphs. Some candidates plotted and joined points in order to draw the graphs, which was usually successful. A few candidates did not draw accurately or did not use a ruler. This question used the command word “draw” rather than “sketch”, meaning accurate diagrams were necessary.

(b) Many fully correct solutions were seen. Some candidates misinterpreted the inequality or graphs, stating the answer $1 \leqslant x \leqslant 3$. A few candidates attempted to solve the inequality algebraically rather than graphically, which was not successful.

Question 4

(a) Few fully correct solutions to this question were seen. A few candidates were able to identify the correct term but made an error when evaluating it. Other candidates wrote down a few of the terms from the expansion without identifying the correct term.

(b) (i) This question proved very challenging for candidates. A variety of methods were seen. It was essential that solutions contained sufficient working to demonstrate that a calculator had not been used, as many responses showed correct answers coming from incorrect working. Some candidates expanded brackets and did not use the binomial theorem, as indicated in the question.

(ii) Success in this part of the question depended on candidates having a value for k in **part (b)(i)**. A good number of solutions were awarded full credit, showing sufficient evidence of rationalising and simplifying to demonstrate a calculator had not been used.

Question 5

(a) (i) This question was challenging for some candidates. Stronger responses accurately and concisely rewrote the function as $\sec^2 x + 2\tan^2 x$ and then simplified to the correct form. It was common to see incorrectly manipulated expressions in the first step. For example, the given expression was sometimes rewritten as $1+\dfrac{2\sin^2 x}{\cos^2 x}$.

(ii) Some excellent solutions were seen to this part. Candidates who gave the correct solution in **part (a)(i)** were usually successful. Occasionally candidates did not offer both solutions or gave solutions in degrees. A few responses did not solve the equation and instead often offered an evaluation of f(4).

(iii) Candidates found this part of the question to be challenging and fully correct solutions were rare. Many candidates omitted this part. Some candidates successfully differentiated their expression of the form $a\tan^2 x + b$. Some chose to apply the quotient rule to the original form of the function. However, this often resulted in errors.

(b) A wide variety of responses were seen to this question, with many candidates finding it challenging. Some candidates formed a correct equation in $\sin\theta$ and solved it to find a correct pair of values. Some candidates were unable to form a 3-term quadratic equation in $\sin\theta$, and attempts to rewrite and solve $5\sin\theta(10\sin\theta + 1) = 3$ were seen.

Question 6

(a) Many candidates were able to factorise fully and then write down the required solutions. Some students wrote down the correct answers without any working, which was not credited.

(b) (i) Many fully correct solutions were seen. A few candidates made one error when calculating the value of c but offered an otherwise correct solution. Some candidates used their expression for $p'(x)$ with the stationary values to form simultaneous equations. Others used their expression to form a product of factors whose coefficients could be compared with the derivative.

(ii) Most responses attempted to find the second derivative and a good number of these were correct.

Question 7

(a) Some excellent solutions were seen to this part of the question. Candidates who observed that the given shaded area was a difference of sector areas with radii of 9 cm and 5 cm usually formed a simple equation and solved it correctly. Candidates who stated the angle as a multiple of π, rather than a decimal, were more likely to be accurate in the next part of the question.

(b) Candidates found this part of the question more challenging than the previous part. However, some excellent solutions were seen. The most successful approach was to use the cosine rule. Some candidates did not earn full credit due to inaccurate final answers, usually due to using rounded figures.

Question 8

Candidates found this question challenging, with many candidates omitting the question. A few candidates integrated once to find $\frac{dy}{dx}$ and found the constant of integration for this. Other candidates misinterpreted the information given. These candidates usually attempted to find the constant for $\frac{dy}{dx}$ using the point $\left(\frac{3\pi}{16}, \frac{\pi}{4}\right)$ and made no use of $\frac{dy}{dx} = \frac{3}{4}$.

Question 9

A small number of candidates offered fully correct solutions or solutions that included a correct and actioned plan. A reasonable number of candidates attempted to find the equation of the tangent at A. Some candidates found the gradient of the tangent quite challenging. These candidates offered, for example, $-(3x-1)^{-2}$ or $\frac{(3x-1)^{-2}}{3\times -2}$ for the derivative.

A few candidates incorrectly attempted to find the difference of 'the area between the curve and the x-axis' and 'the area between the tangent and the x-axis' from $x = 1$ to $x = 9$. Other candidates found the integration of the expression for the curve quite challenging and usually offered $4x + \frac{(3x-1)^0}{0}$.

Question 10

Few fully correct solutions to this question were seen. Some candidates found correct values for both λ and μ but did not interpret these correctly to find the values of m and n. A good number of candidates found at least one correct expression for $\overrightarrow{OP}$, usually in terms of λ, **a** and **c.** Some candidates misinterpreted the ratio 2:3 as being OD:OC.