

Cambridge O Level

ADDITIONAL MATHEMATICS**4037/13**

Paper 1

October/November 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **8** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

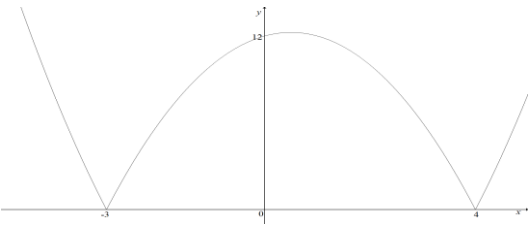
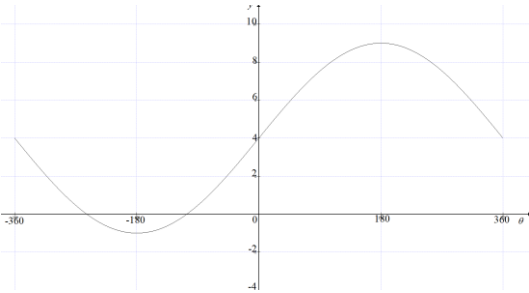
Types of mark

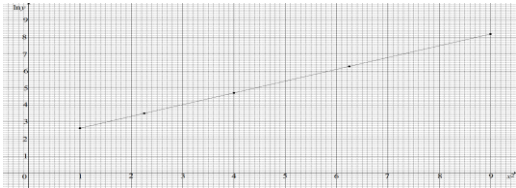
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- | | |
|------|----------------------------|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfw | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |

Question	Answer	Marks	Guidance
1(a)	$y = x^2 - x - 12$ $\frac{dy}{dx} = 2x - 1$ or $(x + 3) + (x - 4)$ or $y = \left(x - \frac{1}{2}\right)^2 - \frac{49}{4}$ or using symmetry $x = \frac{4 - 3}{2}$	M1	For expanding the brackets and differentiate with at least one correct term or for using the product rule or for completing the square or for using symmetry
	$x = \frac{1}{2}$	A1	
	$y = -\frac{49}{4}$ oe	A1	
1(b)		2	B1 for the correct shape. Must have the parabola part of the curve with maximum in the first quadrant and cusps on the x -axis. Ignore labelling of their maximum point if incorrect coordinates B1 for correct intercepts. Must be correct shape
1(c)	$k > \frac{49}{4}$ oe	B1	FT on $\left \text{their } -\frac{49}{4} \right $ excluding $k > 12$
2		4	B1 for correct shape must be a curve with one min in 3 rd quadrant and one max in first quadrant and correct endpoints $(-360, 4)$ and $(360, 4)$ Ignore labelling of their maximum point if incorrect coordinates. depB1 for intercept of 4 on y -axis. Must have the correct shape depB1 for max in correct position of $(180^\circ, 9)$. Must have the correct shape depB1 for min in correct position of $(-180^\circ, -1)$. Must have the correct shape
3	$4x^2 - 4kx - k + 2 [= 0]$	B1	soi
	$k^2 + k - 2$ Critical values $-2, 1$	2	M1 for use of discriminant on <i>their</i> three-term quadratic equation to obtain two critical values
	$-2 < k < 1$	A1	Strict inequality

Question	Answer	Marks	Guidance												
4(a)	$3 = \log_2 8$	B1													
	$\log_2 \frac{8a^4}{b}$	2	M1 for correct use of two operations from multiplication, division or power rule for logs to the base of 2. A1 for log to the base of 2 only												
4(b)	$\lg x = \frac{4}{\lg x}$ or $\frac{1}{\log_x 10} = 4 \log_x 10$	B1	Change of base												
	$(\lg x)^2 = 4$ or $(\log_x 10)^2 = \frac{1}{4}$	B1	Dep on correct change of base Must work with $(\lg x)^2$ or $(\log_x 10)^2$ not $\log x^2$ or $(\log_x 100)$												
	$x = 100$	B1	Dep on correct change of base												
	$x = \frac{1}{100}$ or 0.01	B1	Dep on correct change of base												
5(a)	$p(-2): -8a + 4b + 38 + c = 0$	M1	For substitution of -2 in $p(x)$ and equating to zero. Allow one sign error in evaluating												
	$p(-1): -a + b + 19 + c = 20$	M1	For substitution of -1 in $p(x)$ and equating to 20. Allow one sign error in evaluating												
	$7a - 3b = 39$	A1	AG – must be from correct work												
5(b)	$p'(1): 3a + 2b - 19 = 1$	M1	For substitution of 1 in $p'(x)$ and equating to 1 Allow one sign error in evaluating. Can be unsimplified												
	$a = 6, b = 1, c = 6$	2	M1 dep for solution of <i>their</i> equation with that from (a) to find at least one unknown.												
6(a)	<table border="1" style="margin-bottom: 10px;"> <tr> <td>x^2</td> <td>1</td> <td>2.25</td> <td>4</td> <td>6.25</td> <td>9</td> </tr> <tr> <td>$\ln y$</td> <td>2.64</td> <td>3.51</td> <td>4.72</td> <td>6.28</td> <td>8.18</td> </tr> </table> 	x^2	1	2.25	4	6.25	9	$\ln y$	2.64	3.51	4.72	6.28	8.18	2	M1 for plotting points with one error
x^2	1	2.25	4	6.25	9										
$\ln y$	2.64	3.51	4.72	6.28	8.18										

Question	Answer	Marks	Guidance
6(b)	$\ln y = x^2 \ln b + \ln A$	B1	May be seen in part (a)
	Gradient = $\ln b$ ($\ln b = 0.7$) $b = 2$	2	M1 for attempt to find the gradient and equate to $\ln b$ Gradient must be from linear graph of $\ln y$ vs x^2
	Intercept = $\ln A$ ($\ln A = 1.95$) $A = 7$	2	M1 for attempt to use intercept
6(c)	When $y = 200$, $\ln y = 5.3$ $x^2 = 4.85$ $x = 2.2$ (allow 2.1 or 2.3)	2	M1 for using <i>their</i> linear graph with $\ln y = 5.3$ to obtain a value for x^2 A0 for $x = \pm 2.2$ if -2.2 is not rejected
7(a)	$x^2 + 3x^2 \ln x$	2	M1 for attempt to differentiate a product Allow unsimplified for 2 marks
7(b)	$\int 3x^2 \ln x \, dx = x^3 \ln x - \int x^2 \, dx$	B1	
	$\left[x^3 \ln x - \frac{x^3}{3} \right]_1^2$ $8 \ln 2 - \frac{8}{3} + \frac{1}{3}$	M1	Dep on B1 M1 for correct use of limits
	$\ln 256 - \frac{7}{3}$	2	A1 for one correct term
8(a)	$5x^2 + 3x - 14 = 0$ or $5y^2 - 4y - 57 = 0$	M1	soi
	$x = \frac{7}{5}$, $y = \frac{19}{5}$ $x = -2$, $y = -3$	3	M1 for attempt to solve <i>their</i> quadratic to obtain either $x = \dots$ or $y = \dots$ A1 for one correct pair both x or both y or one correct (x, y) point
	Midpoint $\left(-\frac{3}{10}, \frac{2}{5} \right)$	B1	Must be correct midpoint
	Gradient of perpendicular $-\frac{1}{2}$	B1	Must be correct
	Perp bisector: $y - \frac{2}{5} = -\frac{1}{2} \left(x + \frac{3}{10} \right)$	M1	Must be using <i>their</i> midpoint and a gradient $= -\frac{1}{2}$
	$k = -\frac{4}{5}$	A1	

Question	Answer	Marks	Guidance
8(b)	$\left(\frac{9}{2}, -2\right)$	2	B1 for one correct FT on $2 \times (\text{their } k) - \text{their } \frac{2}{5}$
	$\left(-\frac{51}{10}, \frac{14}{5}\right)$	2	B1 for one correct FT on $\left[\left(3 \times \text{their } \frac{2}{5}\right) - (2 \times \text{their } k)\right]$
9(a)	$\mathbf{c} - 2\mathbf{a}$	B1	
9(b)	$4\mathbf{a} + \frac{2}{3}(\text{their } (\mathbf{c} - 2\mathbf{a}))$ oe	M1	Alternative route: $OC + CB + BD = \mathbf{c} + 2\mathbf{a} - \frac{1}{3} \text{their } AB$
	$\frac{8}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}$	A1	Allow unsimplified
9(c)	$\mu \left(\text{their } \left(\frac{8}{3}\mathbf{a} + \frac{2}{3}\mathbf{c} \right) \right)$	B1	Must be in terms of \mathbf{a} and \mathbf{c} in a valid vector form. Allow unsimplified
9(d)	$\overrightarrow{AC} = \mathbf{c} - 4\mathbf{a}$	B1	
	$\lambda(\text{their } (\mathbf{c} - 4\mathbf{a}))$	B1	Must be in terms of \mathbf{a} and \mathbf{c} in a valid vector form
9(e)	$4\mathbf{a} = \text{their } (\mathbf{c}) - \text{their } (\mathbf{d})$ oe	M1	Must be in terms of \mathbf{a} and \mathbf{c} in a valid vector form
	$\lambda = \frac{1}{2}, \mu = \frac{3}{4}$	3	M1 dep on first M1 for equating like vectors once M1 dep on first M1 for attempt to solve 2 simultaneous equations in λ and μ . leading to $\lambda = \dots$ or $\mu = \dots$ A1 for both
10(a)	$\tan \theta = \frac{2}{7}, \tan \theta = -1$	2	M1 for attempt to factorise or use formula to obtain $\tan \theta = \dots$
	$15.9^\circ, -164.1^\circ, -45^\circ, 135^\circ$	2	A1 for two correct solutions A1 for a further 2 correct solutions and no extras in the range

Question	Answer	Marks	Guidance
10(b)	$\sin(3\phi - 1.5) = \frac{2}{3}$ $3\phi - 1.5 = 0.7297$	M1	Correct order of the operation Do not accept in degrees
	$3\phi - 1.5 = 2.41[12], 7.01[3]$	A1	soi by correct answers with no extras within the range
	0.743, 1.30, 2.84	3	M1 dep on first M1 for correct order of operations or one correct solution A1 for one solution A1 for a further 2 correct solutions and no extras in the range Do not accept in degrees
11(a)	$d = 3\log_x 3$ or $\log_x 3^3$ nfw	B1	Must be exact Allow $d = \log_x 27$ nfw
	$\frac{n}{2}(2\log_x 3 + 3(n-1)\log_x 3)$ nfw	M1	For use of sum formula with <i>their</i> d must be in the form of $\log_x 3$
	$\frac{n}{2}(3n-1)\log_x 3$ or $\left(\frac{3n^2}{2} - \frac{n}{2}\right)\log_x 3$	A1	Must be in the form of $k \log_x 3$
11(b)	$r = 3\tan^2 \theta$	B1	soi
	$ 3\tan^2 \theta < 1$ or $[-1 <] 3\tan^2 \theta < 1$ or $\left[-\frac{1}{3} < \right] \tan^2 \theta < \frac{1}{3}$ or $[0 <] \tan^2 \theta < \frac{1}{3}$	B1	
	$\tan \theta < \frac{1}{\sqrt{3}}$ or $0 < \tan \theta < \frac{1}{\sqrt{3}}$	B1	
	$0 < \theta < \frac{\pi}{6}$	B1	