

# Cambridge O Level

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**ADDITIONAL MATHEMATICS****4037/12**

Paper 1

**October/November 2024**

MARK SCHEME

Maximum Mark: 80

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **10** printed pages.

**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**MARK SCHEME NOTES**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

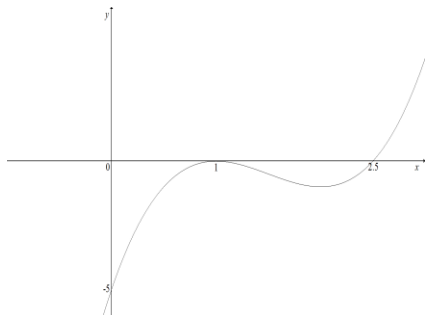
**Types of mark**

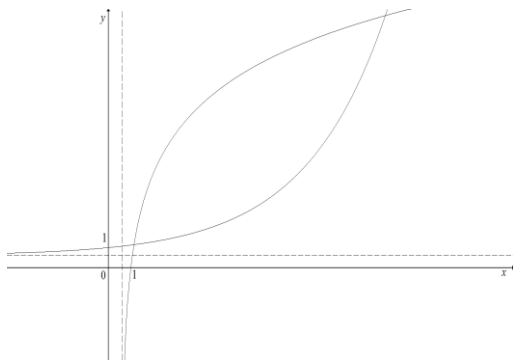
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

**Abbreviations**

- awrt answers which round to  
 cao correct answer only  
 dep dependent  
 FT follow through after error  
 isw ignore subsequent working  
 nfwf not from wrong working  
 oe or equivalent  
 rot rounded or truncated  
 SC Special Case  
 soi seen or implied

Question	Answer	Marks	Guidance
1(a)	$b = 3$	<b>B1</b>	
	Use of $y = a \cos bx + c$ , with <i>their</i> $b$ and either set of given coordinates	<b>M1</b>	<i>their</i> $b \neq \frac{2\pi}{3}$
	$c = -2$	<b>A1</b>	
	$a = 5$	<b>A1</b>	
1(b)	Minimum when $\cos bx = -1$ so	<b>B1</b>	Allow for <i>their</i> $b$ , $b \neq \frac{2\pi}{3}$
	$x = \frac{\pi}{3}$	<b>B1</b>	Allow if $b$ is correct
	$y = -7$	<b>B1</b>	<b>FT</b> on <i>their</i> $c$ – <i>their</i> $a$
	<b>Alternative</b>		
	$\frac{dy}{dx} = -15 \sin 3x$	<b>(B1)</b>	<b>FT</b> on <i>their</i> $a$ , $b$ and $c$ , $b \neq \frac{2\pi}{3}$
	When $\frac{dy}{dx} = 0$ , $x = \frac{\pi}{3}$	<b>(B1)</b>	Allow if $b$ is correct
	$y = -7$	<b>(B1)</b>	<b>FT</b> on <i>their</i> $c$ – <i>their</i> $a$
2(a)	$(f'(x)) = 2(x-1)(2x-5) + 2(x-1)^2$ oe or $6x^2 - 18x + 12$	<b>M1</b>	For use of product rule or expansion and differentiation
	$2(x+1)(2x-5) + 2(x-1)^2 = 0$ oe or $6x^2 - 18x + 12 = 0$ oe	<b>M1</b>	<b>Dep</b> for equating <i>their</i> quadratic $f'(x)$ to zero and attempt to solve to obtain $x = \dots$
	$x = 1, y = 0$ $x = 2, y = -1$	<b>2</b>	<b>A1</b> for any correct pair, must be from correct working only
2(b)		<b>3</b>	<b>B1</b> for a correct cubic shape  <b>B1</b> for a correct cubic shape in the correct position, touching the $x$ -axis once in the 4th quadrant and intersecting once with the positive $x$ -axis  <b>B1</b> for all intercepts and no extras
2(c)	$k < -1$	<b>B1</b>	
	$k > 0$	<b>B1</b>	

Question	Answer	Marks	Guidance
3(a)	$ACB = 2 \tan^{-1} \left( \frac{12}{5} \right)$ oe	<b>M1</b>	
	$ACB = (2 \times 1.176\dots)$ $= 2.35$ to 2 dp	<b>A1</b>	Must see justification to 2 dp
3(b)	Arc length $= 5 \times ACB$	<b>B1</b>	
	Perimeter $= 35.8$	<b>B1</b>	Allow awrt 35.8
3(c)	Area $= (12 \times 5) - \left( \frac{1}{2} \times 5^2 \times 2.35 \right)$	<b>M2</b>	<b>M1</b> for area of kite or area of sector <b>M1 dep</b> for kite area – sector area
	30.6	<b>A1</b>	Allow greater accuracy Any use of fractions gets <b>A0</b>
4(a)(i)	$\frac{2}{3}$	<b>B1</b>	Allow $x > \frac{2}{3}$ , $a = \frac{2}{3}$ , but not $x = \frac{2}{3}$ unless it is replaced with a correct answer
4(a)(ii)	$\mathbb{R}$ oe	<b>B1</b>	Must be using correct notation
4(a)(iii)	$3y - 2 = e^{\frac{x}{3}}$ or $3x - 2 = e^{\frac{y}{3}}$	<b>M1</b>	For valid attempt to reach this stage
	$f^{-1}(x) = \frac{1}{3} \left( e^{\frac{x}{3}} + 2 \right)$	<b>A1</b>	Must be using correct notation
	Domain $x \in \mathbb{R}$ Range $f^{-1} > \frac{2}{3}$	<b>B2</b>	<b>B1</b> for each, must be using the correct notation.
4(a)(iv)		<b>4</b>	<b>B1</b> for the shape of $y = f(x)$ in the first and fourth quadrants only <b>B1 dep</b> on previous <b>B1</b> for (1, 0) <b>B1</b> for a correct shape for $f^{-1}(x)$ , or <b>FT</b> on <i>their</i> $y = f(x)$ with correct shape in first quadrant for symmetry about $y = x$ soi <b>B1 dep</b> on previous <b>B1</b> , for (0, 1) and at least one point of intersection with $y = f(x)$ correct in the first quadrant

Question	Answer	Marks	Guidance
4(b)	$\left(2\left((2x+1)^{\frac{1}{2}}+4\right)+1\right)^{\frac{1}{2}}+4$	<b>B1</b>	
	$(2x+1)^{\frac{1}{2}}=8$	<b>B1</b>	<b>Dep</b>
	$x=31.5$ oe	<b>B1</b>	<b>Dep</b> on both previous <b>B</b> marks
	<b>Alternative</b>		
	$g(x)=9, x=12$	<b>(B1)</b>	
	$g(x)=12$	<b>(B1)</b>	<b>Dep</b>
	$x=31.5$ oe	<b>(B1)</b>	<b>Dep</b> on both previous <b>B</b> marks
5(a)	$\frac{\operatorname{cosec}^2 \theta}{\cot^2 \theta} = \frac{1}{\sin^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta}$ $= \frac{1}{\cos^2 \theta} = \sec^2 \theta$	<b>B1</b>	Sufficient detail is needed Do not award if $\theta$ is consistently omitted
	<b>Alternative 1</b>		
	$\frac{\sin^2 \theta + \cos^2 \theta}{\frac{\sin^2 \theta}{\frac{\cos^2 \theta}{\sin^2 \theta}}}$ $= \frac{1}{\cos^2 \theta} = \sec^2 \theta$	<b>(B1)</b>	Sufficient detail is needed Do not award if $\theta$ is consistently omitted
	<b>Alternative 2</b>		
	$\frac{1}{\cot^2 \theta} + 1$ $\tan^2 \theta + 1$	<b>(B1)</b>	Sufficient detail is needed Do not award if $\theta$ is consistently omitted
5(b)	$\sec^2 \theta$	<b>B1</b>	
5(c)	$\int (\sec^2 \theta - \sin \theta) d\theta$ soi	<b>B1</b>	
	$\tan \theta + \cos \theta$	<b>B2</b>	<b>B1</b> for each
	$\sqrt{3} - \frac{1}{2}$ or exact equivalent	<b>B1</b>	<b>Dep</b> on 3 previous <b>B</b> marks
6(a)	$x^{10} + 20x^7 + 180x^4$	<b>3</b>	<b>B1</b> for each correct term

Question	Answer	Marks	Guidance
6(b)	${}^8C_4(4x^2)^4\left(\frac{1}{2x^2}\right)^4$	<b>M1</b>	May be implied by working to obtain $r = 4$
	1120	<b>A1</b>	From correct working
7	$\left(\frac{dy}{dx}\right) = \frac{(x+2)\left(\frac{6x}{3x^2-1}\right) - \ln(3x^2-1)}{(x+2)^2}$ or $\frac{6x}{(3x^2-1)}(x+2)^{-1} - (x+2)^{-2}\ln(3x^2-1)$ oe	<b>3</b>	<b>B1</b> for $\frac{6x}{3x^2-1}$ <b>M1</b> for correct attempt at differentiation of a quotient or a correct product <b>A1</b> for all terms apart from $\frac{6x}{3x^2-1}$ correct.
	When $x = 1$ , $\frac{dy}{dx} = \frac{9 - \ln 2}{9}$	<b>M1</b>	For use of $x = 1$ in <i>their</i> $\frac{dy}{dx}$ , must see a substitution if in decimal form unless 0.923 obtained from a correct derivative
	$\frac{dx}{dt} = \frac{9h}{9 - \ln 2}$ or exact equivalent	<b>2</b>	<b>M1</b> for $\frac{h}{\text{their}\left(\frac{9 - \ln 2}{9}\right)}$ , with $x = 1$ substituted in <b>A0</b> if using small changes
8	$\frac{dy}{dx} = e^x k(2x+5)^{\frac{1}{2}} + e^x(2x+5)^{\frac{1}{2}}$	<b>M1</b>	
	$\frac{dy}{dx} = e^x(2x+5)^{\frac{1}{2}} + e^x(2x+5)^{\frac{1}{2}}$	<b>A1</b>	
	When $x = 2$ , $\frac{dy}{dx} = \frac{10e^2}{3}$	<b>M1</b>	<b>Dep</b> allow unsimplified Allow for using <i>their</i> $\frac{dy}{dx}$
	When $x = 2$ , $y = 3e^2$	<b>B1</b>	
	Tangent: $y - 3e^2 = \frac{10e^2}{3}(x - 2)$	<b>M1</b>	Allow for using <i>their</i> $\frac{dy}{dx}$ and <i>their</i> $y$
	When $y = 0$ , $x = \frac{11}{10}$	<b>A1</b>	Must be simplified Must be from correct work
	When $x = 0$ , $y = -\frac{11e^2}{3}$	<b>A1</b>	
	$\left(\frac{11}{20}, -\frac{11e^2}{6}\right)$	<b>A1</b>	<b>FT</b> on <i>their</i> coordinates for $x$ and $y$ , but must be exact and simplified

Question	Answer	Marks	Guidance
9	$\frac{8}{2x+1} = 6x+1$ $12x^2 + 8x - 7 = 0$	<b>M1</b>	Attempt to obtain a 3-term quadratic in one variable equated to zero.
	$x = \frac{1}{2}$	<b>2</b>	<b>M1 dep</b> for solution, see guidance
	$k \ln(2x+1)$	<b>M1</b>	
	Area under curve = $[k \ln(2x+1)]_0^{\text{their } \frac{1}{2}}$ $= k \ln(2(\text{their } x)+1) (-0)$	<b>M1</b>	<b>Dep</b> on previous <b>M1</b> for correct application of limits using <i>their</i> $x$ , $k$ and zero Allow unsimplified
	Area under curve = $2 \ln 2$	<b>A1</b>	Not from incorrect work
	Area under straight line = $\frac{5}{8}$ or 0.625 oe	<b>B1</b>	
	Shaded area = $\ln 4 - \frac{5}{8}$	<b>A1</b>	Not from incorrect work



Question	Answer	Marks	Guidance
9	<b>Alternative</b>		
	<b>Either</b> $\frac{8}{2x+1} = 6x+1$ $12x^2 + 8x - 7 = 0$	(M1)	Attempt to obtain a 3-term quadratic in one variable equated to zero.
	$x = \frac{1}{2}$	(2)	<b>M1 dep</b> for solution, see guidance
	$y = 2$	(A1)	Award only if attempt at integration with respect to $y$ is subsequently seen
	<b>Or</b> $x = \frac{2}{y} - \frac{1}{2}$ and $x = \frac{2y-1}{6}$ oe	(M1)	For rearranging both equations to obtain $x$ or $2x$ in terms of $y$
	$2y^2 + 2y - 12 = 0$	(M1)	<b>Dep</b> for attempt to obtain a 3-term quadratic in one variable equated to zero.
	$y = 2$	(2)	<b>M1 dep</b> for solution, see guidance
	<b>Then</b> area enclosed between curve, $y$ -axis and the line $y = 2 = \left[ k \ln y - \frac{1}{2} y \right]_{\text{their } 2}^4$ $= k \ln 4 - 2 - k \ln 2 + 1$	(M1)	For correct application of limits using <i>their</i> $y = 2$ , $k$ and 4 Allow unsimplified
	$2 \ln 2 - 1$	(A1)	Not from incorrect work
	Area enclosed by straight line, the $y$ axis and the line $y = 2$ , $= \frac{3}{8}$	(B1)	
Shaded area $= \ln 4 - \frac{5}{8}$	(A1)	Not from incorrect work	
10(a)	$\frac{30}{2} (4 \tan 2x + (29 \times 3 \tan 2x)) = 455\sqrt{3}$	M1	For attempt to use sum formula with correct $a$ and $d$
	$\tan 2x = \frac{\sqrt{3}}{3}, \frac{455\sqrt{3}}{1365}$	A1	
	$x = -165^\circ, -75^\circ, 15^\circ, 105^\circ$	3	<b>M1</b> for 1 correct solution (allow if in radians or from use of $\tan 2x = 0.577$ or $\tan 2x = 0.58$ e.g. $14.99^\circ, 15.06^\circ$ .) <b>A1</b> for a second correct solution <b>A1</b> for 2 further correct solutions and no extras in the range

Question	Answer	Marks	Guidance
10(b)	$r = 4 \cos^2 \left( \theta - \frac{\pi}{2} \right)$	<b>B1</b>	
	$4 \cos^2 \left( \theta - \frac{\pi}{2} \right) < 1$ or $-1 < 4 \cos^2 \left( \theta - \frac{\pi}{2} \right) < 1$ or $0 \leq 4 \cos^2 \left( \theta - \frac{\pi}{2} \right) < 1$	<b>M1</b>	For use of sum to infinity condition
	$\theta = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$	<b>2</b>	<b>M1 dep</b> for one correct critical value <b>A1</b> for all critical values and no extras in the range $-\frac{\pi}{6} \leq \theta \leq \frac{7\pi}{6}$
	$-\frac{\pi}{6} < \theta < \frac{\pi}{6}$ (excluding 0) $\frac{5\pi}{6} < \theta < \frac{7\pi}{6}$ (excluding $\pi$ )	<b>2</b>	<b>A1</b> for each correct set of values