



Cambridge International AS & A Level

CANDIDATE NAME



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FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

October/November 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.







- 1 Ellie is investigating the heights of two types of beech tree, A and B , in a certain region. She has chosen a random sample of 60 beech trees of type A in the region, recorded their heights, x m, and calculated unbiased estimates for the population mean and population variance as 35.6 m and 4.95 m² respectively.

Ellie also chooses a random sample of 50 beech trees of type B in the region and records their heights, y m. Her results are summarised as follows.

$$\Sigma y = 1654 \quad \Sigma y^2 = 54850$$

Find a 95% confidence interval for the difference between the population mean heights of type A and type B beech trees in the region. [6]

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2 A school with a large number of students is updating its logo. Each student has designed a new logo and two teachers have each awarded a mark out of 50 for each logo. The marks awarded to a random sample of 12 students are shown in the following table.

Student	A	B	C	D	E	F	G	H	I	J	K	L
Teacher 1	36	38	40	36	22	34	45	44	48	35	28	30
Teacher 2	38	42	32	41	32	41	42	50	36	44	42	41

One of the students claims that Teacher 2 is awarding higher marks than Teacher 1.

(a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the claim. [7]

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It was later discovered that Teacher 1 had entered her mark for student *C* incorrectly. Her intended mark was 24 not 40. This was corrected.

(b) Determine whether this correction affects the conclusion of the test carried out in part (a). [2]

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3 A statistician believes that the number of telephone calls received by an advice centre in a 10-minute interval can be modelled by the Poisson distribution $Po(1.9)$. The number of calls received in a randomly chosen 10-minute interval was recorded on each of 100 days. The results are summarised in the table, together with some of the expected frequencies corresponding to the distribution $Po(1.9)$.

Number of calls	0	1	2	3	4	5	6 or more
Observed frequency	10	18	35	21	11	4	1
Expected frequency	14.957	28.418	26.997				1.322

(a) Complete the table. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to determine whether the statistician's belief is reasonable. [6]

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4 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx^3 & 0 \leq x < 1, \\ k(5-x) & 1 \leq x \leq 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Sketch the graph of f . [1]

(b) Show that $k = \frac{4}{33}$. [2]

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(c) Find the cumulative distribution function of X . [3]

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(d) Find the median value of X . [4]

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5 Nikita has three coins. One coin is fair, one coin is biased so that the probability of obtaining a head is $\frac{1}{3}$ and the third coin is biased so that the probability of obtaining a head is $\frac{1}{5}$. The random variable X is the number of heads that Nikita obtains when he throws all three coins at the same time.

(a) Find the probability generating function of X . [3]

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Rajesh has two fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6. The random variable Y is the number of 4s that Rajesh obtains when he throws the two dice.

The random variable Z is the sum of the number of heads obtained by Nikita and the number of 4s obtained by Rajesh.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [4]

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(c) Use your answer to part (b) to find $E(Z)$. [2]

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6 Ansal is investigating the wingspans of Monarch butterflies in two different regions, *X* and *Y*. He takes a random sample of 8 Monarch butterflies from region *X* and records their wingspans, *x* cm. His results are as follows.

8.2 7.0 7.3 8.8 7.8 8.5 9.2 7.4

Ansal also takes a random sample of 9 Monarch butterflies from region *Y* and records their wingspans, *y* cm. His results are summarised as follows.

$$\Sigma y = 71.10 \quad \Sigma y^2 = 567.13$$

Ansal suspects that the mean wingspan of Monarch butterflies from region *X* is greater than the mean wingspan of Monarch butterflies from region *Y*. It is known that the wingspans of Monarch butterflies in regions *X* and *Y* are normally distributed with equal population variances.

Test, at the 10% significance level, whether Ansal's suspicion is supported by the data. [8]

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