Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **October/November 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the set of values of k for which the system of equations

$$\begin{aligned} x+5y+6z &= 1, \\ kx+2y+2z &= 2, \\ -3x+4y+8z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = \cos^{-1} t \quad \text{for } 0 < t < 1.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{t^2}{\sqrt{1-t^2}}$. [2]

(b) Show that $\dfrac{d^2y}{dx^2} = -t^a\left(1-t^2\right)^b\left(2-t^2\right)$, where a and b are constants to be determined. [4]

3 A curve has equation $y = e^x$ for $\ln\frac{4}{3} \leqslant x \leqslant \ln\frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x-axis is denoted by A.

(a) Use the substitution $u = e^x$ to show that

$$A = 2\pi\int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2}\,du.$$ [2]

(b) Use the substitution $u = \sinh v$ to show that

$$A = \pi\left(\frac{904}{225} + \ln\frac{5}{3}\right).$$ [6]

4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

(a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

(b) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the other eigenvalues of $\mathbf{A}$. [3]

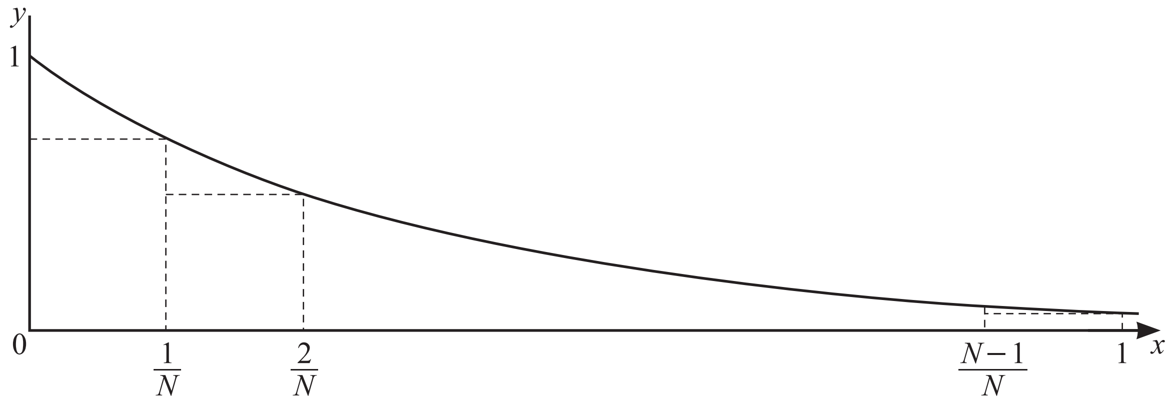
(c) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [4]

5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{dx}{dt} = -6$. [10]

6



The diagram shows the curve with equation $y = \left(\frac{1}{2}\right)^x$ for $0 \leqslant x \leqslant 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x > L_N$, where

$$L_N = \frac{1}{2N\left(2^{\frac{1}{N}} - 1\right)}.$$ [4]

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(b) Use a similar method to find, in terms of N, an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x$. [4]

(c) Find the least value of N such that $U_N - L_N \leqslant 10^{-3}$. [2]

(d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x = \frac{1}{2\ln 2}$, use the value of N found in part **(c)** to find upper and lower bounds for $\ln 2$. [4]

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7 **(a)** Show that an appropriate integrating factor for

$$\sqrt{x^2+16}\,\frac{\mathrm{d}y}{\mathrm{d}x} + y = x\sqrt{x^2+16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2+16}$. [4]

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(b) Hence find the solution of the differential equation

$$\sqrt{x^2+16}\,\frac{\mathrm{d}y}{\mathrm{d}x}+y=x\sqrt{x^2+16}$$

for which $y=6$ when $x=3$. [6]

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8 **(a)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)^7$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^7\theta = a\cos 7\theta + b\cos 5\theta + c\cos 3\theta + d\cos\theta,$$

where a, b, c and d are constants to be determined. [5]

Let $I_n = \int_0^{\frac{1}{4}\pi} \cos^n \theta \, d\theta$.

(b) Show that

$$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}.$$ [4]

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(c) Using the results given in parts **(a)** and **(b)**, find the exact value of I_9. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.