



Cambridge International AS & A Level

CANDIDATE NAME



CENTRE NUMBER

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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.







4 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [3]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by \mathbf{CAB} . [5]

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(d) Sketch C , stating the coordinates of any intersections with the axes and labelling the asymptote. [3]

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(e) Sketch the curve with equation $y = \frac{x^2 + 1}{x^2 + 3}$ and find the set of values of x for which $\frac{x^2 + 1}{x^2 + 3} < \frac{1}{2}$. [4]

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The curve C_2 with polar equation $r = a\theta$ intersects C_1 at the pole and the point with polar coordinates $(a\phi, \phi)$.

(c) Verify that $1.25 < \phi < 1.26$. [2]

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(d) Show that the area of the smaller region enclosed by C_1 and C_2 is equal to

$$\frac{1}{2}a^2\left(\frac{3}{4}\pi + \frac{1}{3}\phi^3 - \phi + \frac{1}{2}\cos 2\phi\right)$$

and deduce, in terms of a and ϕ , the area of the larger region enclosed by C_1 and C_2 . [7]

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Additional page

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