CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$0.3^3 + 0.7^3$ or $1 - (3\times 0.3^2 \times 0.7 + 3\times 0.3\times 0.7^2) = [1 - 0.63]$	**M1**	$p^3 + q^3,\ p+q=1,\ p,q>0$ or $1-(3\times p^2\times q + 3\times p\times q^2),$ $p+q=1, p,q>0.$
	0.37[0]	**A1**	$\frac{37}{100}$.
		2	

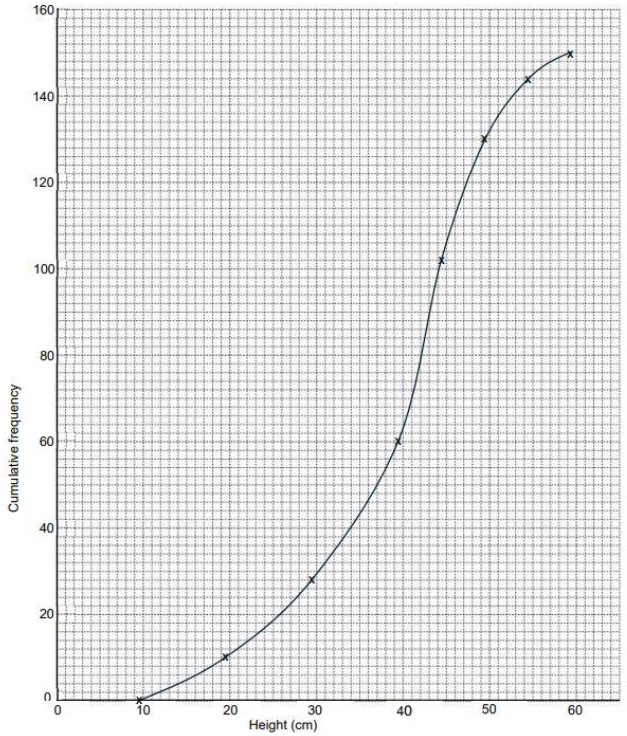
Question	Answer	Marks	Guidance
1(b)	[Mean = $125\times0.3=$] 37.5 [Variance = $125\times0.3\times0.7=$] 26.25	**B1**	37.5 or 37½ and 26.25, 26¼ seen, allow unsimplified. May be seen in standardisation formula. ($[\sigma=]5.12, \frac{\sqrt{105}}{2}$ implies correct variance).
	P(X > 45) = P($Z > \frac{45.5-37.5}{\sqrt{26.25}}$)	**M1**	Substituting their *mean* and *their* positive standard deviation into the ±standardising formula (any number for 45.5), not *their* σ^2, not *their* $\sqrt{\sigma}$.
		M1	Use continuity corrections 44.5 or 45.5 in *their* standardisation formula Note: $\frac{\pm8}{\sqrt{26.25}}$ or $\frac{\pm8}{5.123}$ seen gains M2 BOD
	[1 − Φ(*their* 1.5614)] = 1 – *their* 0.9407	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5. Note: appropriate final answer implies this M1.
	0.0593	**A1**	$0.0592 \leqslant p \leqslant 0.0593$.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">2(a)</td><td rowspan="3">
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>$P(X = x)$</td><td>$\frac{6}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td></tr>
<tr><td></td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr>
<tr><td></td><td>0.167</td><td>0.333</td><td>0.167</td><td>0.167</td><td>0.167</td></tr>
</table>
</td><td>B1</td><td>Table with correct x values and at least one correct probability linked with the correct x-value.
Values need not be in order, lines may not be drawn, may be vertical,
x and $P(X)$ may be omitted.
Condone any additional x values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>4 correct probabilities linked with the correct x-values, need not be in table, accept unsimplified.</td></tr>
<tr><td>B1</td><td>5 correct probabilities linked with correct x-values, may not be in table.
Decimals correct to at least 3 SF.
SC B1 4 or 5 probabilities summing to 1 placed in a probability distribution table with 4 or 5 x-values between 1 and 6 inclusive.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="2">2(b)</td><td>$[E(X) = \frac{1}{36}(6 + 24 + 18 + 24 + 36) =] \ 3$</td><td>B1 FT</td><td>FT their table with 4 or 5 probabilities ($0 < p < 1$) summing to 1.</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(a)	$[P(X > 170) = P(Z >]\frac{170-176}{4.8})$	**M1**	Using ± standardisation formula with 170, 176 and 4.8 substituted appropriately. Condone σ^2 and $\sqrt{\sigma}$. No continuity correction.
	$\left[\Phi(1.25)\right] = 0.894$	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer > 0.5.
		A1	0.894 or $0.89435 \leqslant p \leqslant 0.8944$. If A0 scored, **SC B1** for 0.894 or $0.89435 \leqslant p \leqslant 0.8944$, WWW.
		3	
3(b)	$P(h < 170) = 1 - 0.8944 = 0.1056$	**M1**	1 – *their* **3(a)** seen or implied by 0.7056 or 0.2944
	$\frac{k-176}{4.8}\left[=\Phi^{-1}(0.1056+0.6)\right]=0.541$	**B1**	$0.540 < z \leqslant 0.541$ or $-0.541 \leqslant z < 0.540$ seen.
		M1	Use of ±standardisation formula with *k*, 176, 4.8 equated to a *z*-value (**not** 1.25, 0.7601, 0.2399, 0.7056, 0.7257, 0.8313, 0.253 ± 0.894, 0.6, 0.4), not 4.8^2, not $\sqrt{4.8}$, no continuity correction.
	$k = 178.6$	**A1**	CAO (answer required to 1 dp).
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">4(a)</td><td>

H(cm)	10–19	20–29	30–39	40–44	45–49	50–54	55–59
UB	19.5	29.5	39.5	44.5	49.5	54.5	59.5
cf	[10]	28	60	102	130	144	150

</td><td>B1</td><td>Cf values 28, 60, 102, 130, 144, 150 seen,
Condone omission of 10.
May be implied by accurate plotting (scale no less than 1cm = 10).
May be by data table.</td></tr>
<tr><td rowspan="3"> 
</td><td>B1</td><td>Linearly scaled axes correctly labelled cumulative frequency (cf) (from 0 to 150) and height (h) and centimetres (cm) (from 9.5 to 59.5) with at least 3 values identified on each.
Axes can be the other way round.</td></tr>
<tr><td>M1</td><td>At least 4 points plotted at upper boundary ± 0.5, (e.g. allow (19, 19.5 or 20, 10) etc.) on correctly scaled axes.
(9.5,0), (19.5,10), (29.5, 28), (39.5, 60), (44.5,102),
(49.5, 130), (54.5, 144), (59.5,150).</td></tr>
<tr><td>A1</td><td>All points plotted correctly, curve drawn (within tolerance), joined to (9.5, 0) and not going beyond above 150 vertically.
A0 if straight line segments used.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	[150 × 0.3 = 45]	**M1**	Use of graph must be seen.
	Line drawn from 45 on cf axis to meet graph at $h = 36$	**A1 FT**	Must be an increasing cf graph. Expect an answer in range $35 \leqslant h \leqslant 37$ for a correct graph.
		2	
4(c)	Midpoints 14.5, 24.5, 34.5, 42, 47, 52, 57	**B1**	At least 6 correct midpoints seen, may be unsimplified, may be in calculation, may be by data table.
	Mean = $\dfrac{10\times14.5+18\times24.5+32\times34.5+42\times42+28\times47+14\times52+6\times57}{150}$ $\left[=\dfrac{145+441+1104+1764+1316+728+342}{150}\right]$	**M1**	Correct unsimplified mean formula with *their* midpoints (not ub, lb, upper limits, lower limits, cw, fd, f or cf and must be within class). If midpoints correct, accept partially evaluated.
	$=\dfrac{5840}{150}, \dfrac{584}{15}, 38\dfrac{14}{15}, 38.9$	**A1**	Accept answers wrt 38.9 WWW If M1 withheld, **SC B1** for $\dfrac{5840}{150}, \dfrac{584}{15}, 38\dfrac{14}{15}, 38.9$.
	$sd^2 = \dfrac{10\times14.5^2+18\times24.5^2+32\times34.5^2+42\times42^2+28\times47^2+14\times52^2+6\times57^2}{150}-\left(\textit{their}\ \dfrac{5840}{150}\right)^2$ $\left[=\dfrac{244285}{150}-\left(\textit{their}\ \dfrac{5840}{150}\right)^2\right]$	**M1**	Correct unsimplified variance formula with *their* midpoint (not ub, lb, upper limits, lower limits, cw, fd, f or cf and must be within class). If midpoints correct, accept partially evaluated
	[= 112.76] standard deviation $\left[\sqrt{112.76}\right]$ = 10.6	**A1**	AWRT 10.6 WWW. If second M1 withheld, **SC B1** for 10.6 WWW.
		5	

Question	Answer	Marks	Guidance
5(a)	**Method 1**		
	[P(0, 1, 2) =] $^8C_2(0.75)^6(0.25)^2 + {}^8C_1(0.75)^7(0.25)^1 + (0.75)^8$ [= 0.31146 + 0.26697 + 0.10011] =	**M1**	One term ^8C_x . $(p)^x(1-p)^{8-x}, 0<p<1, 0<x<8$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.679	**B1**	AWRT.
		3	
	Method 2		
	[P(0, 1, 2) = 1 – P(3, 4, 5, 6, 7, 8) =] $1 - \{{}^8C_3(0.75)^5(0.25)^3 + {}^8C_4(0.75)^4(0.25)^4 + {}^8C_5(0.75)^3(0.25)^5 + {}^8C_6(0.75)^2(0.25)^6 + {}^8C_7(0.75)(0.25)^7 + (0.25)^8\}$	**M1**	One term $^8C_x\ (p)^x(1-p)^{8-x}$, $0<p<1, \quad 0<x<8$.
		A1	Correct expression, accept unsimplified, condone omission of up to 3 ‘middle’ terms, leading to final answer.
	= 0.679	**B1**	AWRT.
		3	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	$1-0.75^6$	**M1**	$1-0.75^n, n=6,7$.
	$= 0.822, \frac{3367}{4096}$	**A1**	0.82202148… to at least 3SF.
	Method 2		
	$0.25+0.25\times0.75+0.25\times0.75^2+0.25\times0.75^3+0.25\times0.75^4+0.25\times0.75^5$	**M1**	Summing 6 or 7 terms – condone extra term 0.25×0.75^6 .
	=0.822	**A1**	
	Method 3		
	$1 - 0.75^7 - 0.25\times0.75^6$	**M1**	Correct expression.
	=0.822	**A1**	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="11">5(c)</td><td colspan="3">Method 1 P(2nd gold $\cap$ 5th is first unwrapped).
R G RorG RorG U</td></tr>
<tr><td rowspan="2">$0.25\times0.3\times0.55\times0.55\times0.45[=0.01021]$
on its own or as a numerator</td><td>M1</td><td>$a\times0.3\times b\times c\times0.45$ $0 < a, b, c < 1$.
$a \neq 0.3, 0.45$ $b, c \neq 0.45$.
multiplied in that order, or correct.</td></tr>
<tr><td>A1</td><td>5 correct probabilities multiplied.</td></tr>
<tr><td colspan="3">Method 2 P(2nd gold $\cap$ 5th is first unwrapped).4 possible scenarios</td></tr>
<tr><td rowspan="6"><table>
<tr><td>R G R G U</td><td>$0.25\times0.3\times0.25\times0.3\times0.45$</td><td>[= 0.00253125]</td></tr>
<tr><td>R G R R U</td><td>$0.25\times0.3\times0.25\times0.25\times0.45$</td><td>[= 0.002109375]</td></tr>
<tr><td>R G G R U</td><td>$0.25\times0.3\times0.3\times0.25\times0.45$</td><td>[= 0.00253125]</td></tr>
<tr><td>R G G G U</td><td>$0.25\times0.3\times0.3\times0.3\times0.45$</td><td>[= 0.0030375]</td></tr>
<tr><td></td><td>[Total</td><td>0.010209375]</td></tr>
</table></td><td>M1</td><td>$a\times0.3\times b\times c\times0.45$ $0 < a, b, c < 1$.
$a \neq 0.3, 0.45$ $b, c \neq 0.45$.
4 terms in this form seen added on their own or as a numerator.</td></tr>
<tr><td>A1</td><td>All probabilities correct and attempt to sum the 4 scenarios.</td></tr>
<tr><td colspan="3">For either approach</td></tr>
<tr><td>[P(5^{th} is first unwrapped) =] $(0.55)^4(0.45)[=0.041178]$</td><td>B1</td><td></td></tr>
<tr><td>[P(2^{nd} is first gold | 5^{th} is first unwrapped) =] $\dfrac{0.25\times0.3\times0.55\times0.55\times0.45}{(0.55)^4(0.45)}$
$\left[=\dfrac{0.010209375}{0.0411778125}\right]$</td><td>M1</td><td>$\dfrac{\textit{their}\ \mathrm{P}(\text{2nd gold}\cap\text{5th is first unwrapped})}{\textit{their}\ \mathrm{P}(\text{5th is first unwrapped})}$.
Their probabilities must be clearly identified if incorrect.</td></tr>
<tr><td>= 0.248, $\dfrac{30}{121}$</td><td>A1</td><td>0.24793…</td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	**Method 3** First chocolate is red and second gold P(red given that it is wrapped) × P(gold given that it is wrapped) = $\frac{0.25}{0.55}\times\frac{0.3}{0.55}$	**M1**	Either $\frac{0.25}{0.55 or 0.45}$ or $\frac{0.3}{0.55 or 0.45}$.
		A1	Either $\frac{0.25}{0.55}$ or $\frac{0.3}{0.55}$.
		B1	Both probs correct, can be unsimplified.
		M1	Multiplying their identified P(red given wrapped) by their identified P(gold given wrapped) or correct.
	$=0.248, \frac{30}{121}$	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!2!}=\right]$ 90720	**B1**	
		1	

Question	Answer	Marks	Guidance
6(b)	**Method 1** Total arrangements – arrangements with repeated letters at ends		
	$\frac{9!}{2!2!}-\frac{7!}{2!}\times 2$	**M1**	$a-\frac{7!}{2!}\times b$ a = *their* 6(a) or correct, b = 1,2.
		M1	$a-\frac{7!}{c!}\times 2$ a = *their* 6(a) or correct, c = 1,2.
	85680	**A1 FT**	ft *their* **6(a)** – 5040.
	Method 2 Adding no of different ways		
	P and S at ends $2\times 7!=$ 10080 P or S at one end only $4\times 5\times\frac{7!}{2!}=$ 50400 Neither P nor S at an end $5\times 4\times\frac{7!}{2!2!}=$ 25200	**M1** **M1**	Finding correct number of ways for one of these correctly identified scenarios. Adding no of ways for 3 correctly identified scenarios.
	Total 85680	**A1**	
	Method 3		
	P at beginning $7\times\frac{7!}{2!}$ = 17640 S at beginning $7\times\frac{7!}{2!}$ = 17640 Neither P nor S at beginning $5\times\frac{8!}{2!2!}$ = 50400	**M1** **M1**	Finding correct number of ways for one of these correctly identified scenarios. Adding no of ways for 3 correctly identified scenarios.
	Total 85680	**A1**	
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1** arrangements with PP between Ss { S P P S ^ ^ ^ ^ ^ } add arrangements with PP not between Ss { (S ^ ^ S) ^ P P ^ }		
	$6!+5!\times5\times4$	**M1**	6! + *d*, *d* an integer ≥ 1, may be implied.
	$6!+5!\times5\times4$	**M1**	$e+5!\times f$, *e*, *f* integers ≥ 1, may be implied.
		M1	$e+g!\times\left(5\times4 \text{ or } {}^{5}\mathrm{P}_{2}\right)$, *e* an integer ≥ 1, *g* = 4,5,6.
	[Total]= 3120	**A1**	
	Method 2 - considers the 6 positions for S ^^S		
	Positions 1 and 6 there are $5\times5!$ ways	**M1**	Identifying no of ways if S^^S is in position 1 or 6.
	Positions 2, 3, 4 and 5 there are $4\times5!$ ways	**M1**	Identifying no of ways if S^^S is in position 2, 3, 4 or 5.
	$2\times5\times5!+4\times4\times5!$	**M1**	Adding no of ways for 6 scenarios (or 26 × 5!).
	[Total] = 3120	**A1**	SC B1 for 3120 if any method marks are withheld.
		4	

Question	Answer	Marks	Guidance
6(d)	**Method 1** Either PP in the group of 5 or PP in the group of 4		
	$\frac{{}^5C_3}{{}^9C_5}+\frac{{}^5C_2}{{}^9C_5}$,	**M1**	$a\times\ {}^5C_2$, $a\times\ {}^5C_3$, or ${}^5C_2+{}^5C_3$ seen as a numerator of one or two fractions where a is 1 or 2, no extra terms.
	$\frac{{}^5C_3+{}^5C_2}{{}^9C_5}$	**M1**	${}^9C_5$ *or* ${}^9C_4$ seen (no addition, multiplication) as a denominator of one or two fractions.
	Probability = $\frac{20}{126}, \frac{10}{63}$, 0.159	**A1**	
	Method 2 Considering the positions of P and then S		
	$\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{5}{7}\times\frac{4}{6}\times\frac{4}{9}\times\frac{3}{8}\right)$	**M1**	$a \times 5 \times 4 \times 4 \times 3$ seen as a numerator of a fraction. where a = 1 or 2.
	$\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{5}{7}\times\frac{4}{6}\times\frac{4}{9}\times\frac{3}{8}\right)$	**M1**	$9 \times 8 \times 7 \times 6$ seen as a denominator of a fraction.
	$=\frac{10}{63}$	**A1**	
		3	