

Cambridge International AS & A Level

MATHEMATICS

9709/52 October/November 2024

Paper 5 Probability & Statistics 1 MARK SCHEME Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- Μ Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method Α mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- **DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above). .
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 . decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column. .
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise. .
- Square brackets [] around text or numbers show extra information not needed for the mark to be awarded. •

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Abbreviations

- AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
- CWO Correct Working Only
- ISW Ignore Subsequent Working

SOI Seen Or Implied

- SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
- WWW Without Wrong Working
- AWRT Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$[P(X N) =] \frac{12}{40}$	B 1	0.3, $\frac{3}{10}$, 30% OE.
		1	
1(b)	$[P(N X) =] \frac{12}{50}$	B 1	0.24, $\frac{6}{25}$ OE.
		1	
1(c)	$P(N \cap X) = \frac{12}{120}, \ P(N) = \frac{40}{120}, \ P(X) = \frac{50}{120}$ $\frac{40}{120} \times \frac{50}{120} = \frac{5}{36}, 0.138[8] \neq \frac{12}{120}, 0.1 \text{ Not independent}$	B1	$P(N), P(X) \text{ and } P(N \cap X) \text{ or } P(N \text{ and } X) \text{ notation seen}$ and equated to the values for $P(N), P(X) \text{ and } P(N \cap X) \text{ or } P(N \text{ and } X) \text{ .}$ Calculation stated and evaluated. Not independent clearly stated. $\frac{5}{36} \neq \frac{12}{120} \text{ does not need to be stated.}$ All values OE. Condone consistent use of <i>A</i> , <i>B</i> etc. If values for P(<i>N</i>), P(<i>X</i>) stated, accept $P(N) \times P(X) = \frac{5}{36}.$
		1	

Question	Answer	Marks	Guidance
1(d)	Method 1		
	$ \begin{bmatrix} P(0, 1, 2) =] (0.85)^8 + {}^8C_1(0.85)^7(0.15) + {}^8C_2(0.85)^6(0.15)^2 \\ = 0.27249 + 0.38469 + 0.23760 \end{bmatrix} $	M1	One term of form ${}^{8}C_{x}(p)^{x}(1-p)^{8-x}$. With $0 or 8.$
		A1	Correct unsimplified expression, no terms omitted leading to final answer.
	= 0.895	B 1	$0.8945 \leqslant p \leqslant 0.895.$
	Method 2		
	$ \begin{bmatrix} P(0, 1, 2) = & 1 - {{}^{8}C_{3}(0.85)^{5}(0.15)^{3} + {}^{8}C_{4}(0.85)^{4}(0.15)^{4} + {}^{8}C_{5}(0.85)^{3}(0.15)^{5} \\ &+ {}^{8}C_{6}(0.85)^{2}(0.15)^{6} + {}^{8}C_{7}(0.85)(0.15)^{7} + (0.15)^{8} \end{bmatrix} $	M1	One term of form ${}^{8}C_{x}(p)^{x}(1-p)^{8-x}$ With $0 or 8.$
		A1	Correct unsimplified expression. Condone omission of final bracket '}'. If other brackets omitted, allow recovery if 1 – 0.1052[] seen.
	= 0.895	B1	$0.8945 \leqslant p \leqslant 0.895.$
		3	

Question	Answer	Marks	Guidance
2(a)	$\left[\frac{9!}{2!}\right] = 181440$	B1	Exact value must be seen. CAO.
		1	

Question		Ans	wer	Mar	rks	Guidance
2(b)	Method 1					
	Scenario number of letters between As				B1	Correct outcome/value for 1 identified scenario, accept unsimplified.
	A A [^ ^ ^ ^ ^ ^ ^]	7! × 8 or 8!	[=40320]	I	M1	Add values of 3 correct scenarios, no incorrect/repeated scenarios.
	A ^ A [^ ^ ^ ^ ^]	7!×7	[=35280]			
	A ^ ^ A [^ ^ ^ ^ ^]	7!×6	[=30240]			
Total: $7! \times (8 + 7 + 6)$						
	= 105 840					If M1 not awarded, SC B1 for 105840 WWW.
	Method 2					
Scenario number of letters between As				B1	Correct outcome/value for 1 identified scenario, accept unsimplified.	
	A A [^ ^ ^ ^ ^ ^ ^]	8!	[=40320]	I	M1	Add values of 3 correct scenarios, no incorrect/repeated scenarios.
	A ^ A [^ ^ ^ ^ ^ ^]	${}^{7}P_{1} \times 7!$ or ${}^{7}C_{1} \times 7!$	[= 35280]			
	A ^ ^ A [^ ^ ^ ^ ^]	${}^7\text{P}_2 \times 6! \text{ or } \\ {}^7\text{C}_2 \times 2 \times 6!$	[= 30240]			
	Total: $8! + {^7P_1} \times 7! + {^7}$ or $8! + {^7C_1} \times 7! + {^7C_2} \times$	$P_2 \times 6!$ $x \times 2 \times 6!$				
	=105 840				A1	If M1 not awarded, SC B1 for 105840 WWW.

Question		Ат	nswer	Marks	Guidance
2(b)	Method 3				
	Scenario number of letters between As			B1	Correct outcome/value for 1 identified scenario, accept unsimplified.
	A ^ ^ ^ A [^ ^ ^]	7! × 5	[=25200]	M1	<i>their</i> 2(a) , or correct, subtract values of 5 correct scenarios, no incorrect/repeated scenarios.
	A ^ ^ ^ ^ A [^ ^ ^]	7! × 4	[= 20160]		
	A ^ ^ ^ ^ A [^ ^]	7! × 3	[= 15120]		
	A ^ ^ ^ ^ ^ A [^]	7! × 2	[= 10080]		
	A	7! [× 1]	[= 5040]		
	$Total = \frac{9!}{2!} - 7 \ltimes (5 + 4)$	+3+2+1)			
	=105 840			A1	If M1 not awarded, SC B1 for 105840 WWW.
				3	

Question	Answer										Guidance
3(a)	x	0	2	4	6	8	10	12		B1	Table with correct <i>x</i> values and at least 2 correct probabilities.
	P(X = x)	$\frac{\frac{3}{12}}{\frac{1}{4}},$ $\frac{1}{\frac{4}{0.25}}$	$ \frac{\frac{2}{12}}{\frac{1}{6}}, $ $ \frac{1}{6} $ 0.167	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$ 0.0833	$\frac{1}{12}$	$\frac{1}{12}$			Condone any additional <i>x</i> values if probability stated as 0.
										B1	Four more probabilities correctly linked to the correct x value, need not be in table, accept unsimplified.
										B1	7 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF. SC B1 7 or more probabilities summing to 1 placed in a probability distribution table.
										3	

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Question	Answer	Marks	Guidance
3(b)	$\begin{bmatrix} \mathbf{E}(X) = \mathbf{i} \begin{bmatrix} 0 \end{bmatrix} + 2 \times \frac{2}{12} + 4 \times \frac{2}{12} + 6 \times \frac{2}{12} + 8 \times \frac{1}{12} + 10 \times \frac{1}{12} + 12 \times \frac{1}{12} \\ \begin{bmatrix} 0 \end{bmatrix} + \frac{4}{12} + \frac{8}{12} + \frac{12}{12} + \frac{8}{12} + \frac{10}{12} + \frac{12}{12} = \frac{54}{12} = 4.5 \end{bmatrix}$	M1	Accept unsimplified expression. May be calculated in variance. Accept $\frac{1}{3} + \frac{2}{3} + 1 + \frac{2}{3} + \frac{5}{6} + 1$ OE for the M mark FT <i>their</i> table with 7 or 8 probabilities summing to $0.999 \le total \le 1$ ($0).FT acceptable at the bold partially evaluated stage.$
	$\begin{bmatrix} Var(X) =] \\ [0] + 2^{2} \times \frac{2}{12} + 4^{2} \times \frac{2}{12} + 6^{2} \times \frac{2}{12} + 8^{2} \times \frac{1}{12} + 10^{2} \times \frac{1}{12} + 12^{2} \times \frac{1}{12} - (their 4.5)^{2} \\ \begin{bmatrix} \underline{[0] + 2 \times 4 + 2 \times 16 + 2 \times 36 + [1 \times] 64 + [1 \times] 100 + [1 \times] 144} \\ 12 \end{bmatrix} - \frac{81}{4} \\ \begin{bmatrix} = 35 - 4.5^{2} \end{bmatrix}$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 6 or more probabilities. $(0 which need not sum to 1 or with an expressionno more evaluated than shown.FT acceptable at the bold partially evaluated stage with theirprobabilities.$
	$= 14.75, 14\frac{3}{4}$	A1	CAO Accept $\frac{59}{4}$. If either or both M marks not awarded, SC B1 for correct answer WWW
		3	

Question	Answer	Marks	Guidance
4(a)	$[P(X < 18.2) = P(Z <] \frac{18.2 - 19.8}{2.4})$	M1	Use of \pm standardisation formula with 18.2, 19.8 and 2.4 substituted appropriately, no continuity correction. Condone $\sigma^2(2.4^2)$ or $\sqrt{\sigma}(\sqrt{2.4})$.
	$= \left[\Phi(-0.6667) = 1 - \Phi(0.6667) \right]$ = 1 - 0.7477	M1	Calculating the appropriate probability areas (leading to their final answer, expect < 0.5). Note: 0.432 is <i>z</i> -value of 0.667 so is not an appropriate probability area (M0).
	= 0.252(3)	A1	AWRT 0.252 SOI, accept 0.2525. If one or both M marks not awarded, SC B1 for AWRT 0.252 SOI, accept 0.2525.
	[Expected number = 0.2523 ×450 = 113.5,] = 113 or 114	B1FT	Strict FT <i>their</i> at least 4 figure probability \times 450. (Check with calculator). One integer answer only. No indication of 'approximation', e.g. $\approx, \cong, about$.
		4	
4(b)	$[P(X > 25.5) = 0.26 \text{ so } P(Z > \frac{25.5 - 23.4}{25.5 - 23.4}) = 0.26]$	B1	$0.643 \le z \le 0.6435$ or $-0.6435 \le z \le -0.643$ seen.
	$\frac{25.5 - 23.4}{\sigma} = 0.643$	M1	± standardisation formula with 25.5, 23.4, σ equating to a <i>z</i> -value, (not 1 – <i>their z</i> -value). Condone continuity correction ±0.05, not σ^2 , not $\sqrt{\sigma}$.
	$\sigma = 3.27$	A1	$3.26 \leq \sigma \leq 3.27$. Do not award for improper fractions.
		3	

Question			Answer			Marks	Guidance
5(a)	$^{10}C_4 \times {}^{6}C_2 >$	× ⁵ C ₁				M1	¹⁰ C _a × ⁶ C _b × ⁵ C _c , $a + b + c = 7$, a,b,c integers. No other terms present but condone × 6 or × 3!.
	[= 210 ×13	$5 \times 5] = 15$	5750			A1	
						2	
5(b)	Scenario	V G P				M1	One product using 2 or 3 combinations with upper numbers correct and lower numbers summing to 5 and linked to a
	VVVVG	410	$^{10}C_4 \times {}^6C_1 \ [\times {}^5C_0]$	[1260]		C	correct identified scenario. Condone the consistent use of permutations.
	VVVGG	320	$^{10}C_3 \times {}^{6}C_2 [\times {}^{5}C_0]$	[1800]		B1	2 identified outcomes evaluated accurately, accept unsimplified.
	VVGGG	230	${}^{10}C_2 \times {}^{6}C_3 [\times {}^{5}C_0]$	[900]		M1	Add values of 5 correct scenarios, no incorrect/repeated scenarios.
	VVVGP	311	${}^{10}C_3 \times {}^6C_1 \times {}^5C_1$	[3600]			
	VVGGP	221	${}^{10}C_2 \times {}^6C_2 \times {}^5C_1$	[3375]			
	Total = 109	935			A1	If either or both Ms not awarded, SC B1 for 10935 WWW	
						4	

Question	Answer	Marks	Guidance
6(a)	Falcons Kites	B1	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e. stem values are
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	B1	Correct Kites labelled on same diagram leaves in order and
	$\begin{vmatrix} 9 & 4 & 6 & 0 & 1 & 3 & 5 \\ 6 & 7 & 6 & 7 \end{vmatrix}$ Key: 1 5 4 means 51 minutes for Falcons and 54 minutes for Kites	B1 B1	 Correct Kites labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation. Penalise each error only once in question. E.g. commas in both sets of data. Correct key, for their diagram, need both teams names and 'mins' at least once here, or in leaf headings or title. <u>If 2 separate diagrams drawn max marks</u> B1 if both stems correct. B1 if Falcons correct to the left of the stem. B1 if both keys correct including 'mins' and team name.
		4	
6(b)	Median = 51 [minutes]	B1	Accept Q2 , must be identified.
	[IQR =] 59 – 44	M1	$58 \leq UQ \leq 64 - 42 \leq LQ \leq 46$. Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	= 15 [minutes]	A1	WWW
		3	

Question	Answer	Marks	Guidance
6(c)	$\left[\text{mean} = \frac{792 + 783}{30} = \frac{1575}{30} = \right] 52.5, \ 52\frac{1}{2}, \ \frac{105}{2}$	B1	$\frac{1575}{30}$ is not acceptable for this mark.
	$\Sigma' x^{2} = 85727$ sd ² = [Variance =] $\left[\frac{(43504 + 42223)}{30} - \left(\frac{792 + 783}{30} \right)^{2} \right]$ $\frac{85727}{30} - \left(\frac{1575}{30} \right)^{2} = 101.3167$	M1	Accept unsimplified variance formula. FT <i>their</i> mean. Ignore any square root leading to sd for this mark.
	$\sigma = (\sqrt{101.3167} =) 10.1$	A1	AWRT. Must be identified, e.g. sd, s, std d, σ . Condone 'short' square root signs. If M1 not awarded, SC B1 for, $\sigma = \sqrt{101.3167} \text{ or } \sqrt{\frac{6079}{60}} \text{ oe } =10.1.$
		3	

Question	Answer	Marks	Guidance
7(a)	$[(0.6)^4 \times 0.4 =] 0.0518[4], \frac{162}{3125}$	B1	
		1	

Question	Answer	Marks	Guidance		
7(b)	Method 1				
	$[P(X \le 7) - P(X \le 2) =] (1 - 0.6^7) - (1 - 0.6^2)$	M1	$(1-p^7)-(1-p^2)$ or p^2-p^7 seen, $0 .$		
	[= 0.36 - 0.02799] = 0.332[0], $\frac{25938}{78125}$	A1	If M0 awarded SC B1 0.3320064 or $\frac{25938}{78125}$ CAO.		
	lethod 2				
	[P(X = 3,4,5,6,7) =] 0.4×0.6 ² + 0.4×0.6 ³ + 0.4×0.6 ⁴ + 0.4×0.6 ⁵ + 0.4×0.6 ⁶	M1	$(1-p) \times p^2 + (1-p) \times p^3 + (1-p) \times p^4 + (1-p) \times p^5 + (1-p) \times p^6$ seen, $0 .$		
	$[= 0.144 + 0.0864 + 0.05184 + 0.031104 + 0.0186624]$ $= 0.332[0], \frac{25938}{78125}$	A1	If M0 awarded SC B1 0.3320064 or $\frac{25938}{78125}$ CAO.		
	Method 3 – geometric series	3 – geometric series			
	$[P(X = 3,4,5,6,7) =] \frac{0.144(1-0.6^5)}{1-0.6 \text{ or } 0.4}$	M1	$\frac{0.144(1-p^5)}{1-p} \text{ seen } 0$		
	$=0.332[0], \frac{25938}{78125}$	A1	If M0 awarded SC B1 0.3320064 or $\frac{25938}{78125}$ CAO.		
		2			

Question	Answer			Marks	Guidance
7(c)	Method 1				
	2nd goal scored on:			M1	2 correct unsimplified outcomes. Condone not identified but not incorrectly identified.
	2nd attempt	$(0.4)^2$	[=0.16]	M1	Add values for 4 identified correct scenarios.
	3rd attempt	$(0.4)^2 (0.6) \times (2 \text{ or } {}^2C_1)$	[= 0.192]		Condone adding values of 2nd, 3rd and 4th attempts only. No incorrect scenarios.
	4th attempt	$(0.4)^2 (0.6)^2 \times (3 \text{ or } {}^3C_1)$	[=0.1728]		
	5th attempt	$(0.4)^2 (0.6)^3 \times (4 \text{ or } {}^4C_1)$	[=0.13824]		
	$= 0.663, \frac{2072}{3125}$			A1	If either or both M marks not awarded, SC B1 for 0.663, $\frac{2072}{3125}$ WWW condone 1 index error.
	Method 2				
	$\label{eq:constraint} \begin{split} ^5C_2(0.4)^2(0.6)^3 + {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6)^1 + {}^5C_5(0.4)^5 \\ [0.3456 + 0.2304 + 0.0768 + 0.01024] \\ \text{or} \\ 1 - ({}^5C_0 \ (0.6)^5 + {}^5C_1(0.4)^1(0.6)^4) \end{split}$				At least 2 correct unsimplified terms.
					Add values for 4 terms of the form ${}^{5}C_{a}(0.4)^{a}(0.6)^{5-a}$ or $1 - \text{sum of 2 terms of the form } {}^{5}C_{a}(0.4)^{a}(0.6)^{5-a}$.
	$=0.663, \frac{2072}{3125}$			A1	If either or both M marks not awarded, SC B1 for 0.663 www condone 1 index error.
				3	

Question	Answer	Marks	Guidance
7(d)	[Mean = 75 ×0.4 =] 30 [Variance = 75 ×0.4 × 0.6 =] 18	B1	30 and 18 seen, allow unsimplified. May be seen in standardisation formula. ($\sigma = \sqrt{18}, 3\sqrt{2}, 4.2426 \le \sigma \le 4.243$ implies correct variance) Withold mark if variance clearly identified as standard deviation.
	$P(28 < X < 35) = P(\frac{28.5 - 30}{\sqrt{18}} < Z < \frac{34.5 - 30}{\sqrt{18}})$	M1	Substituting <i>their</i> μ and positive σ into one ±standardising formula (any number for 28.5 or 34.5), not σ^2 , not $\sqrt{\sigma}$.
		M1	Using continuity corrections 27.5 <i>or</i> 28.5 and 34.5 <i>or</i> 35.5 in <i>their</i> 2 separate standardisation formula.
	$\begin{bmatrix} = \Phi(1.0607) + \Phi(0.3536) - 1 \end{bmatrix}$ = 0.8556 + 0.6383 - 1 Or 0.8556 - (1 - 0.6383) Or 0.8556 - 0.3617 Or (0.8556 - 0.5) + (0.6383 - 0.5) Or 0.3556 + 0.1383	M1	Appropriate area Φ , from final process. Must be a probability.
	= 0.494	A1	AWRT.
		5	