

Scheme of Work

Cambridge International AS & A Level Further Mathematics 9231

Further Mechanics (for Paper 3)

For examination from 2020



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Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. The scheme of work has been separated into four documents, one for each examination Paper: Further Pure Mathematics 1 (for Paper 1); Further Pure Mathematics 2 (for Paper 2); Further Mechanics (for Paper 3); and Further Probability & Statistics (for Paper 4). This document relates only to **Further Mechanics 3 (for Paper 3)**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

There is a separate table for each topic of the Further Mechanics syllabus content (3.1 Motion of a projectile, 3.2 Equilibrium of a rigid body, etc.). Each of the bullet points from the syllabus subject content are listed along with teaching suggestions. There is a 'Main theme' for each, which is the focus activity/activities for the content. Where possible, this is supported by an 'Introduction' activity to set the context. Suggestions for independent study (**I**) and formative assessment (**F**) are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that Mathematics is fundamentally problem solving and representing systems and models in different ways. The key concepts are highlighted as a separate item in the new syllabus and teachers should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. They are not referred to specifically in the Scheme of Work as they are essential to tackling problems in all topics.

The key concepts are as follows:

- **Problem solving**
Mathematics is fundamentally problem solving and representing systems and models in different ways.
These include:
 - Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
 - Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
 - Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
 - Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
 - Statistical methods: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

- **Communication**
Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.
- **Mathematical modelling**
Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

Recommended prior knowledge

Knowledge of the Cambridge IGCSE® Mathematics 0580 syllabus (or equivalent) is required for the Cambridge International AS & A Level Further Mathematics 9231 course. All topics from the Cambridge International AS & A Level Mathematics 9709 course are also needed as prior knowledge. However, it is possible to teach both A Level courses alongside each other in parallel, as not all of the Further Mathematics topics have direct dependencies. See *Parallel teaching – A two-year plan to co-teach Cambridge International AS & A Level Mathematics 9709 and Cambridge International AS & A Level Further Mathematics 9231* for guidance. This is available on the School Support Hub www.cambridgeinternational.org/support

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours will vary depending on local practice and your learners' previous experience of the subject.

It is recommended that you spend about 90 hours teaching the content for each Paper: Further Pure Mathematics 1 (for Paper 1); Further Pure Mathematics 2 (for Paper 2); Further Mechanics (for Paper 3); and Further Probability & Statistics (for Paper 4).

The table below gives some guidance about how many hours are recommended for each topic within Further Mechanics (for Paper 3).

Suggested teaching order of syllabus content	Suggested teaching time (hours)
3.6 Momentum	12
3.2 Equilibrium of a rigid body	18
3.3 Circular motion	15
3.1 Motion of a projectile	18
3.4 Hooke's law	15
3.5 Linear motion under a variable force	12

Resources

The textbooks endorsed by Cambridge International for use with this course are listed at www.cambridgeinternational.org. Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. There is also a support resource available for co-teaching the Cambridge International AS & A Level Further Mathematics 9231 course alongside the Cambridge International AS & A Level Mathematics 9709 course: *Parallel teaching – A two-year plan to co-teach Cambridge International AS & A Level Mathematics (9709) and Cambridge International AS & A Level Further Mathematics 9231*, which is available on the School Support Hub.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge International teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format from the School Support Hub. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Websites

This scheme of work includes website links providing access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only particular resources are recommended.

Useful websites include:

www.stem.org.uk The National STEM Learning Network provides access to a range of resources.

<http://integralmaths.org> The Integral® website provides resources developed by a curriculum development project called MEI. Since these schemes were first written, this website has become available only through paid subscription.

www.mmlsoft.com/index.php/products/tarsia The TARSIA software is free to download. It can be used to download and create puzzles to test manipulation.

Important notice about past papers

The 2020 syllabus (for examination in 2020) includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the Specimen Papers and mark schemes for the 2020 series.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Further Mathematics 9231 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Syllabus subject content lists the subject content bullet points from the syllabus, making it clear the knowledge your learners need to build. Pass these on to your learners by expressing them as 'We are learning to / about...'.
(I)

Suggested teaching activities give you lots of ideas about how you can present learners with new information. Try more active methods that get your learners motivated and practising new skills. Where possible, the activities are separated into 'Introduction' ideas to set the context, and 'Main themes' that form the core of the teaching.
(I)

Syllabus subject content

use the method of mathematical induction to establish a given

Suggested teaching activities

Introduction: To put this topic in context, give some examples of deductive proofs, such as proving the formula for the sum of an algebraic or geometric series; or the quadratic formula. It is important to make sure learners understand the need to show mathematical rigour at every step. (I)

Extension activity: Proof by contradiction and by exhaustion could also be examined as an extension activity from the same resource. Some classical proofs are illustrated on <https://undergroundmathematics.org>, search for 'Divisibility by Induction'.
(I)

Main theme: Proving familiar results such as the sum of the positive integers is a good place to start. Some useful examples and exercises can be found on the Integral website (<http://integralmaths.org>). There is a good matching activity on the STEM website (www.stem.org.uk) called 'Creating Connections Between Topics: Proof by Induction'. (F)

Divisibility tests, inequalities, calculus, geometry and series may all be used as contexts and it's always important that the deductive step is written out fully with a rigorous argument. Note: some questions involving calculus may require techniques from Cambridge International AS & A Level Mathematics (9709) Pure Mathematics 3, so take this into consideration when planning.
(F)

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Past and specimen papers

Past papers, specimen papers and mark schemes are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is on-going assessment that informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'.

3.6 Momentum

Syllabus subject content	Suggested teaching activities
recall Newton's experimental law and the definition of the coefficient of restitution, the property $0 \leq e \leq 1$, and the meaning of the terms 'perfectly elastic' ($e = 1$) and 'inelastic' ($e = 0$)	<p>Prior knowledge: Learners should already be familiar with the content of the topics '4.3 Momentum' and '4.5 Energy, work and power' from the Cambridge International AS & A Level Mathematics 9709 syllabus (content for Paper 4).</p> <p>Introduction: For a simple introduction to the principles in this topic, there is a vast array of material to allow learners to become familiar with basic concepts. Examples include, a worked introduction on 'Newton's Law of Restitution (Experimental Law): ExamSolutions Maths' on Youtube (www.youtube.com/watch?v=l6FYmoBB87w) and a worked example on 'Applying Newton's Law of Restitution to a collision: ExamSolutions Maths', also on Youtube (www.youtube.com/watch?v=WjHoYdvairw).</p> <p>Main theme: Learners should be familiar with the use of equations based on the conservation of linear momentum and Newton's experimental laws. They need to be familiar with the meaning of the terms 'perfectly elastic' and 'inelastic' when $e = 1$ and 0 respectively. They should appreciate that $e = 0$ implies that the particles coalesce, while $e = 1$ results in no loss of kinetic energy.</p>
use conservation of linear momentum and/or Newton's experimental law to solve problems that may be modelled as the direct or oblique impact of two smooth spheres, or the direct or oblique impact of a smooth sphere with a fixed surface	<p>Prior knowledge: It is vital that learners appreciate the key importance of using a consistent and straightforward style of notation that they, and others, can follow easily. In problems involving a fixed surface, learners should be familiar with either using components of velocity for a single impact or considering the components of velocity in the course of more than one impact.</p> <p>There is a useful presentation (L2: Equation of equilibrium) on the NPTEL website (http://nptel.ac.in/courses/112103109/2) that includes oblique impacts.</p> <p>(I) Simple examples can be examined using Geogebra (www.geogebra.org), which is a free online graphing package. For example, a great interactive activity where learners can experiment by changing initial conditions, is called 'Collisions' (www.geogebra.org/m/m1boNBIV)</p> <p>As with all work in this area of applied mathematics, the importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty. The past examination paper questions detailed below provide some examples of the type of contexts in which questions on this topic may be set and learners should gain</p>

Syllabus subject content	Suggested teaching activities
	<p>experience through seeing as wide a range of problems as possible.</p> <p>Extension activity: There are a range of problems involving a variety of unusual contexts available on the NRICH website (www.nrich.maths.org), for example https://nrich.maths.org/9059 (I)</p>
Past and specimen examination papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>Specimen Paper 3 Q4 oblique impact</p>	

3.2 Equilibrium of a rigid body

Syllabus subject content	Suggested teaching activities
<p>calculate the moment of a force about a point for questions involving coplanar forces only; understanding of the vector nature of moments is not required</p>	<p>Prior knowledge: Learners need to have covered the content of '4.1 Forces and equilibrium' in the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 4).</p> <p>Main theme: In almost every case involving equilibrium, a clearly labelled diagram is essential. The diagram should show all the forces that are acting on the system and the direction, marked with an arrow, of each of these forces. These forces will include some or all of the weight, the normal reaction, the friction force and any external forces specified in the problem. Learners should already have met the idea of resolving forces in two directions for any system that is in equilibrium, and will now be learning an additional tool, namely taking moments.</p> <p>It is a good idea to for learners to start by understanding what is meant by the moment of a force, and how to calculate it for a given force. The Teaching Advanced Physics (TAP) website (http://tap.iop.org) provides some good ideas for introducing the concept of moments, with practical activities that you can use to enhance understanding for learners. (I) From the Home page, select 'Mechanics', then 'Statics' and then 'Episode 203: Turning effects'. Note: you should ignore any reference to 'couples' as they are not within the scope of this specification.</p> <p>A useful video tutorial ('<i>The moment of a force (torque) Part 1 of 3</i>') that defines moment of forces and demonstrates how to calculate the moment of a force can be found on Youtube (www.youtube.com/watch?v=ikB1SVvZTvU).</p> <p>There is also a helpful tutorial video ('<i>What is the moment of a force?</i>') on the Exam Solutions website (www.examsolutions.net). From the Home page, click on 'Cambridge International Examinations', then 'M2 Tutorials' and then scroll down to the heading 'Equilibrium in a rigid body' and select '1. What is the moment of a force?'.</p>

Syllabus subject content	Suggested teaching activities
<p>use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry</p>	<p>Main theme: The Teaching Advanced Physics (TAP) website (http://tap.iop.org) has good practical activities which illustrate the concept of the centre of mass of an object (http://tap.iop.org/mechanics/static/203/page_46264.html). (I) From the Home page, select 'Mechanics', then 'Statics' and then 'Episode 203: Turning effects'.</p> <p>Learners now need to understand how to identify the centre of mass of certain specified uniform bodies, either by using symmetry or by using the appropriate formula that is given in the list provided in the examination. For example, the centre of mass of a square lamina is at its centre, where the two diagonals meet.</p> <p>A series of video tutorials on this topic can be found on the Exam Solutions website (www.examsolutions.net). From the Home page, click on 'Cambridge International Examinations', then 'M2 Tutorials' (www.examsolutions.net/international-exams/cie/m2-tutorials).</p>
<p>use given information about the position of the centre of mass of a triangular lamina and other simple shapes; proofs of results given in the MF19 List of formulae and statistical tables are not required</p>	<p>Main theme: A couple of good video tutorials can be found on the Exam Solutions website (www.examsolutions.net). From the Home page, click on 'Cambridge International Examinations', then 'M2 Tutorials', then scroll down to 'Uniform Laminas' and click on</p> <ul style="list-style-type: none"> • 'Triangular laminas' to see a video of how to find the centre of mass of a triangular lamina (www.examsolutions.net/tutorials/centre-of-mass-triangular-laminas/?level=International&board=CIE&module=M2&topic=1690) • 'Sectors and semi-circles' to see a video of how to find the centre of mass for a uniform lamina in the form of a sector and semi-circle (www.examsolutions.net/tutorials/centre-of-mass-sectors-semi-circles/?level=International&board=CIE&module=M2&topic=1690). <p>You should make learners aware that formulae for centres of mass of some shapes are given to them in examinations. It would be good practice for you to show learners, at least the more mathematically confident, where these formulae come from.</p>

Syllabus subject content	Suggested teaching activities
<p>determine the position of the centre of mass of a composite body by considering an equivalent system of particles; simple cases only, e.g. a uniform L-shaped lamina, or a uniform cone joined at its base to a uniform hemisphere of the same radius</p> <p>use the principle that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this</p>	<p>Main theme: The Exam Solutions website (www.examsolutions.net) has a short series of video tutorials demonstrating the process involved in finding centres of mass of composite bodies and frameworks. From the Home page, click on 'Cambridge International Examinations', then 'M2 Tutorials' (www.examsolutions.net/international-exams/cie/m2-tutorials/).</p> <p>Please note that for the purposes of this syllabus, only simple cases of composite bodies will be required, for example, a uniform L-shaped lamina, or a uniform cone joined at its base to a uniform hemisphere of the same radius. Usually, a moments equation will be the means to a solution, using known results from the Notation list for the positions of the centres of mass of the separate parts.</p> <p>You can find several video tutorials here at the link given above. These tutorials take you through the method in step-by-step, showing the method that can be applied to any similar problem.</p> <p>The Mathedup! website (www.mathedup.co.uk) has some interactive lessons on the 'Centre of mass' (www.mathedup.co.uk/key-stage-5/applied-maths/mechanics-2/centre-of-mass/). Also see the Integral website (http://integralmaths.org)</p>
<p>solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding</p>	<p>Main theme: Learners need to know that a system will not always be in equilibrium. For example, if a cone is placed on its base on a slope, various possibilities can occur. If the slope is small, the cone may rest there in equilibrium. If the slope is steeper, the cone may slide down the slope, or it may topple over. It is possible to determine mathematically which of these situations occurs by resolving forces and taking moments. Physically, the steepness of the slope and the roughness of the surface are key factors in the outcome. There is a large amount of material available in which the conditions for toppling and sliding are discussed. For example, a video tutorial on Youtube (www.youtube.com) called 'Sliding or Toppling' (www.youtube.com/watch?v=T4XjwOc5Uqg) takes you through the different scenarios of sliding and toppling.</p> <p>Other relevant video tutorials can be found on Exam Solutions (www.examsolutions.net) website, for example, from the Home Page, select 'Cambridge International Examinations', then 'M2 Tutorials' then scroll down to the</p>

Syllabus subject content	Suggested teaching activities
	<p>heading 'Hanging and Toppling Problems' and select 'Freely hanging suspended lamina' (www.examsolutions.net/tutorials/freely-hanging-suspended-lamina/?level=International&board=CIE&module=M2&topic=1704) and 'Lamina topping on an inclined plane' (www.examsolutions.net/tutorials/lamina-toppling-on-an-inclined-plane/?level=International&board=CIE&module=M2&topic=1704).</p> <p>Extension activity: Learners can extend their understanding of this topic by looking at a selection of the available internet material, by searching 'toppling and sliding'. (I)</p>
Past and specimen examination papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)(I)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>Specimen Paper 3 Q1</p>	

3.3 Circular motion

Syllabus subject content	Suggested teaching activities
<p>understand the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$</p>	<p>Prior knowledge: Learners should know and be able to use Newton's laws of motion and understand forces. (This is covered in '4 Mechanics (for Paper 4)' of the Cambridge International AS & A Level Mathematics 9709 course.</p> <p>Main theme: Having previously studied the motion of a particle moving in a straight line under the effect of a force, learners will now consider the motion of a particle in a circle. There are two different cases to be considered. Firstly, a particle moving in a horizontal circle, for example on the end of a string. Secondly, the motion of a particle moving in a vertical circle; again this could be on the end of a string, or on the surface of a sphere, for example. There are many practical applications of circular motion in the real world: satellites orbiting the earth, fairground rides, etc.</p> <p>'Episode 223: Preparation for circular motion topic' on the Teaching Advanced Physics (TAP) website (http://tap.iop.org) provides some good ideas for practical activities, which you can use with learners to bring out the most important ideas in circular motion.</p> <p>The main purpose of any of the indicated activities is to enable learners to grasp the real-life situation that is being explored, and realise that by modelling it in fairly simple mathematical terms, predictions can be made, and the value of physical quantities estimated.</p> <p>It is important for learners to understand the difference between linear motion and angular motion, and the relation between the linear speed of a particle and its angular speed: $v = r\omega$. In questions, either speed could be given but the other can easily be found provided the radius of the circle is known.</p> <p>More mathematically confident learners may find the theoretical and mathematical background interesting. You can find useful material on the Exam Solutions website (www.examsolutions.net). From the Home Page, select 'Cambridge International Examinations', then 'M2 Tutorials' and then 'Angular speed and acceleration' (www.examsolutions.net/tutorials/angular-speed-and-acceleration/?level=International&board=CIE&module=M2&topic=11477).</p>

Syllabus subject content	Suggested teaching activities
<p>understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$; proof of the acceleration formulae is not required</p>	<p>Main theme: Learners need to know the formula for the acceleration of a particle moving in a circle. The expression for this acceleration should be quoted whenever it is required.</p> <p>The resource 'Episode 223: Preparation for circular motion topic' on the TAP (http://tap.iop.org) website gives some ideas on practical activities for investigating circular motion (http://tap.iop.org/mechanics/circular/223/page_46471.html). The resource 'Angular speed and acceleration' on the same website (select 'Cambridge International Examinations', then 'M2 Tutorials' and then 'Angular speed and acceleration') gives a video tutorial comparing linear and circular motion (www.examsolutions.net/tutorials/angular-speed-and-acceleration/?level=International&board=CIE&module=M2&topic=11477) gives a video tutorial comparing linear and circular motion.</p> <p>The main purpose of any of the indicated activities is to enable learners to grasp the real-life situation that is being explored, and realise that by modelling it in fairly simple mathematical terms, predictions can be made, and the value of physical quantities estimated.</p> <p>Please note that proof of these acceleration formulae is not required.</p>
<p>solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed</p>	<p>Main theme: For learners to be successful, you will need to help them to understand clearly what forces are involved in each problem and to choose the correct directions in which to resolve them. It is also key to stress the importance of drawing a diagram, with all the given information clearly indicated: points, speeds, each force and its direction.</p> <p>It is worth giving learners time to practise as many different situations as possible to give them wide experience in analysing the forces involved. Situations will differ, but the method of solution is usually very similar, involving the application of Newton's laws.</p> <p>Video tutorials on different scenarios can be found on the Exam Solutions website (www.examsolutions.net) – from the Home Page, select 'Cambridge International Examinations', then 'M2 Tutorials' (www.examsolutions.net/international-exams/cie/m2-tutorials) – and also in the document 'Mechanics 3 Revision notes' on Mr Barton's Maths website (www.mrbartonmaths.com); click on 'Student's', then 'A Level', then 'Notes, videos and examples' and then 'Mechanics 3 Revision Notes'.</p>

Syllabus subject content	Suggested teaching activities
	<p>There are some good ideas for class discussion and extension studies on the NRICH website (http://nrich.maths.org). For example, 'Whirlyball' (http://nrich.maths.org/4756).</p> <p>Another useful link can be found on the Physics & tutor website (www.physicsandmathstutor.com). Select the 'Maths' link under the heading 'Revision'. Then under the heading 'A-levels' select 'Mechanics 3' and click on the 'Notes' link www.physicsandmathstutor.com/maths-revision/a-level-mechanics-3/.</p>
<p>solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy; including finding a normal contact force or the tension in a string, locating points at which these are zero, and conditions for complete circular motion.</p>	<p>Main theme: When a particle is moving under gravity in a vertical circle, the speed is no longer constant, but we can apply the methods that we learned when considering horizontal circles as our starting-point. The following two links give good summaries of the principles and methods involved, using examples of the type that are covered by this syllabus. These are predominantly, though not exclusively, the vertical motion of a particle attached to the end of a string and the vertical motion of a particle on the smooth inner or outer surface of a sphere ...</p> <ul style="list-style-type: none"> • 'Motion in a vertical circle' (this is on the revision maths website (https://revisionmaths.com); select 'Mechanics' under the heading 'A-Levels', then 'Motion in a vertical circle') • 'Mechanics 3 Revision notes' (this is on Mr Barton's Maths website (www.mrbartonmaths.com); click on 'Student's', then 'A Level', then 'Notes, videos and examples' and then 'Mechanics 3 Revision Notes') <p>... finding a normal contact force or the tension in a string, locating points at which these are zero, and conditions for complete circular motion are all included.</p> <p>As with all applied mechanics topics the fundamental skills must be practised using as many different contexts as possible and a complete and fully labelled diagram essential for success. The importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty.</p> <p>Another useful link can be found on the 'Physics & tutor' website (www.physicsandmathstutor.com). Select the 'Maths' link under the heading 'Revision'. Then under the heading 'A-levels' select 'Mechanics 3' and click on the 'Notes' link www.physicsandmathstutor.com/maths-revision/a-level-mechanics-3/.</p>

Syllabus subject content	Suggested teaching activities
	<p>The past examination paper questions detailed below provide some examples of the type of contexts in which questions on this topic may be set and learners should gain experience through seeing as wide a range of problems as possible.</p> <p>Extension activity: Supplementary notes and extension material called 'Maths Mechanics 3' are available from the TES website (www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759).</p> <p>Additional material can also be found on the STEM website (www.stem.org.uk/), providing additional notes and a range of suitable extension problems. (I)</p>
Past and specimen examination papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)(I)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>2020 Specimen 9231 Paper 3 Q5</p>	

3.1 Motion of a projectile

Syllabus subject content	Suggested teaching activities
<p>model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of the model; vector methods are not required</p>	<p>Prior knowledge: Learners should have covered the content of the '4.2 Kinematics of motion in a straight line' topic in the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 4).</p> <p>Main theme: Look at ways of modelling the motion of an object in two dimensions. You will be building on the basic ideas covered when considering the motion of a particle in a straight line, using the <i>suvat</i> equations to find unknown quantities when others are given. Now, you need to consider motion in two dimensions, under the action of gravity, considering the horizontal and vertical components of the motion separately. The horizontal motion is usually the easier place to start, because the horizontal acceleration is zero. The vertical motion uses a constant acceleration of g, which is usually taken as 9.8 ms^{-2}.</p> <p>There are some good ideas for class discussion and examples on the modelling of projectiles on the NRIC website (www.nrich.maths.org). (I) Also, you could do a search for 'Modelling projectiles with quadratic equations' on You tube (www.youtube.com).</p> <p>The main purpose of any of the indicated activities is to enable learners to grasp the real-life situation that is being explored, and realise that by modelling it in fairly simple mathematical terms, predictions can be made, and the value of physical quantities estimated.</p>
<p>use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached</p>	<p>Prior knowledge: Learners should have covered the content of the '4.2 Kinematics of motion in a straight line' topic in the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 4).</p> <p>Main theme: Investigate the motion of a projectile by considering the horizontal and vertical components. This will usually involve resolving the velocity into its horizontal and vertical components and using the equations of motion for constant acceleration, in this case, the acceleration due to gravity. Care should be taken with the signs. For example, when a particle is projected at an angle above the horizontal, and its initial speed is taken as positive, then its acceleration must have a negative sign because gravity acts vertically downwards. This can usually be achieved by defining clearly at the beginning of a problem which direction you are taking as positive.</p> <p>A summary of the above and a simple example can be found on the revision maths website (https://revisionmaths.com). Click on 'A-Level maths' then 'Mechanics' then 'Projectiles'. There are</p>

Syllabus subject content	Suggested teaching activities
	<p>many other tutorials and videos on this topic on the internet, found by searching 'projectile motion'. For example, the 'Projectiles and equations of motion – Mechanics 1 for A-level' video here on Youtube: www.youtube.com/watch?v=tJJpKVSqMcw</p> <p>The Exam Solutions website (www.examsolutions.net) has a video tutorial which sets out clearly the essential terminology and ideas for analysing projectile motion. This is the first in a series of tutorials which look at some different cases of projectile motion, for example a particle projected from a horizontal surface above or below the horizontal, a particle projected from a cliff top. You can find the others by searching for 'projectiles' on this website.</p> <p>There are also PowerPoint presentations with activities on the Mathed Up! website (www.mathedup.co.uk). Select 'Key Stage 5', then 'Applied Maths', then 'Mechanics 1' and 'Projectiles'.</p>
<p>derive and use the Cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown; knowledge of the 'bounding parabola' for accessible points is not included.</p>	<p>Prior knowledge: Learners should have covered the content of the '4.2 Kinematics of motion in a straight line' topic in the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 4).</p> <p>Main theme: Learners should derive and/or use the equation of the trajectory of a projectile; which can be found by considering the general case of the method learned in the previous section.</p> <p>The Exam Solutions website (www.examsolutions.net) has a clear explanation of the method for deriving the equation of the trajectory. Also included are a useful set of problems for learners to solve; look in the 'Cambridge International Examinations' link. (I)</p> <p>Extension activity: Learners can extend their understanding of this topic by looking at a selection of the available internet material found by searching for 'projectile motion'. (I) For example, the 'What is a projectile?' page on the 'Physics Classroom' website (www.physicsclassroom.com); click on 'physics tutorial', then 'Vectors – Motion and Forces in Two Dimensions', then 'Lesson 2 – Projectile Motion' and then 'What is a projectile' (www.physicsclassroom.com/Class/vectors/u3l2a.cfm) investigates the links with Physics.</p> <p>Learners should gain experience of questions in different contexts through seeing as wide a range of problems and past paper questions as possible.</p>

Past and specimen examination papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)(I)**

The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.

Specimen 9231 Paper 3 Q6

3.4 Hooke's law

Learning objectives	Suggested teaching activities
<p>use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term modulus of elasticity</p> <p>use the formula for the elastic potential energy stored in a string or spring; proof of the formula is not required</p>	<p>Prior knowledge: Learners should have a working knowledge of the content of the '4.1 Forces and equilibrium' topic of the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 4).</p> <p>Main theme: The TAP website (http://tap.iop.org) provides a good introduction to this topic, with some practical activities: from the Home page, select 'Mechanics', then Episode 227: Hooke's law'. (I)</p> <p>There is a large amount of material available on this topic. More confident learners may find the background and physics knowledge illuminating but, for the purposes of this syllabus, such an understanding is not essential. What is very important is an understanding of the mathematics involved, and an ability to apply it to worked examples. The 'Maths Mechanics 3' resource on the TES website (www.tes.com) gives a clear explanation of the mathematics involved, with a series of worked examples (www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759). Another helpful source is Mr Barton's Maths website (www.mrbartonmaths.com) click on 'Student's', then 'A Level', then 'Notes, videos and examples' and then 'Mechanics 3 Revision Notes'.</p>
<p>solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed, e.g. a particle moving horizontally or vertically or on an inclined plane while attached to one or more strings or springs, or a particle attached to an elastic string acting as a 'conical pendulum'.</p>	<p>Main theme: Learners need to be able to apply the knowledge and methods learned in the previous sections to a variety of problem situations. For example, a particle moving horizontally or vertically or on an inclined plane while attached to one or more strings or springs, or a particle attached to an elastic string acting as a 'conical pendulum'. This list is not exhaustive, though it is representative of the type of problem that will be set.</p> <p>You can find notes with examples on the Physics & tutor website (www.physicsandmathstutor.com). Select the 'Maths' link under the heading 'Revision'. Then under the heading 'A-levels' select 'Mechanics 3' and click on the 'Notes'.</p>
Past and specimen examination papers	

Learning objectives**Suggested teaching activities**

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Specimen 9231 Paper 3 Q2

3.5 Linear motion under a variable force

Learning objectives	Suggested teaching activities
<p>solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation; including use of $v \frac{dv}{dx}$ for acceleration, where appropriate; calculus required is restricted to content from Pure Mathematics 3 in Cambridge International A Level Mathematics 9709; only differential equations in which the variables are separable are included.</p>	<p>Prior knowledge: Learners need to be able to differentiate and integrate a variety of expressions; these will be restricted to content from Pure Mathematics 3 in the Cambridge International AS & A Level Mathematics 9709 syllabus. Learners also need to be able to solve differential equations in which the variables are separable.</p> <p>Main theme: Differentiation and integration will enable the learner to move between expressions for displacement, velocity and acceleration, starting at any one of these. The force acting on the particle may be given in terms of x or t. The learner will then need to use Newton's second law of motion to set up a first order differential equation. This will then usually be solved, subject to given initial conditions.</p> <p>This will involve using $\frac{dx}{dt}$ for velocity, and $\frac{dv}{dt}$ or $v \frac{dv}{dx}$ for acceleration, as appropriate. Learners need to be clear that the two differential expressions for acceleration are equivalent, but the choice will depend on whether the variable is expressed with respect to t or x. You can find notes with examples here on the Physics & tutor website (www.physicsandmathstutor.com). Select the 'Maths' link under the heading 'Revision'. Then under the heading 'A-levels' select 'Mechanics 3' and click on the 'Notes'. Section '1 Further kinematics' is the relevant section for detailed notes with examples.</p> <p>Extension activity: There are some good ideas for class discussion and extension studies on the NRICH website (http://nrich.maths.org). For example, 'Mechanical Mindgames' (http://nrich.maths.org/7110). (I) The main purpose of any of the indicated activities is to enable learners to grasp the real-life situation that is being explored, and realise that by modelling it in fairly simple mathematical terms, predictions can be made, and the value of physical quantities estimated</p>

Past and specimen examination papers

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Specimen 9231 Paper 3 Q3

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