

Scheme of Work

Cambridge International AS & A Level Further Mathematics 9231

Further Probability & Statistics (for Paper 4)

For examination from 2020



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Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. The scheme of work has been separated into four documents, one for each examination Paper: Further Pure Mathematics 1 (for Paper 1); Further Pure Mathematics 2 (for Paper 2); Further Mechanics (for Paper 3); and Further Probability & Statistics (for Paper 4). This document relates only to **Further Probability & Statistics (for Paper 4)**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

There is a separate table for each topic of the Further Probability & Statistics content (4.1 Continuous random variables, 4.2 Inference using normal and t -distributions, etc.). Each of the bullet points from the syllabus subject content are listed along with teaching suggestions. There is a 'Main theme' for each, which is the focus activity/activities for the content. Where possible, this is supported by an 'Introduction' activity to set the context. Suggestions for independent study (**I**) and formative assessment (**F**) are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that Mathematics is fundamentally problem solving and representing systems and models in different ways. The key concepts are highlighted as a separate item in the new syllabus and teachers should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. They are not referred to specifically in the Scheme of Work as they are essential to tackling problems in all topics.

The key concepts are as follows:

- **Problem solving**

Mathematics is fundamentally problem solving and representing systems and models in different ways.

These include:

- Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
- Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
- Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
- Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
- Statistical methods: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

- **Communication**

Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.

- **Mathematical modelling**

Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

Recommended prior knowledge

Knowledge of the Cambridge IGCSE® Mathematics 0580 syllabus (or equivalent) is required for the Cambridge International AS & A Level Further Mathematics 9231 course. All topics from the Cambridge International AS & A Level Mathematics 9709 course are also needed as prior knowledge. However, it is possible to teach both A Level courses alongside each other in parallel, as not all of the Further Mathematics topics have direct dependencies. See *Parallel teaching – A two-year plan to co-teach Cambridge International AS & A Level Mathematics 9709 and Cambridge International AS & A Level Further Mathematics 9231* for guidance. This is available on the School Support Hub www.cambridgeinternational.org/support

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours will vary depending on local practice and your learners' previous experience of the subject.

It is recommended that you spend about 90 hours teaching the content for each Paper: Further Pure Mathematics 1 (for Paper 1); Further Pure Mathematics 2 (for Paper 2); Further Mechanics (for Paper 3); and Further Probability & Statistics (for Paper 4).

The table below gives some guidance about how many hours are recommended for each topic within Further Probability & Statistics (for Paper 4).

Suggested teaching order of syllabus content	Suggested teaching time (hours)
4.1 Continuous random variables	12
4.2 Inference using normal and <i>t</i> -distributions	18
4.3 χ^2 -tests	12
4.4 Non-parametric tests	18
4.5 Probability generating functions	15

Resources

The textbooks endorsed by Cambridge International for use with this course are listed at www.cambridgeinternational.org. Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. There is also a support resource available for co-teaching the Cambridge International AS & A Level Further Mathematics 9231 course alongside the Cambridge International AS & A Level Mathematics 9709 course: *Parallel teaching – A two-year plan to co-teach Cambridge International AS & A Level Mathematics (9709) and Cambridge International AS & A Level Further Mathematics 9231*, which is available on the School Support Hub.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge International teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format from the School Support Hub. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Websites

This scheme of work includes website links providing access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only particular resources are recommended.

Useful websites include:

www.stem.org.uk The National STEM Learning Network provides access to a range of resources.

<http://integralmaths.org> The Integral® website provides resources developed by a curriculum development project called MEI. Since these schemes were first written, this website has become available only through paid subscription.

www.mmlsoft.com/index.php/products/tarsia The TARSIA software is free to download. It can be used to download and create puzzles to test manipulation.

Important notice about past papers

The 2020 syllabus (for examination in 2020) includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the Specimen Papers and mark schemes for the 2020 series.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Further Mathematics 9231 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Syllabus subject content lists the subject content bullet points from the syllabus, making it clear the knowledge your learners need to build. Pass these on to your learners by expressing them as ‘We are learning to / about...’.

Suggested teaching activities give you lots of ideas about how you can present learners with new information. Try more active methods that get your learners motivated and practising new skills. Where possible, the activities are separated into ‘Introduction’ ideas to set the context, and ‘Main themes’ that form the core of the teaching.

Syllabus subject content

use the method of mathematical induction to establish a given

Suggested teaching activities

Introduction: To put this topic in context, give some examples of deductive proofs, such as proving the formula for the sum of an algebraic or geometric series; or the quadratic formula. It is important to make sure learners understand the need to show mathematical rigour at every step. **(I)**

Extension activity: Proof by contradiction and by exhaustion could also be examined as an extension activity from the same resource. Some classical proofs are illustrated on <https://undergroundmathematics.org>, search for ‘Divisibility & Induction’.

Main theme: Proving familiar results such as the sum of the positive integers is a good place to start. Some useful examples and exercises can be found on the Integral website (<http://integralmaths.org>). There is a good matching activity on the STEM website (www.stem.org.uk) called ‘Creating Connections Between Topics: Proof by Induction’. **(F)**

Divisibility tests, inequalities, calculus, geometry and series may all be used as contexts and it’s always important that the deductive step is written out fully with a rigorous argument. Note: some questions involving calculus may require techniques from Cambridge International AS & A Level Mathematics (9709) Pure Mathematics 3, so take this into consideration when planning.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Past and specimen papers

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is on-going assessment that informs you about the progress of your learners. Don’t forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, ‘mind maps’, or ‘concept maps’.

4.1 Continuous random variables

Syllabus subject content	Suggested teaching activities
use a probability density function which may be defined piecewise	<p>Prior knowledge: Learners should be familiar with continuous probability density functions, but the definition of a distribution function cannot be assumed at this stage; this is covered in the Cambridge International AS & A Level Mathematics 9709 topic '6 Probability & Statistics 2' (for Paper 6).</p> <p>Main theme: This section focusses on the definition, properties and uses of the probability density function and the cumulative distribution. Learners will use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution. They will also locate the position of the median and percentiles of a distribution by consideration of an appropriate area.</p> <p>The 'probability density functions' video on the Khan Academy website (www.khanacademy.org) gives a simple introduction to the probability density function, and explains the relevance of area (www.khanacademy.org/math/statistics-probability/random-variables-stats-library/discrete-and-continuous-random-variables/v/probability-density-functions).</p> <p>Learners should be able to extend all of these ideas to a probability density function that is defined piecewise.</p>
use the general result $E(g(X)) = \int f(x)g(x) dx$ where $f(x)$ is the probability density function of the continuous random variable X and $g(X)$ is a function of X	<p>Main theme: This is an extension of the result for the calculation of the mean for a probability density function, $f(x)$: $E(X) = \int x f(x) dx$ covered in the previous section. For worked examples and exercises, see https://integralmaths.org/course/view.php?id=121&section=4.</p>
understand and use the relationship between the probability density function (PDF) and the cumulative distribution function (CDF), and use either to evaluate probabilities or percentiles	<p>Main theme: When considering a frequency distribution, the corresponding cumulative frequencies are obtained by summing all the frequencies up to a particular value. In this section, the definition is extended to a continuous random variable, with integration replacing the discrete summation. The following 'Cumulative distribution function – probability' video on Youtube (www.youtube.com) by Engineer Clearly, gives an introduction to the cumulative distribution function, and how it relates to the PDF: www.youtube.com/watch?v=bGS19PxlGC4</p>

Syllabus subject content	Suggested teaching activities
<p>use cumulative distribution functions (CDFs) of related variables in simple cases, e.g. given the CDF of a variable X, find the CDF of a related variable Y, and hence its PDF, e.g. where $Y = X^3$.</p>	<p>Main theme: Notes on this topic can be found on the Integral website (http://integralmaths.org/). In particular, see https://integralmaths.org/course/view.php?id=121&section=3</p> <p>It is essential that learners show full detail when deducing the distribution function of the transformed random variable. It is perfectly possible to arrive at the correct answer even though crucial steps have been omitted and they must appreciate that this is not acceptable in this topic. It is also important that correct notation is used throughout. Random variables must appear in upper case while the distribution function or PDF should be expressed in lower case.</p>
Past and specimen examination papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)(I)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>Specimen Paper 4 Q5 piecewise probability density function</p>	

4.2 Inference using normal and t -distributions

Learning objectives	Suggested teaching activities
<p>formulate hypotheses and apply a hypothesis test concerning the population mean using a small sample drawn from a normal population of unknown variance, using a t-test</p>	<p>Prior knowledge: Learners should be familiar with the structure of a hypothesis test and with the formulae for unbiased estimates of both mean and variance from data given in a variety of forms. Learners should already be familiar with the content of the '6.5 Hypothesis Tests' and the '6.4 Sampling and estimation' topics from the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 6).</p> <p>Main theme: In this section, the basic ideas of hypotheses tests that have already been covered will be extended to include different situations. So far, a z-test has been used for large samples or small samples where the population variance is known. Now we will consider small samples from a normal population of unknown variance. In this case, a t-test will be used. Learners should be aware that the t-distribution approximates to the normal distribution for large values of the variable, but they do not need to prove this. Basic notes, examples and exercises are available on the Integral website (http://integralmaths.org/).</p> <p>The link below to 'Hypothesis testing and p-values' on the Khan Academy website (www.khanacademy.org) relates to an introductory set of short videos covering much of the material in this section. Learners should be familiar with the different approaches to carrying out a hypothesis test, although it is likely that they will focus on the process that best suits their own style of working: www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/hypothesis-testing-and-p-values</p>
<p>calculate a pooled estimate of a population variance from two samples; calculations based on either raw or summarised data may be required.</p>	<p>Main theme: Learners need to be able to calculate an estimate of the variance of a sample by pooling the values from two samples. The following link gives the formula and examples on calculating a pooled estimate for the population variance, but please note that learners do NOT need to know or understand the proof of the formula: search Google for 'PennState Two-sample Pooled t-interval'.)</p>

Learning objectives	Suggested teaching activities
<p>formulate hypotheses concerning the difference of population means, and apply, as appropriate</p> <ul style="list-style-type: none"> • a 2-sample t-test, • a paired sample t-test, • a test using a normal distribution <p>the ability to select the test appropriate to the circumstances of a problem is expected</p>	<p>Main theme: In this section, hypotheses tests are used to test whether there is a significant difference between the population means of two distributions. For example, the means of the lengths of a particular species of fish with samples taken from two lakes.</p> <p>Learners must know the method involved for each of the specified tests, but it is very important that they gain as much experience as possible in selecting the appropriate test for data supplied in a variety of forms as well as appreciating the conditions necessary for a particular test to be valid. Past examination papers are very helpful in helping candidates to gain this experience.</p> <p>The links below to the Khan Academy(www.khanacademy.org) are also available as an introduction to the principles of hypothesis testing and can be used as a starting point for a discussion of the concept.</p> <ul style="list-style-type: none"> • one-tailed and two-tailed tests: www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/one-tailed-and-two-tailed-tests • Z-statistics vs T-statistics: www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/z-statistics-vs-t-statistics • small sample hypothesis test: www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/small-sample-hypothesis-tests <p>It is important to emphasise the situations in which $\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}$ is appropriate rather than the alternative</p> $\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}$
<p>determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a t-distribution</p>	<p>Prior knowledge: Learners should already be familiar with the content of the '6.4 Sampling and estimation' topic from the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 6). Here, the idea of a confidence interval was introduced, and simple calculations were carried out where the population is normally distributed with known variance or where a large sample is used.</p> <p>Main theme: In this section, confidence intervals are calculated using small samples from a normal population with unknown variance. A simple introduction to the idea of a confidence interval for a population mean, 'Understanding confidence intervals: statistics help' by the Statistics Learning Centre, can be found at on Youtube (www.youtube.com): www.youtube.com/watch?v=tFWsuO9f74o</p>

Learning objectives	Suggested teaching activities
	<p>This is good revision of the topic.</p> <p>A more detailed explanation can be found on the website for Online Statistics Education: An Interactive Multimedia Course of study (http://onlinestatbook.com), developed by Rice University. The direct link is http://onlinestatbook.com/2/estimation/confidence.html (If the link fails, go to the Home page and select 'Web Version (2.0)', then 'X: Estimation' from the left-hand column, then under 'E. Confidence Intervals' select '1.Introduction'.)</p>
<p>determine a confidence interval for a difference of population means, using a t-distribution or a normal distribution, as appropriate</p>	<p>Main theme: Here, the confidence interval is calculated for a difference of population means. The following links give examples of this topic.</p> <ul style="list-style-type: none"> • 'Creating a confidence interval for the difference of two means with known standard deviations' by Deborah J. Rumsey (on the dummies.com website): www.dummies.com/education/math/statistics/creating-a-confidence-interval-for-the-difference-of-two-means-with-known-standard-deviations/ • 'Difference between means' by David M. Lane on the Online Statistics Education website (http://onlinestatbook.com): http://onlinestatbook.com/2/estimation/difference_means.html (If the link fails, go to the Home page and select 'Web Version (2.0)', then 'X: Estimation' from the left-hand column, then under 'E. Confidence Intervals' select '5. Confidence interval for the difference between means'.) <p>By this stage, learners need to be familiar with the calculation of a confidence interval for a mean or the difference of means, with large and small samples, and be able to discern which is the appropriate method in each case. They should gain as much practice as possible. Past examination papers contain a wealth of examples.</p> <p>A large amount of material can be found on the internet by searching 'confidence intervals', in addition to the links already given.</p>

Past and specimen examination papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)(I)**

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Specimen Paper 4 Q3 paired t -test

Specimen Paper 4 Q4 confidence interval for difference in means

4.3 χ^2 -tests

Syllabus subject content	Suggested teaching activities
<p>fit a theoretical distribution, as prescribed by a given hypothesis, to given data; questions will not involve lengthy calculations</p> <p>use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding goodness of fit analysis; classes should be combined so that each expected frequency is at least 5</p>	<p>Prior knowledge: Learners should be familiar with discrete uniform, binomial, Poisson, geometric, normal and other continuous random variables defined by their PDF or distribution function. These have been covered in the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 5 and Paper 6).</p> <p>Main theme: In this section, learners will use the χ^2-test to carry out goodness of fit tests. Frequencies obtained from an experiment or situation are compared with the frequencies that would be obtained from a specified distribution. The basic principles can be found in the video 'Pearson's chi square test (goodness of fit)' on the Khan Academy website (www.khanacademy.org). (Direct link: www.khanacademy.org/math/statistics-probability/inference-categorical-data-chi-square-tests/chi-square-goodness-of-fit-tests/v/pearson-s-chi-square-test-goodness-of-fit)</p> <p>Some simple explanations can be found on Youtube (www.youtube.com):</p> <ul style="list-style-type: none"> • 'Simple explanation of Chi-squared' by J David Eisenberg (www.youtube.com/watch?v=VskmMgXmkMQ) • 'How to ... perform a Chi-square test (by hand)' by Eugene O'Loughlin (www.youtube.com/watch?v=V4SRgabFbz0) <p>When using the χ^2 distribution, the approximation used is not valid if the expected frequency is less than 5. This applies to both goodness of fit and contingency tables. Where appropriate, either rows or columns should be combined so that the expected frequency in each cell is at least 5. Yates' correction is not required.</p> <p>An example of a hypothesis for a goodness of fit test is</p> <p>H_0 : model fits the data H_1 : model does not fit the data</p> <p>For all χ^2-tests, it is important to set out the working in tabular form, and the rules for calculating the number of degrees of freedom must be fully understood. In general, the number of degrees of freedom is the number of independent variables involved in calculating the statistic. This is found by subtracting the number of restrictions from the number of classes.</p>

Syllabus subject content	Suggested teaching activities
<p>use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a contingency table; Yates' correction is not required; where appropriate, either rows or columns should be combined so that the expected frequency in each cell is at least 5.</p>	<p>Main theme: The χ^2-test should be used to carry out tests for independence in a contingency table. The basic principles of a χ^2-test for independence in a contingency table can be found on the Khan Academy website (www.khanacademy.org); search for 'Contingency table chi-square test' (www.khanacademy.org/math/statistics-probability/inference-categorical-data-chi-square-tests/chi-square-tests-for-homogeneity-and-association-independence/v/contingency-table-chi-square-test).</p> <p>Another explanation with a simple example, 'Chi square contingency tables', can be found on the Online Statistics Education website: http://onlinestatbook.com/2/chi_square/contingencyM.html</p> <p>An example of a suitable hypothesis in a contingency table test is H_0 : factors are not associated H_1 : factors are associated (in context)</p> <p>For all χ^2-tests, it is important to set out the working in tabular form and the rules for calculating the number of degrees of freedom must be fully understood.</p> <p>The minimum number in an expected frequency is 5; this applies to both goodness of fit and contingency tables. Where appropriate, either rows or columns should be combined so that the expected frequency in each cell is at least 5. Yates' correction is not required.</p> <p>Extension material: Learners could investigate the nature of the χ^2 distribution.</p>

Past and specimen examination papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)(I)**

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Specimen Paper 4 Q2 goodness of fit test

4.4 Non-parametric tests

Learning objectives	Suggested teaching activities
<p>understand the idea of a non-parametric test and appreciate situations in which such a test might be useful, e.g. when sampling from a population which cannot be assumed to be normally distributed</p>	<p>Prior knowledge: Learners should be familiar with the structure of a hypothesis test and with the formulae for unbiased estimates of both mean and variance from data given in a variety of forms. Learners should already be familiar with the content of the '6.5 Hypothesis Tests' and the '6.4 Sampling and estimation' sections of the Cambridge International AS & A Level Mathematics 9709 syllabus (for Paper 6).</p> <p>Main theme: So far, learners will have become familiar with parametric tests, although they will have simply referred to them as 'tests'. In this section, the focus moves to distributions that cannot be assumed to be normal, so the usual parameters (mean and variance) will not be appropriate. Often, though not always, these non-parametric tests are based on the use of ranks: some aspect of the data is ranked. The hypothesis test usually concerns the population median rather than the mean.</p>
<p>understand the basis of the sign test, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test; including knowledge that Wilcoxon tests are valid only for symmetrical distributions</p> <p>use a single-sample sign test and a single-sample Wilcoxon signed-rank test to test a hypothesis concerning a population median; including the use of normal approximations where appropriate; questions will not involve tied ranks or observations equal to the population median value being tested</p>	<p>Main theme: This section focuses on single sample sign tests. An example using the simple sign test involves determining the median of a set of data, then splitting the sample data into two categories, plus and minus, depending on whether the item is above or below the median. The Wilcoxon tests are more sophisticated, and put responses into rank order.</p> <p>'Chapter 13 The Wilcoxon signed rank test' on the MEI website (http://mei.org.uk/files/pdf/Wilcoxonrerevised.pdf) gives a simple introduction to the sign test and the Wilcoxon signed-rank test.</p> <p>Information can also be found on the Integral website (http://integralmaths.org/).</p>

Learning objectives	Suggested teaching activities
	<p>There is also an introduction to these tests, a thorough discussion of the material and some examples on the PennState Eberly College of Science website, STAT 414/415 (https://onlinecourses.science.psu.edu):</p>
<p>use a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and a Wilcoxon rank-sum test, as appropriate, to test for identity of populations; including the use of normal approximations where appropriate; questions will not involve tied ranks or zero-difference pairs.</p>	<p>Main theme: Examples using matched pairs will require the use of the Wilcoxon signed-rank test; examples using non-matched pairs will use the Wilcoxon rank-sum test. Learners may note that these two tests correspond to a t-test in cases where the population can be assumed to be normal or near-normal.</p> <p>An example can be found on the PennState Eberly College of Science website STAT 464 (https://onlinecourses.science.psu.edu): '4.3 Wilcoxon rank sum test' https://onlinecourses.science.psu.edu/stat464/node/36</p> <p>'Chapter 10 The Wilcoxon Rank-Sum test' (www.stat.auckland.ac.nz/~wild/ChanceEnc/Ch10.wilcoxon.pdf) is another good example. This can be found on The University of Auckland website (www.stat.auckland.ac.nz).</p> <p>Please note that questions will not involve tied ranks or zero-difference pairs.</p> <p>Information can also be found on the Integral website (http://integralmaths.org/).</p>
Past and specimen examination papers	

Learning objectives	Suggested teaching activities
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)(I)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>Specimen Paper 4 Q1 Wilcoxon signed-rank test</p>	

4.5 Probability generating functions

Syllabus subject content	Suggested teaching activities
understand the concept of a probability generating function (PGF) and construct and use the PGF for given distributions; including the discrete uniform, binomial, geometric and Poisson distributions	<p>Prior knowledge: Learners need to be familiar with the distributions covered in Cambridge International AS & A Level Mathematics 9709 syllabus (for Papers 5 and 6). In particular, this will include the discrete uniform, binomial, geometric and Poisson distributions.</p> <p>Main theme: Learners will become familiar with the concept of a probability generating function, noting that it summarises the probabilities with which a discrete random variable takes its possible values. They will be required to construct and use the probability generating function for a distribution which will be given, or a type with which they are familiar, and understand the uniqueness of the relationship between a distribution and its probability generating function.</p> <p>The following links each give an introduction and examples for all the distributions listed above. No proofs are required, but more confident learners might like to look at these. (I)</p> <p>Detailed notes with examples and exercises:</p> <ul style="list-style-type: none"> • ‘6 Probability generating functions’ from the University of Cambridge Department of Computer Science and Technology (www.cl.cam.ac.uk/teaching/0708/Probability/prob06.pdf); please note that this link includes material beyond the requirements for Paper 4, but it can provide extra depth for confident learners (I) • ‘Chapter 4 Generating functions’ from The University of Auckland, Department of Statistics www.stat.auckland.ac.nz/~fewster/325/notes/ch4.pdf <p>Information can also be found on the Integral website (http://integralmaths.org/).</p>

Syllabus subject content	Suggested teaching activities
<p>use formulae for the mean and variance of a discrete random variable in terms of its PGF, and use these formulae to calculate the mean and variance of a given probability distribution</p>	<p>Main theme: Learners need to be able to derive the mean and variance of a discrete random variable from its PGF. The links given in the previous section are also helpful here:</p> <ul style="list-style-type: none"> • ‘6 Probability generating functions’ from the University of Cambridge Department of Computer Science and Technology (www.cl.cam.ac.uk/teaching/0708/Probability/prob06.pdf); please note that this link includes material beyond the requirements for Paper 4, but it can provide extra depth for confident learners (I) • ‘Chapter 4 Generating functions’ from The University of Auckland, Department of Statistics www.stat.auckland.ac.nz/~fewster/325/notes/ch4.pdf <p>See also ‘Probability generating function introduction’ by Mark Willis on You tube: www.youtube.com/watch?v=GL5DVQ2-5Fs</p>
<p>use the result that the PGF of the sum of independent random variables is the product of the PGFs of those random variables</p>	<p>Main theme: Learners need to know that the probability generating function of a sum of two or more independent discrete random variables is the product of their probability generating functions. The links given in the previous section are again helpful here:</p> <ul style="list-style-type: none"> • ‘6 Probability generating functions’ from the University of Cambridge Department of Computer Science and Technology (www.cl.cam.ac.uk/teaching/0708/Probability/prob06.pdf); please note that this link includes material beyond the requirements for Paper 4, but it can provide extra depth for confident learners (I) • ‘Chapter 4 Generating functions’ from The University of Auckland, Department of Statistics www.stat.auckland.ac.nz/~fewster/325/notes/ch4.pdf
Past and specimen examination papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)(I)</p> <p>The 2020 syllabus includes changes to the assessment structure, assessment objective weightings and syllabus content when compared to the 2017–2018 and 2019 syllabuses. Therefore, if you use past papers and mark schemes from earlier series, please do so with caution. It is still possible to help your learners understand what the examination papers look like and to give an idea of the required standard but please be aware that some of the content, the assessment structure and nature of the mark scheme has changed. Please also use the specimen papers and mark schemes for the 2020 series.</p> <p>Specimen Paper 4 Q6 probability generating function</p>	

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