Cambridge Assessment International Education

Example Candidate Responses – Paper 3

Cambridge International AS & A Level Further Mathematics 9231

For examination from 2022

Cambridge Pathway

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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Further Mathematics 9231 and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet, candidate responses have been chosen from the June 2022 series to exemplify a range of answers for all the questions on the question paper.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Please also refer to the June 2022 Examiner Report for further detail and guidance.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

9231 June 2022 Question Paper 33

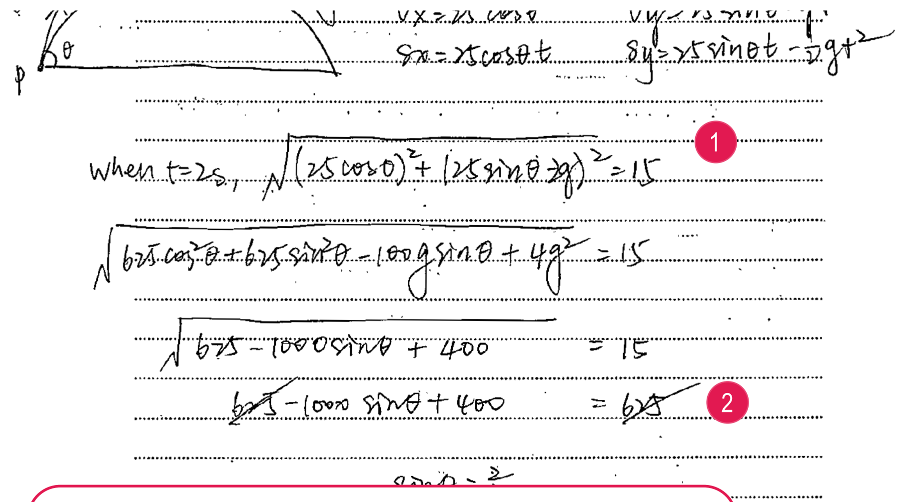
9231 June 2022 Mark Scheme 33

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

This booklet goes through the paper one question at a time. The candidate answers are set in a table. In the left-hand column are the candidate answers, and in the right-hand column are the examiner comments.

Example Candidate Response – 1	Examiner comments
	1 This was a very common error. The force acting on P is a resistive force and the acceleration is measured in a positive direction, so there needs to be a minus sign in the N2L equation. 2 The method of simplifying the integrand is correct, but a coefficient of 3 has been dropped

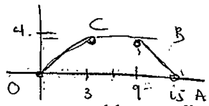
Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

Question 1

Example Candidate Response – 1	Examiner comments
1 A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x-y plane. The coordinates of the vertices are $O(0,0)$, $A(15,0)$, $B(9,4)$ and $C(3,4)$. Find the x-coordinate of the centre of mass of the lamina. (1) [4] $M_1 = 3 \times 4 \times \frac{1}{2} = 6$ $M_2 = 6 \times 4 = 24$ $M_3 = 6 \times 4 \div 2 = 12$ $d_{1x} = 3 \div 3 = 1$ (2) $d_{2x} = 3 + 3 = 6$ $d_{3x} = 6 + 3 + 2 = 11$ $\bar{x}(6 + 24 + 12) = 6 \times 1 + 24 \times 6 + 12 \times 11$ (3) $42\bar{x} = 282$ $\bar{x} = \frac{47}{7}$	(1) From the first line of working, the candidate clearly divides the lamina into two triangles and a rectangle. The areas of the three parts have been calculated correctly. However, it is always wise to draw a diagram to illustrate the situation. Those candidates who omitted to draw a diagram often made the error which this candidate has made in the second line of working. (2) The distance required here is the distance of the centre of mass of the triangle *OCD*, where *D* is the point (3, 0), from the y-axis. It should be two-thirds of the length of *OD*, measured from *O*, that is 2. It was a common error for this distance to be taken as one-third of the length of *OD* from *O*. A diagram would have been likely to help the candidate to avoid this error. (3) The moments equation is correct apart from the error already noted in line 2 of the working **Total mark awarded = 2 out of 4**

Example Candidate Response – 2



2

1 A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x-y plane. The coordinates of the vertices are $O(0,0)$, $A(15,0)$, $B(9,4)$ and $C(3,4)$.

Find the x-coordinate of the centre of mass of the lamina. [4]

	Area	Centre of mass
Rectangle	$15 \times 4 = 60$	$\frac{15}{2}$ 1
Triangle 1	$\frac{1}{2} \times 4 \times 3 = 6$	$\frac{4+4+0}{3} = \frac{8}{3}$
Triangle 2	$\frac{1}{2} \times 6 \times 4 = 12$	$\frac{4}{3}$ $\cancel{4}$
Lamina	$60 - 6 - 12 = 42$	$\bar{x}$

$\therefore 42\bar{x} = 60\left(\frac{15}{2}\right) - 6\left(\frac{8}{3}\right) - 12\left(\frac{4}{3}\right)$ 2

$42\bar{x} = 450 - 16 - 16$

$\therefore \bar{x} = \frac{418}{42} = \frac{209}{21} = 9.95$

Examiner comments

1 The candidate considers the area of the lamina by imagining it as a rectangle with a triangle subtracted at each end. These areas are correct. The centre of mass of the rectangle is also correct. However, the centres of mass of the two triangles are incorrect. This illustrates a common error when finding the centres of mass of the component areas of the lamina. The centre of mass of a triangular lamina is two-thirds of the distance from a vertex along one of the sides. So, for triangle 1 this should be $\frac{1}{3}$ of 3 and for triangle 2 it should be $9 + \frac{2}{3}$ of 6. The distances in the table should be 1 for triangle 1 and 13 for triangle 2. This example illustrates the type of error made in finding the centres of mass of the component parts of a composite lamina.

2 The moments equation is correct following through with the errors in the distances of the centres of mass from the y-axis.

Total mark awarded = 2 out of 4

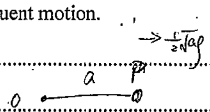
Question 2

Example Candidate Response – 1

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

friction: $f = \mu R = \frac{1}{3}mg$



Elastic energy gained $= E = \frac{\lambda x^2}{2l}$

$= \frac{\frac{4}{3}mg\,x^2}{2a}$

KE loss: $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times m \times \frac{1}{4}ga = \frac{1}{8}mga.$

Energy gain through friction $= \frac{1}{3}mgx$

$\cancel{\frac{\frac{4}{3}mgx^2}{2a} = \frac{1}{3}mgx + \frac{1}{8}mga}$

$\cancel{\frac{\frac{4}{3}x^2}{2a} = \frac{1}{3}x + \frac{1}{8}a}$

$\cancel{\frac{4}{3}x^2 = \frac{2}{3}ax + \frac{1}{4}a^2}$

$\frac{\frac{4}{3}mgx^2}{2a} + \frac{1}{3}mgx = \frac{1}{8}mga$ **1**

$\frac{\frac{4}{3}x^2}{2a} + \frac{1}{3}x = \frac{1}{8}a$

$\frac{2x^2}{3a} + \frac{1}{3}x - \frac{1}{8}a = 0$

$\frac{2}{3}x^2 + \frac{a}{3}x - \frac{a^2}{8} = 0$ **2**

Examiner comments

1 The candidate forms the energy-work done equation. Each term is correct, and the sign attached to each is correct. Three marks are awarded at this stage.

2 The candidate does some simplification and obtains a correct homogeneous equation in x and a. This is the point where some candidates stopped, not recognising that multiplication through by 24 will give a quadratic equation which can easily be factorised. This type of quadratic equation, with one variable, x, and one fixed quantity (here, a) given in the question is common in Mechanics. It can be solved, either by factorisation or by use of the formula to give x in terms of a.

Total mark awarded = 3 out of 5

Example Candidate Response – 2

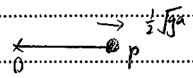
2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

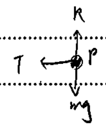
$l = a$

$\lambda = \frac{4}{3}mg$

$\mu = \frac{1}{3}$



$\cancel{T = \frac{4}{3}mg}$ $R = mg$



IME = FME

$\frac{1}{2}m\left(\frac{1}{4}\right)(ga) = \frac{\frac{4}{3}mg(x)^2}{2a} + \frac{1}{3}mg$ 1

$\frac{1}{8}mga = \frac{2}{3}\frac{mgx^2}{a} + \frac{1}{3}mg$

$\frac{1}{8}a = \frac{2}{3}\left(\frac{x^2}{a}\right) + \frac{1}{3}$

$\cancel{\frac{1}{8}a \leftarrow \frac{2}{3}}$

$\frac{2x^2}{3a} = \frac{1}{8}a - \frac{1}{3}$

$2x^2 = \frac{3a^2}{8} - \frac{3a}{3}$

$= \frac{9a^2 - 24a}{24}$

$x^2 = \frac{9a^2 - 24a}{48}$

$= \frac{3a^2 - 8a}{16}$

$x = \frac{\sqrt{3a^2 - 8a}}{4}$

Examiner comments

1 The candidate attempts to form the energy-work done equation for this situation. There is a loss in Kinetic Energy, a gain in Elastic Potential Energy and work done against friction. The first two of these are correct, but the last is not. This is a common error, where the frictional force, $\frac{1}{3}mg$, is used instead of the work done term. The frictional force needs to be multiplied by the distance moved by the particle, in this case, x. A mark is awarded for the correct Elastic Potential Energy term, but no further marks for an energy-work done equation in which one of the terms is simply a force.

Total mark awarded = 1 out of 5

Question 3

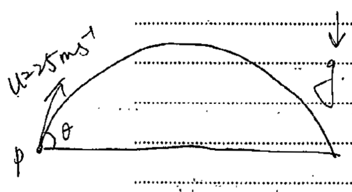
Example Candidate Response – 1

$v=u+at$

3 A particle P is projected with speed $25\,\text{m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2 s the speed of P is $15\,\text{m s}^{-1}$.

u $t=2s$ $v=15$

(a) Find the value of $\sin\theta$. [5]



$u_x = 25\cos\theta$ $u_y = 25\sin\theta$

$v_x = 25\cos\theta$ $v_y = 25\sin\theta - gt$

$S_x = 25\cos\theta\, t$ $S_y = 25\sin\theta\, t - \frac{1}{2}gt^2$

When $t=2s$, $\sqrt{(25\cos\theta)^2 + (25\sin\theta - 2g)^2} = 15$ **1**

$\sqrt{625\cos^2\theta + 625\sin^2\theta - 100g\sin\theta + 4g^2} = 15$

$\sqrt{625 - 1000\sin\theta + 400} = 15$

$\cancel{625} - 1000\sin\theta + 400 = \cancel{625}$ **2**

$\sin\theta = \frac{2}{5}$

5

(b) Find the range of the flight. [3]

R.

$S_y = 25\sin\theta\, t - \frac{1}{2}gt^2 = 0$

$25 \cdot \frac{2}{5} t - 5t^2 = 0$

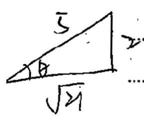
$10t - 5t^2 = 0$

$5t(2-t) = 0$

$\therefore$ when $t=0$ & 2, the particle P will touch the ground. **3**

$t=2s$: $S_x = 25\cos\theta \cdot 2 = 25 \cdot \frac{\sqrt{21}}{5} \cdot 2$

$= 10\sqrt{21}$ m



Examiner comments

1 This equation is correct. The horizontal and vertical components of the velocity are squared and added and equated to the given speed.

2 The simplification of the left-hand-side is correct, but 15 squared, on the right-hand-side is not equal to 625. Errors in this part were not common, but this is an example of the type of arithmetic error that was seen.
Mark for (a) = 4 out of 5

3 The method in this part is correct, but the arithmetical error in part (a) has led to further inaccuracy.
Mark for (b) = 1 out of 3

Total mark awarded = 5 out of 8

Question 4

Example Candidate Response – 1	Examiner comments

(a) Find, in terms of m and g, the magnitude of the tension in the string at A.

$\cancel{T\cos\theta = mg}$

$T_1 = mg\cos\theta$ 1

$T_2 = \cancel{mg\cos\alpha}\ \frac{mu^2}{r} - mg\cos\alpha$

$T = \frac{m\cdot(2u)^2}{a} - mg\cos\alpha$. 2

$T = \frac{4mu^2}{a} - mg\cos\alpha$

$\Delta KE = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = \frac{1}{2}m\cdot 4u^2 - \frac{1}{2}mu^2$

$= \cancel{\frac{3}{2}u}\ \frac{3}{2}mu^2$

$\Delta GPE = mgh = mg\cdot(a\cos\theta + a\cos\alpha)$.

$\frac{3}{2}mu^2 = mg(a\cos\theta + \cos\alpha)$ 3

$3u^2 = 2ga(\cos\theta + \cos\alpha)$. $\quad \cos\alpha = \frac{3u^2}{2ga} - \cos\theta$

$\cancel{T =}\ 10mg\cos\theta = \frac{4mu^2}{a} - mg\cos\alpha$

$10g\cos\theta = \frac{4u^2}{a} - g\cos\alpha$

$10ga\cos\theta = 4u^2 - \cancel{ag\cos\theta}\ ag\cos\alpha$

$10ga\cos\theta = 4u^2 - ag\cdot\left(\frac{3u^2}{2ga} - \cos\theta\right)$

$10ga\cos\theta = 4u^2 - \frac{3u^2}{2} + ag\cos\theta$.

$9ag\cos\theta = 4u^2 - \frac{3u^2}{2}$

$9ag\cos\theta = \frac{5}{2}u^2$

$9ag\cos\theta = \frac{5}{2}\cdot\frac{2}{3}ga$

$9ag\cos\theta = \frac{5}{3}ga$.

$\cos\theta = \frac{5}{27}$.

(b) Find the value of $\cos\alpha$.

$\cos\alpha = \frac{3u^2}{2ga} - \frac{5}{27}\ \cancel{mg}$.

$= \frac{3\cdot\frac{2}{3}ga}{2ga} - \frac{5}{27}$

$= 1 - \frac{5}{27}$

$= \frac{22}{27}$ 4

Examiner comments

1 This is an attempt to use Newton's second law (N2L) at the point *A*, but it lacks an acceleration term. This would suggest that the candidate thinks that there is no initial velocity. However, this initial velocity does appear in the energy equation, so there is an inconsistency. The first step in answering any question on vertical circular motion is to draw a diagram with all the given information added.

2 This is an attempt to use N2L at the point *B*. It includes the tension, the correct component of the weight and the correct acceleration term. However, there is a sign error. The final term should be positive. This error was often seen when a candidate attempts to solve the problem without first drawing a diagram. It cannot be emphasised enough that a diagram is an essential tool in solving a mechanics problem involving motion in a vertical circle. A simple diagram with the particle shown in two different positions and forces added will greatly reduce the possibility of sign errors.

3 This is a correct expression of the energy equation.
Mark for (a) = 3 out of 6

4 The method mark is awarded here. The candidate uses the incorrect value from part (a) in a correct equation found in part (a).
Mark for (b) = 1 out of 2

Total mark awarded = 4 out of 8

Example Candidate Response – 2

(a) Find, in terms of m and g, the magnitude of the tension in the string at A.

$T_B - mg\cos\alpha = m\frac{u^2}{a}$ 1

$T_B = m\frac{(2\sqrt{\frac{2}{3}ga})^2}{a} + mg\cos\alpha$

$T_B = \cancel{mg(\frac{8}{3}\cos\alpha)}\ mg(\frac{8}{3} + \cos\alpha)$

$\cancel{T_A = \frac{1}{10}T_B = \frac{1}{10}mg(\frac{8}{3}+\cos\alpha)}$

$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2 + mgh$ 2

$\frac{1}{2}m(\sqrt{\frac{2}{3}ga})^2 + mga(1+\cos\theta) = \frac{1}{2}m(2\sqrt{\frac{2}{3}ga})^2 + mga\cos\alpha$

$-mga = mga(\cos\alpha - 1 - \cos\theta)$

$\cos\alpha - 1 - \cos\theta = -1$

$\cos\alpha = \cos\theta$

$T_A + mg\cos\theta = m\frac{u^2}{a}$ 3

$T_A = m\frac{\frac{2}{3}ga}{a} - mg\cos\theta$

$= mg(\frac{2}{3} - \cos\theta)$

$mg(\frac{8}{3} + \cos\alpha) = 10mg(\frac{2}{3} - \cos\theta)$ 4

$\frac{8}{3} + \cos\theta = \frac{20}{3} - 10\cos\theta$

$\cos\theta = \frac{4}{11}$ 5

$T_A = mg(\frac{2}{3} - \frac{4}{11}) = \frac{10}{33}mg$

(b) Find the value of $\cos\alpha$.

$\cos\alpha = \cos\theta = \frac{4}{11}$

6

Examiner comments

1 This is a correct application of N2L at the second position of the string, named *B*.

2 This is an energy equation with the energy at *B* equated to the energy at *A*. There is an error in calculating the potential energies. The potential energy at *A*, on the left-hand-side, is measured from the lowest point in the circular path, whereas the potential energy at *B* is measured from the centre of the circle. Either of these 'base' positions would be fine, but they must be consistent. In this case, the term on the right-hand-side should be $mga(1 - \cos\alpha)$. Sign errors were common in the energy equation.

3 This is a correct application of N2L at position *A*.

4 This uses the information that the magnitude of the tension in the string at *B* is 10 times its magnitude at *A*.

5 The candidate uses their equations concisely to find an expression for *T* in terms of m and g. Other candidates had no clear strategy for elimination and were unable to complete this step.
Mark for (a) = 4 out of 6

6 Because of an earlier error, this is an incorrect, and simpler, connection between the two cosines and no marks can be awarded.
Mark for (b) = 0 out of 2

Total mark awarded = 4 out of 8

Question 5

Example Candidate Response – 1

5 A particle P of mass 4 kg is moving in a horizontal straight line. At time t s the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and the displacement of P from a fixed point O on the line is x m. The only force acting on P is a resistive force of magnitude $(4e^{-x}+12)e^{-x}$ N. When $t=0$, $x=0$ and $v=4$.

(a) Show by integration that $v=\frac{1+3e^{x}}{e^{x}}$. [4]

4kg

P O

$4v\frac{dv}{dx} = -e^{-x}(4e^{-x}+12)$ 1

$\int 4v\,dv = -\int (4e^{-2x}+12e^{-x})\,dx$

$2v^2 = +2e^{-2x} + 12e^{-x} + C$

$v=4,\ x=0$

$2\times 16 = +2+12+C \quad \therefore C = \cancel{46}\ 18$

$2v^2 = +2e^{-2x} + 12e^{-x} + \cancel{46}\ 18$ 2

$v^2 = e^{-2x} + 6e^{-x} + \cancel{23}\ 18$ 3

$\cancel{v^2 = e^{-x}(e^{-x} - 6 + 23e^{x})}$

$v^2 = e^{-x}(e^{-x} + 6 + 18e^{x})$ 4

$v^2 = e^{-x^2}(9e^{x^2} + 6e^{x} + 1)$ 5

$\therefore v = e^{-x}(3e^{x}+1)$

$v = \frac{3e^{x}+1}{e^{x}}$

Examiner comments

1 This is a correct application of N2L to this problem.

2 The integration is correct and the constant c has been found correctly.

3 The following lines illustrate incorrect work which do not legitimately obtain the given answer. Manipulations of this type and others were commonly seen as candidates showed a determination to arrive at the given answer regardless of errors.

4 The division by 2 on the previous line is incomplete, so this line is incorrect.

5 The incorrect 18 now becomes the correct 9 and this might have been credited as ‘recovery’ except that another error has occurred in the power of the exponential. The first term should have $2x$ and not x^2. Mark for (a) = 3 out of 4

Example Candidate Response – 1, continued	Examiner comments
(b) Find an expression for x in terms of t. $V = \frac{dx}{dt} = \frac{1+3e^x}{e^x}$ $\int \frac{e^x}{1+3e^x}\,dx = \int dt$ $\ln(1+3e^x) = t + A$ 6 $t=0,\ x=0$. $\ln(1+3) = A \quad A = \ln 4$. $\ln(1+3e^x) = t + \ln 4$ 7 $1+3e^x = e^{t+\ln 4}$ $1+3e^x = 4e^t$ $3e^x = 4e^t - 1$ $e^x = \frac{4}{3}e^t - \frac{1}{3}$ $x = \ln\left(\frac{4}{3}e^t - \frac{1}{3}\right)$ 8 $x = \frac{t + \ln\frac{4}{3}}{\ln\frac{1}{3}}$	6 The basic integration is correct, but a factor of 3 has been dropped from the denominator of the left-hand-side. 7 This candidate has found an expression for t but then proceeds to find x in terms of t. A common error is for candidates to leave t in terms of x as their final answer. Sometimes this may be sufficient, but candidates must read the question carefully to see what is required, here x in terms of t. 8 This answer is correct apart from the missing 3 in the power of the exponential. The final line comes from some incorrect manipulation of the log terms, but is disregarded. Mark for (b) = 2 out of 4 **Total mark awarded = 5 out of 8**

Example Candidate Response – 2

5 A particle P of mass 4 kg is moving in a horizontal straight line. At time t s the velocity of P is $v\,\text{m s}^{-1}$ and the displacement of P from a fixed point O on the line is x m. The only force acting on P is a resistive force of magnitude $(4e^{-x}+12)e^{-x}$ N. When $t = 0$, $x = 0$ and $v = 4$.

(a) Show by integration that $v = \dfrac{1+3e^{x}}{e^{x}}$. [4]

$$a = \frac{F}{m} = \frac{(4e^{-x}+12)e^{-x}}{4} = v\cdot\frac{dv}{dx}$$ 1

$\int e^{-x}dx$ $\quad$ $-1e^{-x}$ $\quad$ e^{-x} $\quad$ $(e^{x})' = -e^{-x}$

$$\int (e^{-x}+3)\,e^{-x}dx = \int v\cdot dv$$

$$\frac{1}{2}v^2 = \int \frac{\frac{1+3e^{x}}{e^{x}}}{e^{x}}dx$$ 2

$$= \int \frac{1+e^{x}}{e^{2x}}dx$$

$$= \int \frac{1}{e^{2x}}dx + \int \frac{1}{e^{x}}dx$$

$$= -\frac{1}{2}e^{-2x} - e^{-x} + c$$ 3

when $x=0$ $v=4$

$$\frac{1}{2}\cdot 4^2 = -\frac{1}{2}e^{-0} - e^{-0} + c$$

$$c = \frac{1}{2}4^2 + \frac{1}{2} + 1 = \frac{19}{2}$$

$$v^2 = -e^{-2x} - 2e^{-x} + 19$$

$$v^2 = \left(\frac{1+3e^{x}}{e^{x}}\right)^2$$ 4

$$v = \frac{1+3e^{x}}{e^{x}}$$

Examiner comments

1 This was a very common error. The force acting on P is a resistive force and the acceleration is measured in a positive direction, so there needs to be a minus sign in the N2L equation.

2 The method of simplifying the integrand is correct, but a coefficient of 3 has been dropped at this step.

3 A method mark can be awarded because although there are two errors in the integration, these are an arithmetical error and a sign error, so the method required for the integration is as intended in the question.

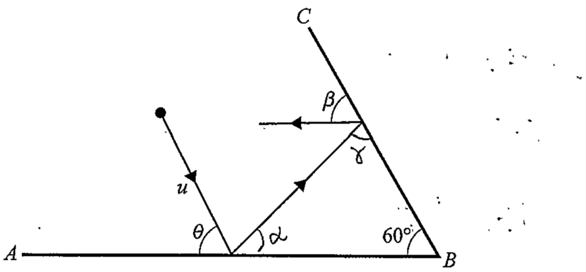
4 This is incorrect. The candidate has simply written down the given answer to the question and abandoned the previous incorrect work.

Mark for (a) = 1 out of 4

Example Candidate Response – 2, continued	Examiner comments
(b) Find an expression for x in terms of t. $v = \frac{dx}{dt} = \frac{1+3e^x}{e^x}$ $\int \frac{e^x}{1+3e^x} \cdot dx = \int dt$ $t = \frac{1}{3}\ln(1+e^x) + C$ 5 when $t=0$ $x=0$ $0 = \frac{1}{3}\ln 2 + C$ $C = -\frac{1}{3}\ln 2$ $t = \frac{1}{3}\ln\left(\frac{(1+e^x)}{2}\right)$ $e^{3t} = \frac{1+e^x}{2}$ $2e^{3t} - 1 = e^x$ $x = \ln(2e^{3t} - 1)$ 6	5 The form of the integrated term is correct, but a factor of 3 has been dropped. 6 This answer would be correct except for the earlier error. This type of error was common. A candidate shows that they know how to set up the mechanics, and how to integrate, but then spoils their answer with arithmetic or algebraic errors. Mark for (b) = 2 out of 4 **Total mark awarded = 3 out of 8**

Question 6

Example Candidate Response – 1



AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan\theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of e. [4]

$u_\perp = \cancel{u\sin\theta}\ u\sin\theta$, $u_{//} = u\cos\theta = v_{//}$

$v_\perp = eu\sin\theta$ 1

$\cancel{u_{//}^2 + v_\perp^2}$ 2

$v^2 = v_{//}^2 + v_\perp^2 = u^2\cos^2\theta + e^2u^2\sin^2\theta = u^2(\cos^2\theta + e^2\sin^2\theta)$

$\frac{1}{2}mu^2 \times \frac{2}{5} = \frac{1}{2}mv^2$

$\frac{1}{5}u^2 = \frac{\cos^2\theta + e\sin^2\theta}{2}u^2$ 3 $\quad e = \frac{\frac{2}{5} - \cos^2\theta}{\sin^2\theta} = \frac{\frac{2}{5} - \frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$ 4

$\cancel{5\cos^2\theta + 5e}$

$\cancel{5e\sin^2\theta = 2 - 5\cos^2\theta}$

$\cancel{e = \frac{2 - 5\cos^2\theta}{5\sin^2\theta}}$

$\cancel{e\sin^2\theta = \frac{2}{5} - \cos^2\theta}$

$\cancel{e = \frac{2}{5\left(1 - \frac{1}{\tan^2\theta + 1}\right)} - \frac{1}{\tan^2\theta} = 2 - \frac{1}{4} =}$

(b) Find the size of angle β.

$\cancel{v = u\sqrt{\frac{1}{5} + \frac{1}{4}\cdot\frac{4}{5}} = u\sqrt{\frac{2}{5}}}$

$\cancel{v = u\sqrt{\frac{1}{5} + \frac{1}{16}\cdot\frac{4}{5}} = \frac{\sqrt{5}}{2\sqrt{5}}}$

$\tan\alpha = \frac{v_\perp}{v_{//}} = \frac{\frac{1}{4}u\sin\theta}{u\cos\theta} = \frac{1}{4}\tan\theta = \frac{1}{2}$; ~~sin~~ 5

$\cancel{\alpha}$

$\tan\gamma = \tan(180° - 60° - \alpha) = \frac{\tan(120°) - \tan\alpha}{\cancel{1 + \tan 120° \tan\alpha}} = -\frac{\sqrt{3} + \frac{1}{2}}{1 - \frac{\sqrt{3}}{2}} = -\frac{2\sqrt{3}+1}{2-\sqrt{3}}$

$\tan\beta = \frac{v_\perp'}{v_{//}'} = \frac{\frac{1}{4}v\sin\gamma}{v\cos\gamma} = \frac{1}{4}\tan\gamma = -\frac{2\sqrt{3}+1}{8-4\sqrt{3}}$

6

Examiner comments

1 The candidate uses conservation of linear momentum and Newton's law of restitution to find the components of the velocity after the impact. This is always a good starting point, and candidates who did not do this rarely made any progress in the whole question.

2 The candidate finds the components of the velocity and squares and adds them to give the square of the velocity.

3 A transcription error has occurred here, with the power of the *e* omitted.

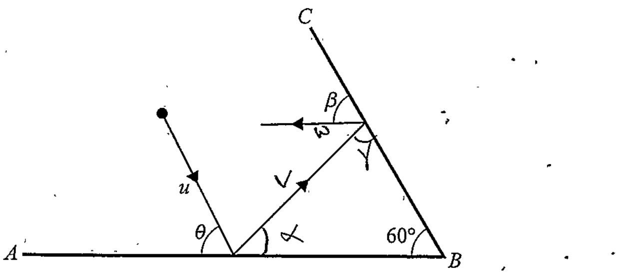
4 This incorrect answer was seen quite often following inaccurate work.
Mark for (a) = 3 out of 4

5 The method here is correct, but the answer is incorrect because of the error in part (a).

6 Again, the method here is correct.
Mark for (b) = 2 out of 2

Total mark awarded = 5 out of 8

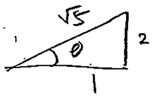
Example Candidate Response – 2



AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan\theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of e. [4]



$\tan\theta = 2$ $\beta = 60°$ $\dfrac{v\sin\alpha}{u\sin\theta} = e.$

$u\cos\theta = v\cos\alpha$ **1** $v\sin\alpha = eu\sin\theta$ $\sin\theta = \dfrac{2}{\sqrt5}$

$\cancel{u}\cos\theta = \dfrac{e\cancel{u}\sin\theta\cos\alpha}{\sin\alpha}$ $v = \dfrac{eu\sin\theta}{\sin\alpha}$

$\dfrac{1u}{\tan\theta} = \dfrac{e}{\tan\alpha}$ $E_{KF} = 0.4\,E_{KI}$

$E_{KF} = \frac{2}{5}\left(\frac{1}{2}\right).m\,(u\sin\theta)^2$

$\cancel{\tan\theta =}$ $\frac{1}{2}\cancel{m}v^2\sin^2\alpha = \frac{1}{5}\cancel{m}u^2\sin^2\theta$ **2**

3 $\tan\alpha = e\tan\theta$ $5v^2\sin^2\alpha = 2u^2\left(\frac{4}{5}\right)$

$\tan\alpha = 2e$ $25v^2\sin^2\alpha = 8u^2$

$25(eu\sin\theta)^2 = 8u^2$

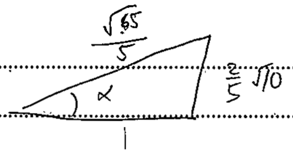
$25e^2\cancel{u}^2\left(\frac{4}{5}\right) = 8\cancel{u}^2$

$20e^2 = 8$

$e^2 = \frac{8}{20}$

$e = \dfrac{\sqrt{10}}{5}$

(b) Find the size of angle β.



$\tan\alpha = 2e$ **4**

$\tan\alpha = \frac{2}{5}\sqrt{10}$

$\alpha = 51.67$

$\gamma = 180 - 60 - 51.67$

$= 68.33$

$v\cos\gamma = w\cos\beta$ $w\sin\beta = ev\sin\gamma$

$v\cos\gamma = \dfrac{\sqrt{10}}{5}v\,\dfrac{\sin\gamma\cos\beta}{\sin\beta}$ $w\sin\beta = \dfrac{\sqrt{10}}{5}v\sin\gamma$

$\dfrac{\cancel{v}1}{\cancel{\cot\gamma}\ \tan\gamma} = \dfrac{\frac{\sqrt{10}}{5}}{\cancel{\cot}\tan\beta}$ $w = \dfrac{\sqrt{10}}{5}v\,\dfrac{\sin\gamma}{\sin\beta}$

$\tan\beta = \dfrac{\sqrt{10}}{5}\tan\gamma$ **5**

$\cancel{\tan}\beta = 57.9°$

Examiner comments

1 The candidate uses conservation of linear momentum and Newton's law of restitution to find the components of the velocity after the impact.

2 This is an incorrect application of the given information about the kinetic energy. The candidate uses only one component of the velocity, before and after the impact, instead of the velocity which must include both components. Another common error at this point was for the 40% to be on the wrong side of the equation. Either of these errors leads to the loss of most of the marks in this part of the question.

3 This result is not required in this part, but it is worth noting that it will prove useful in part (b).
Mark for (a) = 1 out of 4

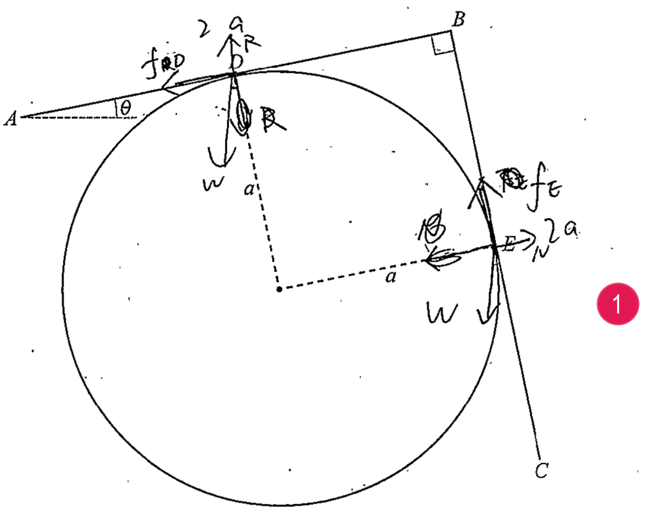
4 This is the expression referred to in the comment in part (a), with the value for tan θ substituted. Now that *e* has been found, the value of the angle alpha can be deduced. This is inaccurate because of the error in part (a).

5 It is worth noting that this is the result from part (a) again, and with the angle γ now found, the final answer can be deduced. This candidate's answer is incorrect following their earlier error, but the method marks can be awarded.
Mark for (b) = 2 out of 4

Total mark awarded = 3 out of 8

Question 7

Example Candidate Response – 1



A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. [6]

$W\sin\theta\, a + f_D a + f_E a = W\cos\theta\, a$ 2

$W\sin\theta\, a + \mu R a + \mu N a = W\cos\theta$

$W\cos\theta + W\cos\theta = \mu N + R$ 3

$2W\cos\theta = \mu N + R$

$\mu R + 2W\sin\theta = N$ 4

$W\sin\theta + \mu R = W\cos\theta - \mu N$

$W\sin\theta + \mu R = W\cos\theta - \mu^2 R + \mu 2W\sin\theta$

$R = 2W\cos\theta - \mu N$ 5

$$\frac{R}{N} = \frac{2W\cos - \mu N}{\mu R + 2W\sin\theta}$$

Examiner comments

1 A good start, with the normal reactions, friction forces and weights annotated on the diagram.

2 It is advisable for candidates to give some indication of where their equations are coming from, rather than leave the examiner to interpret. This is a correct equation from taking moments about the centre.

3 This is a correct equation from resolution of all the forces perpendicular to AB.

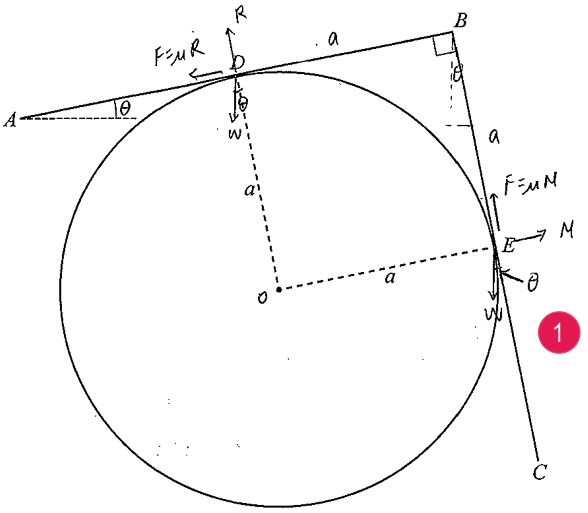
4 This is a correct equation from resolution of all the forces parallel to AB.

5 The candidate now has three equations (two resolutions and one moments) and these are sufficient to obtain the required result. The question asks for the ratio $R : N$ in terms of the coefficient of friction and this should prompt the candidate to eliminate the other quantities and to isolate the terms in R and N. It was not unusual to see candidates display a good understanding of the topic by writing down correct resolution and/or moments equations, but then not have a strategy for combining them to achieve the goal set in the question.

Mark for (a) = 3 out of 6

Example Candidate Response – 1, continued	Examiner comments
$\frac{R}{N}$ **(b)** Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. $W\cos\theta + W\cos\theta = \mu N + R$ $2W\cos\theta = \frac{1}{3}N + R$ $\mu R + 2W\sin\theta = N$ $\frac{1}{3}R + 2W\sin\theta = N$ $\frac{2W\sin\theta}{2W\cos\theta} = \frac{N - \frac{1}{3}R}{\frac{1}{3}N + R}$ $\tan\theta = \frac{N - \frac{1}{3}R}{\frac{1}{3}N + R}$ 6	6 This step is correct, but the candidate makes no further progress because they have not obtained the ratio R : N in part (a). With the given value of the coefficient of friction, this would have enabled completion of this part of the question. The second part of a question often builds on work done in an earlier part. Mark for (b) = 1 out of 3 **Total mark awarded = 4 out of 9**

Example Candidate Response – 2



1

A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. [6]

At point D, $R(\uparrow)$: $R = W\cos\theta$

~~R(→) :~~

2

At point E, ~~R(↑) : μN = W cos θ~~

$R(\rightarrow)$: $N = W\sin\theta$

take moments about centre of cylinder, O :

$\circlearrowleft$ Q : $(W\cos\theta)(a) = (\mu R)(a) + \mu(N)(a) + (W\sin\theta)(a)$ 3

$W\cos\theta = \mu R + \mu N + W\sin\theta$

$R = \mu R + \mu N + N$

$R(1-\mu) = N(1+\mu)$

Examiner comments

1 The candidate has added the normal reactions, the friction forces and the weights at points *D* and *E*. This is an essential first step in approaching any question on equilibrium of a rigid body system. Candidates who did not do this rarely made any real progress in this question.

2 The candidate has made a significant error here. It is not legitimate to resolve perpendicular to each rod, individually. These results are not correct: they are based on an erroneous misunderstanding that parts of the system can be considered in isolation. This is not so. All resolutions of forces must include all the forces acting on the whole rigid body. This invalid approach was adopted by the majority of candidates. These attempted resolution equations should involve also the forces at *E*.

3 This is a correct moments equation. The most efficient solution would have been to take moments also about *B*. These two moments equations are sufficient to solve the problem.

Example Candidate Response – 2, continued	Examiner comments
$\frac{R}{N} = \frac{1+\mu}{1-\mu}$ $R : N = 1+\mu : 1-\mu$ ratio of $R:N$ is $1+\mu : 1-\mu$ 4	4 Although this answer is correct, it comes from incorrect work. Mark for (a) = 2 out of 6
(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. $R(1-\frac{1}{3}) = N(1+\frac{1}{3})$ $\frac{2}{3}R = \frac{4}{3}N$ $R = 2N$ $\frac{N}{R} = \frac{w\sin\theta}{w\cos\theta}$ 5 $\frac{N}{R} = \tan\theta$ $\tan\theta = \frac{N}{(2N)}$ $\tan\theta = \frac{1}{2}$	5 Incorrect equations are used here. Mark for (b) = 0 out of 3 **Total mark awarded = 2 out of 9**

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