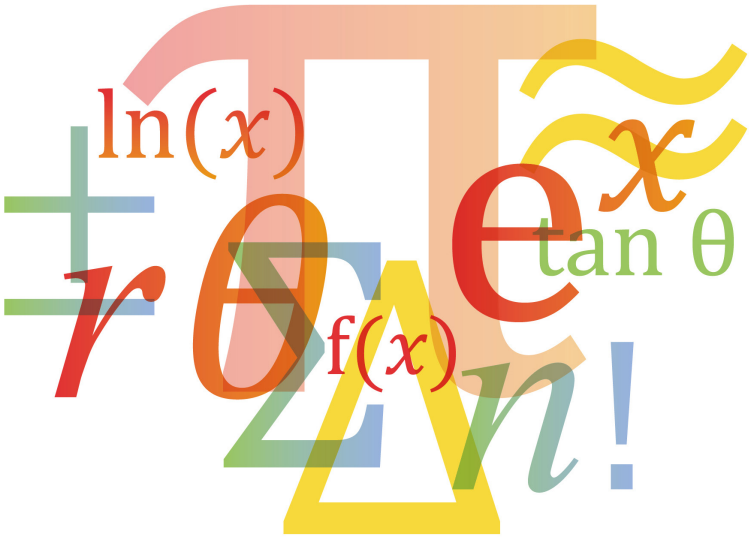
Cambridge Assessment International Education

Specimen Paper Answers – Paper 5

Cambridge International AS & A Level Mathematics 9709

For examination from 2020



Version 2

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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709, and to show examples of model answers to the 2020 Specimen Paper 5. Paper 5 assesses the syllabus content for Probability & Statistics 1. We have provided answers for each question in the specimen paper, along with examiner comments explaining where and why marks were awarded. Candidates need to demonstrate the appropriate techniques, as well as applying their knowledge when solving problems.

Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component. The main focus of examination questions will be the AS & A Level Mathematics subject content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of the syllabus.

There are six to eight structured questions in Paper 5; candidates must answer **all** questions. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Candidates are expected to answer directly on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

The mark schemes for the Specimen Papers are available to download from the School Support Hub at www.cambridgeinternational.org/support

2020 Specimen Mark Scheme 5

Past exam resources and other teacher support materials are available on the School Support Hub (www.cambridgeinternational.org/support).

Assessment overview

There are three routes for Cambridge International AS & A Level Mathematics. Candidates may combine components as shown below.

Route 1 **AS Level only** (Candidates take the AS components in the same series)	**Paper 1** Pure Mathematics 1	**Paper 2** Pure Mathematics 2	**Paper 3** Pure Mathematics 3	**Paper 4** Mechanics	**Paper 5** Probability & Statistics 1	**Paper 6** Probability & Statistics 2
Either	✓		Not available for AS Level		✓	Not available for AS Level
Or	✓			✓		
Or Note this option in Route 1 cannot count towards A Level	✓	✓				

Route 2 **A Level (staged over two years)**	**Paper 1** Pure Mathematics 1	**Paper 2** Pure Mathematics 2	**Paper 3** Pure Mathematics 3	**Paper 4** Mechanics	**Paper 5** Probability & Statistics 1	**Paper 6** Probability & Statistics 2
Either Year 1 AS Level	✓	Not available for A Level		✓		
Year 2 Complete the A Level			✓		✓	
Or Year 1 AS Level	✓				✓	
Year 2 Complete the A Level			✓			✓
Or Year 1 AS Level	✓				✓	
Year 2 Complete the A Level			✓	✓		

Route 3 **A Level** (Candidates take the A Level components in the same series)	**Paper 1** Pure Mathematics 1	**Paper 2** Pure Mathematics 2	**Paper 3** Pure Mathematics 3	**Paper 4** Mechanics	**Paper 5** Probability & Statistics 1	**Paper 6** Probability & Statistics 2
Either	✓	Not available for A Level	✓	✓	✓	
Or	✓		✓		✓	✓

Paper 5 – Probability & Statistics 1

- Written examination, 1 hour 15 minutes, 50 marks
- 6 to 8 structured questions based on the Probability & Statistics subject content
- Candidates answer all questions
- Externally assessed by Cambridge International
- 40% of the AS Level
- 20% of the A Level

Compulsory for A Level.

Assessment objectives

The assessment objectives (AOs) are the same for all papers:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weightings for assessment objectives

The approximate weightings (± 5%) allocated to each of the AOs are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

Question 1

1 The following back-to-back stem-and-leaf diagram shows the annual salaries of a group of 39 females and 39 males.

Females			Males	
(4)	5 2 0 0	20	3	(1)
(9)	9 8 8 7 6 4 | 0 0 0	21	0 0 7	(3)
(8)	8 7 5 3 3 1 0 0	22	0 0 4 5 6 6	(6)
(6)	6 4 2 1 0 0	23	0 0 2 3 3 5 6 7 7	(9)
(6)	7 5 4 0 0 0	24	0 1 1 2 5 5 6 8 8 9	(10)
(4)	9 5 0 0	25	3 4 5 7 7 8 9	(7)
(2)	5 0	26	0 4 6	(3)

Key: 2 \| 20 \| 3 means \$20 200 for females and \$20 300 for males.

(a) Find the median and the quartiles of the females' salaries. [2]

Median value at term $\frac{39+1}{2} = 20$

Median = \$22 700 1

Lower Quartile value at term $\frac{39+1}{4} = 10$

Lower Quartile = \$21 700 2

Upper Quartile value at term $\frac{39+1}{4} \times 3 = 30$

Upper Quartile = \$24 000 2

Examiner comment

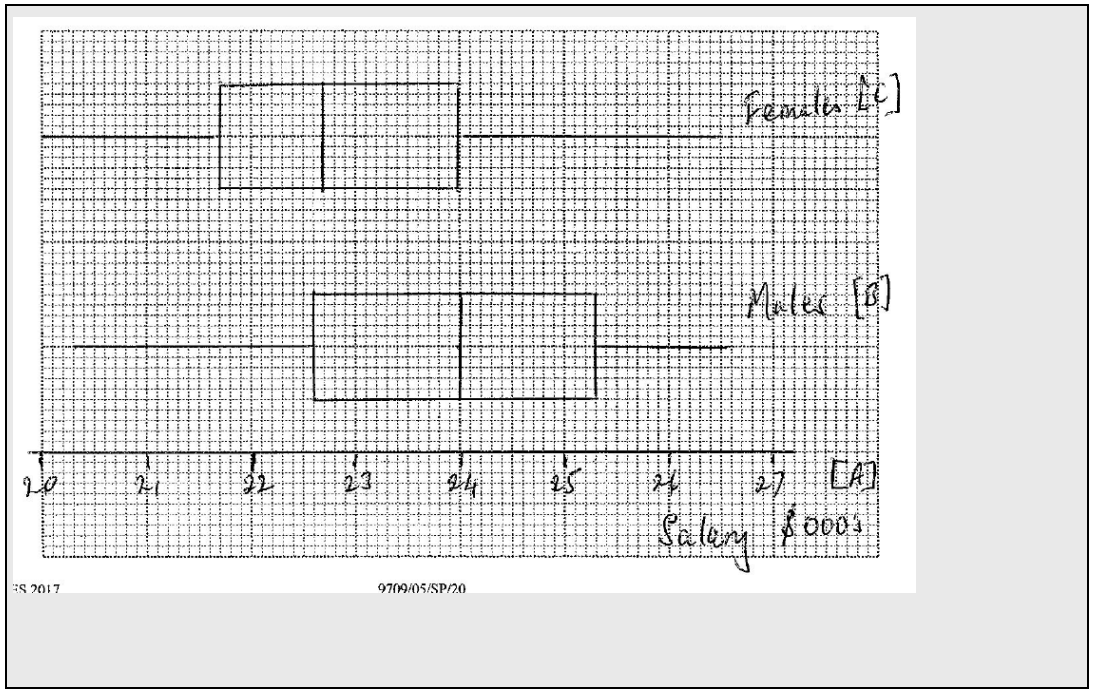
Candidates need to identify the required term and use the standard approach. This means that the 'leaf term numbers' can be used to reduce the counting involved to reach the 20th term, instead of the prior knowledge approach of 'counting in'. Stating the median 1 is awarded the first B1 mark. Both the quartiles must be correctly stated 2 for the second B1 to be awarded.

As the question paper identifies that 2 marks are being awarded for stating 3 values, there is some indication that the method is going to be implied by the answers. It is good practice to identify which value is the upper or lower quartile.

Common errors for this type of question include not interpreting the key correctly, so \$227 may be seen. Candidates also find identifying the correct value on the left section of a back-to-back stem-and-leaf diagram problematic because of not using the value ordering correctly, so \$22 000 may also be seen.

You are given that the median salary of the males is \$24 000, the lower quartile is \$22 600 and the upper quartile is \$25 300.

(b) Draw a pair of box-and-whisker plots in a single diagram on the grid below to represent the data. [3]



3

2

1

Examiner comment

The stem-and leaf diagram has values between 20 000 and 27 000, so the scale must cover those values. Using 2 cm for \$1000 helps to be able to plot accurately. 1 The scale is linear, numbered throughout the range and labelled with both the item 'Salary' and units; this gains the first B1 mark.

2 Using the median and quartiles that are given in the question, the male box and whisker plot is drawn, with the maximum and minimum values taken from the stem-and leaf diagram; this gains the second B1 mark. 3 The candidate should then use their own values for the female median and quartiles, with the maximum and minimum values taken from the stem-and-leaf diagram to plot a second box-and-whisker plot on the same diagram. No additional scale is drawn. Each plot is labelled correctly.

Many candidates penalise themselves by using a scale that makes it difficult to plot accurately, e.g. 4 cm = \$30 000. At this level, accurate graphs are required to gain maximum credit. The omission of units on the axis or the genders on the box-and whisker plot means that the labelling is incomplete so may be penalised. A less frequent, but careless error is extending the 'whiskers' into the 'box', which should be avoidable. The reading of values from the stem-and-leaf diagram can be difficult for the 'Females' side, with a minimum value of \$20 500.

Question 2

2 A summary of the speeds, x kilometres per hour, of 22 cars passing a certain point gave the following information:

$$\Sigma(x - 50) = 81.4 \text{ and } \Sigma(x - 50)^2 = 671.0.$$

Find the variance of the speeds and hence find the value of Σx^2. [4]

Coded Mean $= \frac{\Sigma(x-50)}{22} = \frac{81.4}{22} = 3.7$

Variance from Coded data $= \frac{\Sigma(x-50)^2}{22} - (\text{coded mean})^2$

$= \frac{671}{22} - 3.7^2$ (1) $= 16.81$ (2)

Uncoded mean $= 3.7 + 50 = 53.7$

Variance from uncoded data $\frac{\Sigma x^2}{22} - (\text{uncoded mean})^2 = \frac{\Sigma x^2}{22} - 53.7^2 = 16.81$ (3)

$\Rightarrow \frac{\Sigma x^2}{22} = 16.81 + 53.7^2$

$\Rightarrow \Sigma x^2 = (16.81 + 53.7^2) \times 22 = 63811$ (4)

Examiner comment

The solution requires candidates to recognise that the variance remains constant whether calculated with coded or uncoded data. Candidates find using coded data more challenging, so often use the uncoded data first, which in this example has the required unknown. The attempt to use the variance formula with a value for the coded mean will gain the first method mark. (1) It is often helpful to calculate the coded mean separately and use the value in the variance formula. Good practice is to state the formula being used, as this often helps substitute the values accurately. The accuracy mark is awarded for the correct variance value. (2) As this is not a final answer, it should be stated to 4 significant figures to minimise the possibilities of premature approximation errors.

To gain the second method mark, there needs to be an attempt with values using uncoded data and equated to the calculated variance. (3) Again, calculating the uncoded mean separately and then substituting into a stated formula helps to reduce substitution errors.

There is a need to rearrange the equation to make Σx^2 the subject, using prior knowledge algebra skills. The final accuracy mark is awarded for the final answer. (4) As this is an exact answer, it would be anticipated that this would not be rounded.

Question 3

3 A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable X represents the number of paperback books she chooses.

(a) Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

Number of ways choosing 2 paperbacks = ${}^4C_2$

$P(2 \text{ paperbacks}) = \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} \times {}^4C_2$ 1

$= \frac{3}{14}$ 2

Examiner comment

This common approach develops the prior knowledge of probability without replacement. A method mark is awarded for making a statement that multiplies the number of ways that 2 paperbacks can be chosen by 4 probabilities. 1 A common error is not considering that there is more than one way of choosing the 2 paperbacks. A less common error is assuming that the books are replaced, so that the denominator is always 8. The final answer is awarded the accuracy mark. 2 There is no expectation to convert fractions into decimals, and an unsimplified fraction is often stated as well. The decimal answer of 0.214 would gain credit, but is less accurate.

An alternative approach would be to consider

$$\frac{\text{number of ways to choose 2 paperbacks}}{\text{number of ways of chosing 4 books}} = \frac{{}^6C_2}{{}^8C_4}$$

(b) Draw up the probability distribution table for X. [3]

$P(4 \text{ paperbacks}) = \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} = \frac{3}{14}$ 1

$P(3 \text{ paperbacks}) = 1 - \frac{3}{14} - \frac{3}{14} = \frac{8}{14}$

X	2	3	4
Probability (X)	$\frac{3}{14}$	$\frac{8}{14}$	$\frac{3}{14}$

2 3

Examiner comment

The question instructs the candidate to draw a probability distribution table, and one value has already been shown in part **(a)**. By choosing to calculate P(4 paperbacks), there is no need to consider any hardbacks, simplifying the probability calculation. The correct answer 1 would gain the second B1 mark available. There needs to be consideration of what outcomes are possible, and as there are only 2 hardbacks sent by the book club, the minimum number of paperbacks chosen is 2, and the maximum is 4, so the only other outcome is 3. Using prior knowledge that the sum of probabilities for all outcomes is 1, enables the final probability to be calculated easily. At this stage, the probability distribution table is drawn, with only possible outcomes in the top line 2 which will gain the first B1 mark. With the table completed to gain the final B1 mark. 3

Common errors are to draw the probability distribution table first without considering the possible outcomes, and include the values 0 and 1 for X. This will be penalised, unless a probability of zero is clearly stated. It is also common for the probabilities not to sum to 1, especially if candidates calculate both outstanding values. As the P(2 paperbacks) is stated in **(a)**, the probability distribution table often requires at least one probability to be included to gain credit.

(c) You are given that E(X) = 3.

Find Var(X). [2]

$\text{Var}X = \Sigma X^2 p - \left(E(X)\right)^2 = \frac{2^2 \times 3 + 3^2 \times 8 + 4^2 \times 3}{14} - 3^2$ 1

$= \frac{3}{7}$ 2

Examiner comment

1 Here, using the variance formula from raw data, the method mark is awarded for interpreting their probability distribution table in **(b)**; there is no requirement to calculate the mean as E(X) is stated.

2 When evaluated, the correct answer will gain the accuracy mark.

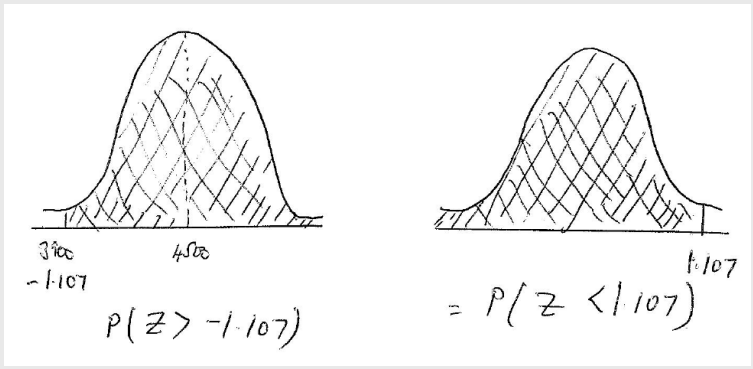
Common errors are to square the probability instead of the variable, and to either subtract E(X) or omit the term completely when calculating the variance.
It is good practice to state the basic formula, which may help to minimise these errors.

Question 4

4 A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(a) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

$P(X > 3900) = P\left(Z > \frac{3900 - 4520}{560}\right)$ 1



$= P(Z > -1.107) = P(Z < 1.107) = \Phi(1.107)$ 2

$= 0.8657$ 3

Number of days = 0.8657 x 365 = 315.98 so 316 days 4

Examiner comment

The question states that the daily sales are normally distributed, this provides clear direction of the approach the candidate should take. The substitution into the standardising formula for normal distributions gains the first method mark. 1 The use of a simple sketch helps to identify the required area for the probability, and the attempt to find it gains the second method mark. 2 As this is not a final answer, it is good practice to state it to at least 4 significant figures (SF) to ensure later accuracy; this value will gain the first accuracy mark. 3

To calculate the number of days that meet the criteria, this probability is multiplied by the standard 365 days in a year. 4 The answer must be stated as a whole number as this is a real-life context; this gains the final independent mark.

Common errors are the use of a continuity correction in the standardisation formula, which is not required as this is a continuous variable, and dividing by the variance. Many candidates state the incorrect probability, which can be avoided by the use of a simple sketch. Candidates are expected to work with at least 4 significant figures from calculated values, so candidates who round their probability to 3SF (or even less accurately) at stage 3 often penalise themselves with an inaccurate final answer. A surprising error that is seen is the use of 360 days in a year.

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(b) Find the value of m. [3]

$P(X > 8000) = 0.122$, from tables $z = 1.165$ 1

So $\frac{8000 - m}{560} = 1.165$ 2

Rearranging $m = 8000 - 1.165 \times 560 = 7347.6 = 7350$ 3

Examiner comment

1 The correct use of tables is a skill expected within the syllabus, so stating the correct value of z gains the first B1 mark. 2 Creating the normal standardisation equation with their z value will gain the method mark.

3 Using anticipated prior knowledge algebra skills to rearrange the equation to calculate the value of m will gain the A1 mark. A more accurate answer will not normally be penalised unless the question requires a specific degree of accuracy or as a requirement of context. However, as the 'rubric' instructions on the examination paper remind candidates, non-exact answers should be given to 3 significant figures, so it is appropriate to state the rounded answer.

(c) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

P(exceeds 8000 on fewer than 2 days) = P(0 or 1 days)

$= (0.878)^6 + {}^6C_1(0.112)(0.878)^5$ 1 2

$= 0.840037$

$= 0.840$ 3

Examiner comment

There is a need to interpret the context as a starting point of the solution, so identifying the number of days which fulfil the criteria. A binomial term (like the second term) 1 in the correct format will gain the second method mark. 2 Having the correct expression stated with values will gain the first method mark. 3 The accuracy mark is awarded with the correct final answer stated.

The most frequent error seen in this type of question is to not interpret the context correctly. Another common error is to include P(2 days) and to omit P(0) days. Care needs to be taken to ensure accurate calculator operation – correct expressions do not always result in correct evaluation suggesting calculator errors are made. The mark scheme indicates that in this question an answer of 0.84 would be acceptable, even though it is only actually 2 significant figures, because of the final zero.

Question 5

5 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown 90 times.

(a) Use an approximation to find the probability that a 3 is obtained fewer than 18 times. [4]

A normal approximation is appropriate to use.

$p = \frac{1}{6}$, $q = \frac{5}{6}$ so mean $= np = 90 \times \frac{1}{6} = 15$ 1

variance $= npq = 15 \times \frac{5}{6} = \frac{75}{6}$ 2

$$P(X < 18) = P\left(Z < \frac{17.5 - 15}{\sqrt{\frac{75}{6}}}\right)$$ 3

$$= P(Z < 0.7071) = 0.760$$ 4

Examiner comment

Candidates need to decide which approximation to use – it is helpful to note this in the answer as it means that when reviewing the answer later it can be checked for consistency. Having decided upon the normal approximation, the mean and variance need to be calculated, and their values are awarded B1 marks at 1 and 2. As the data is discrete, a continuity correction must be used, and the method mark is awarded when the standardisation formula is stated with the appropriate values substituted 3. 4 Having evaluated the expression, Normal tables are used to find the probability for the final accuracy mark.

The most common errors seen in this type of question are not using, or adding, a continuity correction and using the variance in the denominator of the standardisation formula.

(b) Justify your use of the approximation in part **(a)**. [1]

np = 15 and nq = 75. As both np > 5 and nq > 5, the normal approximation is appropriate. 1

On another occasion, the same die is thrown repeatedly until a 3 is obtained.

(c) Find the probability that obtaining a 3 requires fewer than 7 throws. [2]

Geometric approximation appropriate.

$P(\text{fewer than 7 throws}) = 1 - \left(\frac{5}{6}\right)^6$ 2

$= \frac{31031}{46656}$ 3

Examiner comment

Stating the conditions required for the normal distribution to be suitable as well as calculating the values of this data set, ensures that the understanding of the candidate is recorded. This would be awarded the B1 mark 1. It is good practice to finish with a conclusion that is drawn from their work, although in this question no further credit is given.

A common error in this type of question is to only calculate *np*, with *p* being the lower probability value. This may be sufficient if a clear reason is given as to why *nq* will also fulfil the conditions. A less frequent error is to calculate *np* and *npq*, which does not show that the conditions are met.

Examiner comment

A good solution will state the approach that is being taken; it helps to prompt the candidate to use the correct formulae. Although no additional evidence is shown, the solution indicates that the candidate has correctly interpreted the condition of 'fewer than 7 throws'. By stating the unsimplified expression, the solution clearly identifies the method that is being used, so is awarded the method mark 2. As it is possible to use a calculator to evaluate this statement, no further working is required before the accuracy mark 3 is gained. Many candidates find interpreting the condition 'fewer than' challenging, with a common assumption being that '7 throws' is included in the result. Where the answer is a fraction, there is no requirement to convert to a decimal – inaccurate decimal conversions are seen surprisingly frequently from a correct fraction answer.

Question 6

6 A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(a) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi.

Find the number of different ways in which this can be done. [3]

Number of ways Jon and Sarah with 2 others in taxi $P = {}^6C_2$ 1

Number of ways remaining 4 peopled go into the taxi $Q = {}^4C_4$

Taxis P and Q can be swapped.

Total number of ways = ${}^6C_2 \times {}^4C_4 \times 2$ 2 $= 30$ 3

Examiner comment

To solve combination and permutation questions, it is often sensible to consider each condition stated separately before evaluating finally. The significant condition is that Jon and Sarah are travelling together. By making an initial choice of taxi for this group to travel, ensures that the context can be maintained. Their taxi will contain two additional friends from the six remaining, which leads to the first statement in the solution for a method mark 1. There are now four friends left, so they must be in the second taxi, and the ways they can be selected is stated. Because the taxis can be identified, the taxi that Jon and Sarah are travelling can be swapped, which means that when the total number of ways that the friends can be selected to travel in the two taxis is the product of the values. 2 The unsimplified statement is sufficient of the second method mark. A correct answer will gain the accuracy mark 3 .

Working out the number of ways that groups can be made is an indication that this is a 'combination' question, and the most frequent error is to use 'permutations'. As the seat position of the friends is not important, this is an indication that permutations should not be used. Many solutions often present a single expression, which will gain credit, but does not allow the candidate to confirm their approach when reviewing their answer.

Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi *P* and Jon and Sarah sit in the back of taxi *P* next to each other.

(b) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

Taxi P – Mark in front.

One additional passenger required – ${}^5C_1$ possibilities. 1

As Jon and Sarah sit together – there are 2 possible places additional passenger can sit (left or right seats at the rear of the taxi).

Jon and Sarah can swap positions – 2 possible ways.

No of ways passengers in taxi P can be seated: ${}^5C_1 \times 2 \times 2$ 2 *= 20*

No of ways taxi Q passengers can arranged: ${}^4P_4 = 24$

Total number of seating arrangements = 20 x 24 3 *= 480* 4

Examiner comment

As taxi *P* has specific conditions, it is often sensible to start by considering the arrangements for these passengers. As the seating positions are important, and the question has a key word prompt 'arrangements', permutations should be used.

1 As there is only one additional passenger needed, the number of ways they can be chosen is stated, which is awarded the first method mark. The reasoning of where this passenger can sit next to Jon and Sarah is stated, as is also the ways that Jon and Sarah can sit next to each other. To work out the total ways taxi *P* passengers can be seated requires the separate values to be multiplied, gaining the second method mark 2. The remaining four friends must be travelling in taxi *Q*, there are no seating conditions, so this is a permutation calculation. To work out the total different seating arrangements, requires the seating arrangements of taxi *P* and taxi *Q* to be multiplied for the third method mark 3, with the correct final answer being awarded the accuracy mark 4.

Many candidates find permutation problems more accessible. The common errors occur because the conditions are not applied systematically, for example, that Jon and Sarah can sit next to each other in two ways, so that no matter where the additional friend sits, Jon and Sarah will be able to swap. An alternative approach that can be used where there are several limiting conditions applied, is to simply list out possible scenarios for taxi *P* and providing this is systematic and accurate, it would result in a correct final total.

Question 7

7 Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

- Event X is 'exactly two of the selected balls have the same number'.
- Event Y is 'the ball selected from bag A has number 4'.

(a) Find P(X). [5]

Ways Event X can occur

Bag A	Bag B	Bag C		
2	Not 2	2	$= \frac{1}{4} \times 1 \times \frac{1}{7} = \frac{1}{28}$	1
8	8	Not 8	$= \frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} = \frac{6}{140}$	2
8	Not 8	8	$= \frac{1}{4} \times \frac{3}{5} \times \frac{4}{7} = \frac{12}{140}$	
Not 8	8	8	$= \frac{3}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{24}{140}$	3
			P(X) = Total	4
			$= \frac{47}{140}$	5

Examiner comment

In many probability questions, success requires a systematic approach to the stated 'success' conditions. This solution has initially listed the possible success scenarios systematically. This reduces the potential of omitting any required probabilities. An alternative approach could be to use a tree diagram. By setting the work out in a tabulated format, the appropriate probabilities can be linked within the scenarios, avoiding careless errors. As the number '2' is only in Bags A and C, it is the easier scenario to consider first.

The marks can be awarded in any order, so by evaluating the first scenario, the solution has one correct option and will gain the B1 mark 1 2 By having unsimplified expressions for two scenarios, the first method mark is awarded. 3 As a systematic approach to the listing is being used, the second method mark will be awarded at the final line of the table, as all three scenarios have been identified. 4 As Event X is fulfilled by any of the scenarios listed, the required probability is the sum of the separate values, which as there are more than three scenarios listed means that the third method mark is awarded. Although no explicit sum is stated, this is implied by writing 'Total' below the table. 5 Having the correct final answer gains the final accuracy mark.

The most frequent error is the omission of scenarios when considering the success criteria. Using a systematic approach minimises this. The use of a tree diagram in probability questions often assists in clarifying the required options.

(b) Find P($X \cap Y$) and hence determine whether or not events X and Y are independent. [3]

$P(X \cap Y) = P(4,8,8) = \frac{1}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{8}{140}$ 1

Independent if $P(X \cap Y) = P(X) \times P(Y)$

$P(Y) = \frac{1}{4}$

$P(X) \times P(Y) = \frac{47}{140} \times \frac{1}{4} = \frac{47}{560}$ 2 $\neq \frac{8}{140}$ so not independent. 3

Examiner comment

The question instructs to find P($X \cap Y$), so this is the sensible place to start with the solution. As Event Y is that 'the ball from bag A is a number 4', the only possible scenario is 4, 8, 8. 1 Using the standard probability calculation gains the independent mark.

The condition for independence is then stated, so that the candidate can ensure that all the required values are available. This means P(Y) needs to be calculated. Substituting the values into the independence condition will enable the method mark to be awarded 2 as there is a clear attempt to compare the two probability expressions.

3 To gain the final accuracy mark, the expressions have to be evaluated and a conclusion stated; if no value is shown, it is not clear if the condition is being fulfilled.

Failure to evaluate P(X) × P(Y) is a common cause for loss of credit when candidates are required to determine independence, but a failure to actually state a conclusion is also seen. Candidates must be careful of 'circular' arguments, where they calculate P($X \cap Y$) by doing P(X) × P(Y), and then comparing their identical values. Some candidates confuse the requirements for 'independent' and 'mutually exclusive' events, and use the incorrect condition.

(c) Find the probability that two balls are numbered 2, given that exactly two of the selected balls have the same number. [2]

$P(\text{2 balls numbered 2} \mid \text{2 balls have same number}) = \dfrac{P(2, \text{Not } 2, 2)}{P(X)}$

$= \dfrac{\frac{1}{28}}{\frac{47}{140}}$ (1) $= \dfrac{5}{47}$ (2)

Examiner comment

The solution identifies that this is a conditional probability and interprets the information within the conditional probability formula. This has allowed the candidate to identify that all the required values have already been calculated in **(a)** and so can be substituted into the expression. This will place $\frac{1}{28}$ in the numerator for the method mark (1). The correct evaluation will lead to the accuracy mark (2).

It is common for candidates not to recognise that work within previous answers may be used in later parts, and in many solutions calculate the values again at this stage. The allocation of marks should be an indication to candidates that this is more work than anticipated.

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