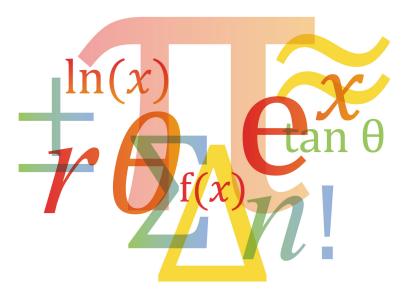


Specimen Paper Answers – Paper 4

Cambridge International AS & A Level Mathematics 9709

For examination from 2020





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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709, and to show examples of model answers to the 2020 Specimen Paper 4. Paper 4 assesses the syllabus content for Mechanics. We have provided answers for each question in the specimen paper, along with examiner comments explaining where and why marks were awarded. Candidates need to demonstrate the appropriate techniques, as well as applying their knowledge when solving problems.

Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component. The main focus of examination questions will be the AS & A Level Mathematics subject content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of the syllabus.

There are 6 to 8 structured questions in Paper 4; candidates must answer **all** questions. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Candidates are expected to answer directly on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

The mark schemes for the Specimen Papers are available to download from the School Support Hub at www.cambridgeinternational.org/support

2020 Specimen Mark Scheme 4

Past exam resources and other teacher support materials are available on the School Support Hub (<u>www.cambridgeinternational.org/support</u>).

Assessment overview

There are three routes for Cambridge International AS & A Level Mathematics. Candidates may combine components as shown below.

Route 1 AS Level only (Candidates take the AS components in the same series)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either	1				1	
Or	4			✓		
Or Note this option in Route 1 cannot count towards A Level	1	√	Not available for AS Level			Not available for AS Level

Route 2 A Level (staged over two years)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either Year 1 AS Level	~			\checkmark		
Year 2 Complete the A Level			✓		1	
Or Year 1 AS Level	~	Not available for			~	
Year 2 Complete the A Level		A Level	✓			✓
Or Year 1 AS Level	~				*	
Year 2 Complete the A Level			✓	\checkmark		

Route 3 A Level (Candidates take the A Level components in the same series)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either	~	Not available	✓	\checkmark	1	
Or	~	for A Level	1		~	✓

Paper 4 – Mechanics

- Written examination, 1 hour 15 minutes, 50 marks
- 6 to 8 structured questions based on the Mechanics subject content
- Candidates answer all questions
- Externally assessed by Cambridge International
- 40% of the AS Level
- 20% of the A Level

Offered as part of AS Level or A Level.

Assessment objectives

The assessment objectives (AOs) are the same for all papers:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weightings for assessment objectives

The approximate weightings (± 5%) allocated to each of the AOs are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

```
1 A particle P is projected vertically upwards with speed 20 m s<sup>-1</sup> from a point on the ground.
```

(a) Find the greatest height above the ground reached by *P*.

```
The acceleration a = -g = -10, the initial velocity is u = 20. Let h m be the greatest height.
```

```
Since the greatest height is required, it can be assumed that the final velocity is v = 0.
```

```
Use v^2 = u^2 + 2as giving 0 = 20^2 + 2 \times (-10) \times h (1)
```

Solving this equation gives h = 20 and so the greatest height above

```
ground is 20 m. 2
```

[2]

Examiner comment

This problem involves the application of the constant acceleration equations.

Use of this equation to find *h* earns M1.

2 Correct evaluation of *h* earns A1.

There are alternative methods for this question, two possible approaches are shown below.

(b) Find the total time from projection until *P* returns to the ground.

Method 1

Using u = 20 and a = -g = -10 gives $0 = 20 + (-10) \times t$ so t = 2 (1)

Doubling this t value gives the total time from projection until P returns to ground as 4 seconds.

Method 2

When the particle returns to the ground the displacement is zero.

Here u = 20, a = -g = -10, s = 0 and we need to find t so use

$$s = ut + \frac{1}{2}at^2$$

This gives $0 = 20t + \frac{1}{2} \times (-10) \times t^2 \to 0 = 20t - 5t^2 = 5t(4 - t)$ (1)

Solutions to this are t = 0 (the initial position) and t = 4 (the required answer).

Hence the total time for the particle to return to the ground is 4 seconds.

Examiner comment

[2]

Time taken to reach the greatest height is found by setting v = 0 in the equation v = u + at.

Correct use of v = u + at to find the time taken to reach the greatest height earns M1.

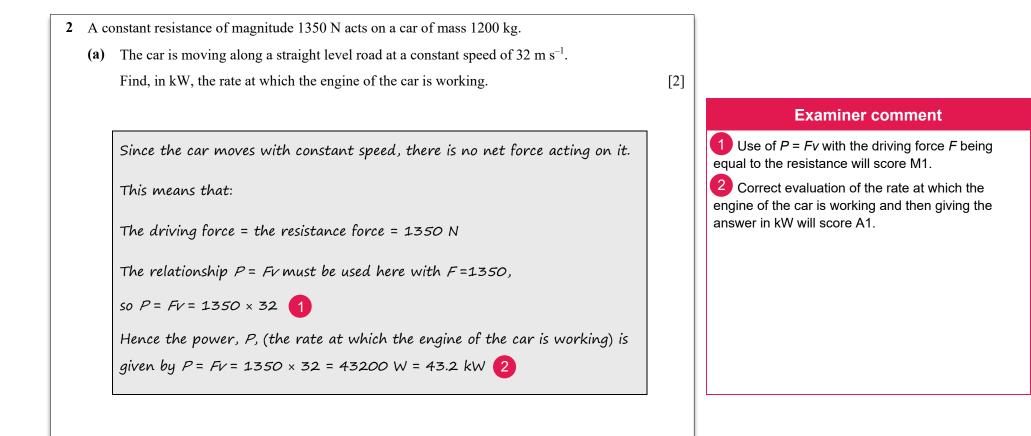
2 Doubling the time to greatest height to find the correct total time scores A1.

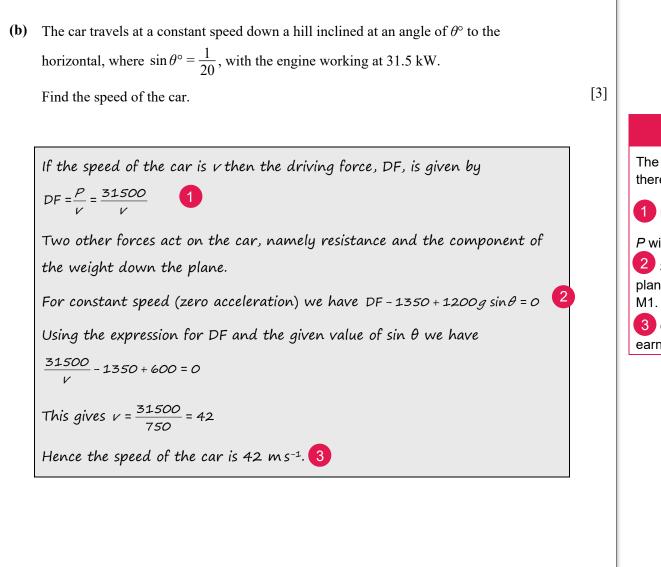
Examiner comment

Again candidates should use one of the constant acceleration equations (often referred to as the *suvat* equations).

1 Use of this equation will earn M1.

Correct evaluation of t = 4 will earn A1.





Examiner comment

The motion is one with constant speed and so again there is no net force on the car.

Using driving force = $\frac{P}{V}$ with the given value of

P will score M1.

2 Stating that the total force acting down the plane = 0, involving the correct three forces earns M1.

Correct solution of the equation giving v = 42 earns A1.

- 3 Three small smooth spheres A, B and C of equal radii and of masses 4 kg, 2 kg and 3 kg respectively, lie in that order in a straight line on a smooth horizontal plane. Initially, B and C are at rest and A is moving towards B with speed 6 m s⁻¹. After the collison with B, sphere A continues to move in the same direction but with speed 2 m s⁻¹.
 - (a) Find the speed of *B* after this collison.

Let the speed of B after the collision be $v \text{ m s}^{-1}$.

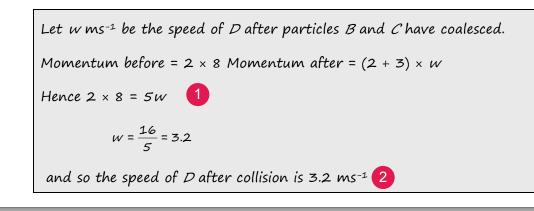
Apply the principal of conservation of momentum to the collision between A and B.

Momentum before = Momentum after $\rightarrow 4 \times 6 = 4 \times 2 + 2\nu$

Solving gives v = 8 and so the speed of B after collision is 8 m s⁻¹. (2)

Sphere *B* collides with *C*. In this collison these two spheres coalesce to form an object *D*.

(b) Find the speed of *D* after this collision.



Examiner comment

Momentum is a new topic for the 2020 syllabus.

1 Application of conservation of momentum to particles *A* and *B* will earn M1.

2 Correct evaluation to find v = 8 will score A1.

[2]

[2]

Examiner comment

Again, candidates need to apply conservation of momentum before and after the collision between B and C.

Correct application of conservation of momentum between particles *B* and *C* earns M1.

Correct evaluation to find the speed of D as 3.2 m s⁻¹ scores A1.

(c) Show that the total loss of kinetic energy in the system due to the two collisions is 38.4 J.

Only sphere A is moving initially and so

the initial kinetic energy = $\frac{1}{2} \times 4 \times 6^2 = 72J$

Finally A is moving with speed 2 m s⁻¹ and D is moving with speed

3.2 m s⁻¹

Hence the final kinetic energy = $\frac{1}{2} \times 4 \times 2^2 + \frac{1}{2} \times 5 \times 3.2^2 = 8 + 25.6 = 33.6 \text{ J}$

Hence the total loss of kinetic energy in the system = 72 - 33.6 = 38.4 J

Examiner comment

When finding the final KE, candidates must remember that A is still moving with speed 2 m s^{-1} .

Using kinetic energy = $\frac{1}{2}mv^2$ applied to any of

the particles will earn M1.

[2]

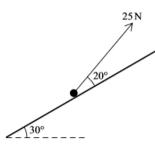
2 Correct evaluation of initial and final kinetic energies and subtracting showing correct loss for A1. The answer is given, so it is vital that candidates show all their working in detail.

As this is a new topic for the 2020 syllabus, candidates will not find any examples such as this one in any of the past papers. The most important element of these problems is the conservation of momentum. A common error seen in problems involving kinetic energy is demonstrated by the problem: 'Let the initial and final velocities of a particle be $u \text{ m s}^{-1}$ and $v \text{ m s}^{-1}$ respectively. Let the particle have mass m kg. What is the loss in kinetic energy of the particle?' The correct loss in kinetic

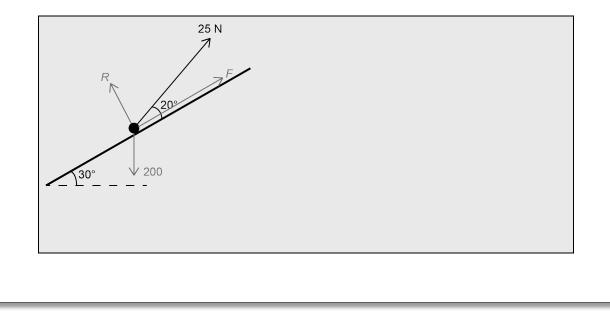
energy of the particle is KE loss = $\frac{1}{2}m(v^2-u^2)$. However, many candidates often wrongly assume that it is given by $\frac{1}{2}m(v-u)^2$. Candidates should check that they use the correct formula for this.

Question 4

- 4 A particle of mass 20 kg is on a rough plane inclined at an angle of 30° to the horizontal. A force of magnitude 25 N, acting at an angle of 20° above a line of greatest slope of the plane, is used to prevent the particle from sliding down the plane. The coefficient of friction between the particle and the plane is μ .
 - (a) Complete the diagram below to show all the forces acting on the particle.



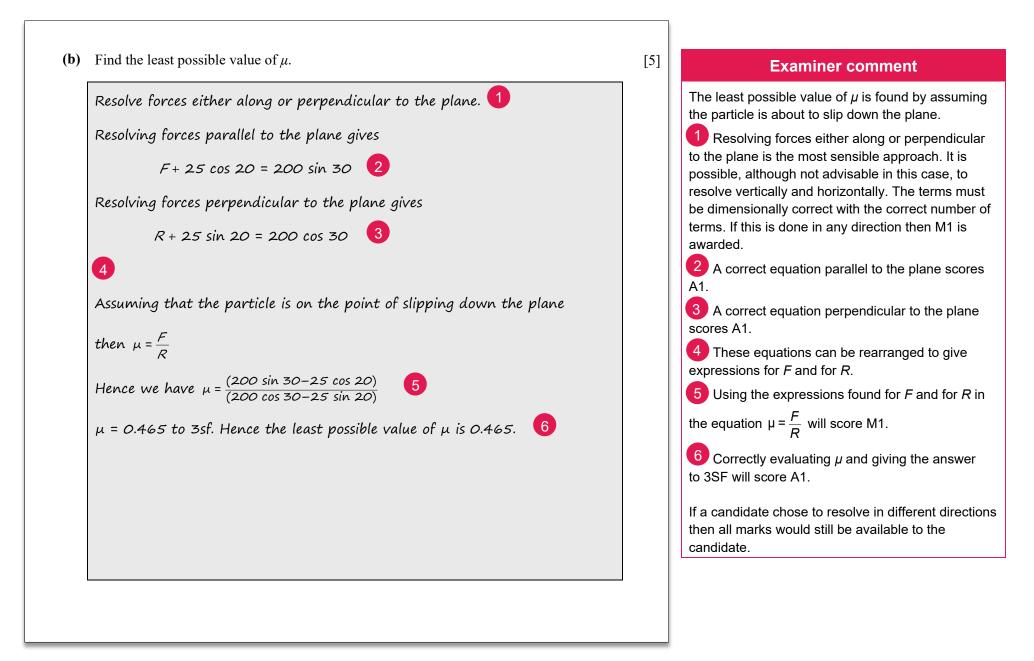
[1]

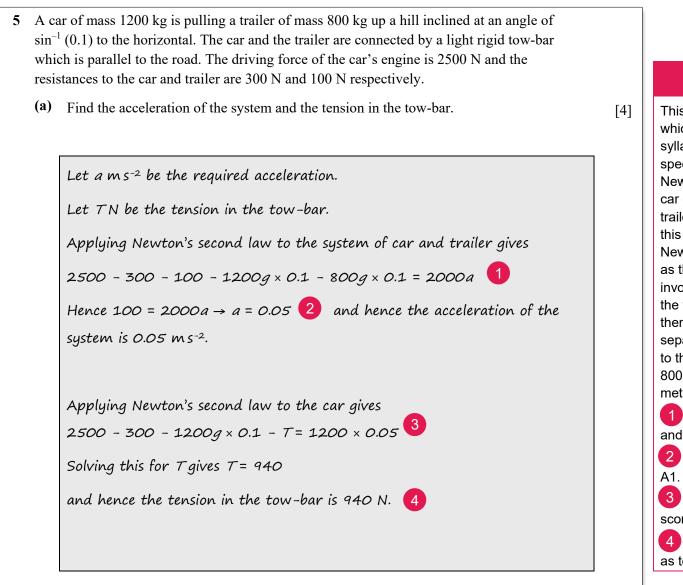


Examiner comment

The following correctly shown on the diagram will be awarded B1:

- The friction force acts up the plane as the particle is about to slip down the plane.
- The weight force is 20g = 200 acting vertically downwards.
- The normal reaction force acts perpendicular to the plane.





Examiner comment

This question is an example of connected particles, which is a topic that is covered in the 2017–2019 syllabus but problems such as the one here are specifically referred to in the 2020 syllabus. Newton's second law can be applied either to the car or to the trailer or to the system of car plus trailer. However, if the acceleration is required, as in this case, then it is generally simpler to apply Newton's second law to the system of car and trailer as this will produce an equation that does not involve the force in the tow-bar. When the tension in the tow-bar is required, then Newton's law must then be applied either to the car or to the trailer separately. If we had applied Newton's second law to the trailer then we have $T - 100 - 800q \times 0.1 =$ 800×0.05 and again this gives T = 940. Either method will score full marks.

If all terms are included, driving force, friction and weight components equated to *ma* then M1.

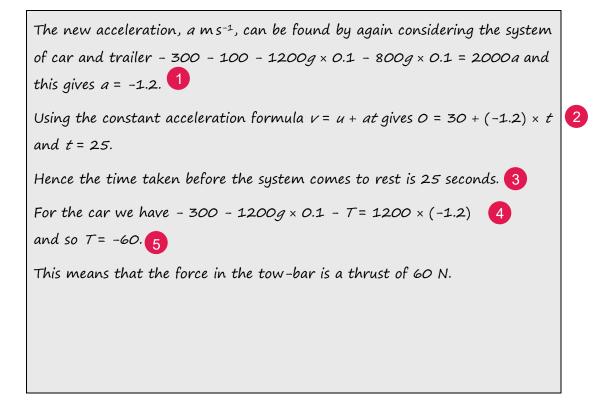
2 Correct evaluation for the acceleration *a* scores

All relevant terms must be included here to score M1.

Correct evaluation of the tension in the tow-bar as tension = 940 N for A1.

(b) When the car and trailer are travelling at a speed of 30 m s⁻¹, the driving force becomes zero.

Find the time, in seconds, before the system comes to rest and the force in the tow-bar during this time.



Examiner comment

[5]

Here the new acceleration must be found and then used in a constant acceleration formula.

A Newton's second law must be applied either to the car or to the trailer in order to find the tension in the tow-bar during this stage of the motion of the car and trailer. If Newton's second law is applied to the trailer rather than the car then we would have: $T = 100 = 8000 \times 0.1 = 800 \times (-1.2)$ and this would

 $T - 100 - 800g \times 0.1 = 800 \times (-1.2)$ and this would give T = -60 as before.

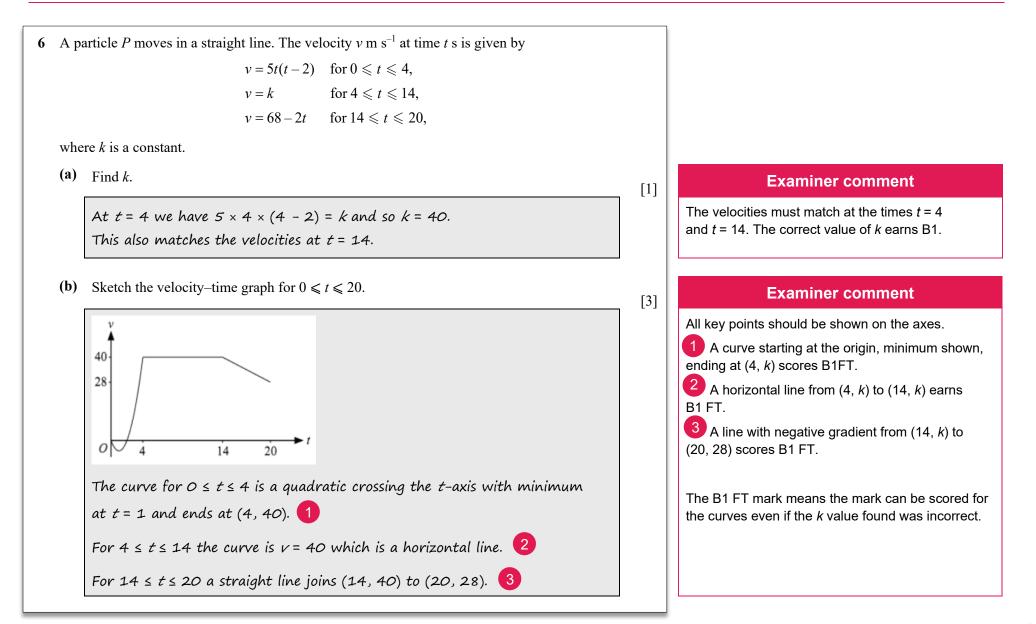
1 Use of Newton's second law with all relevant terms included and *a* found will score M1.

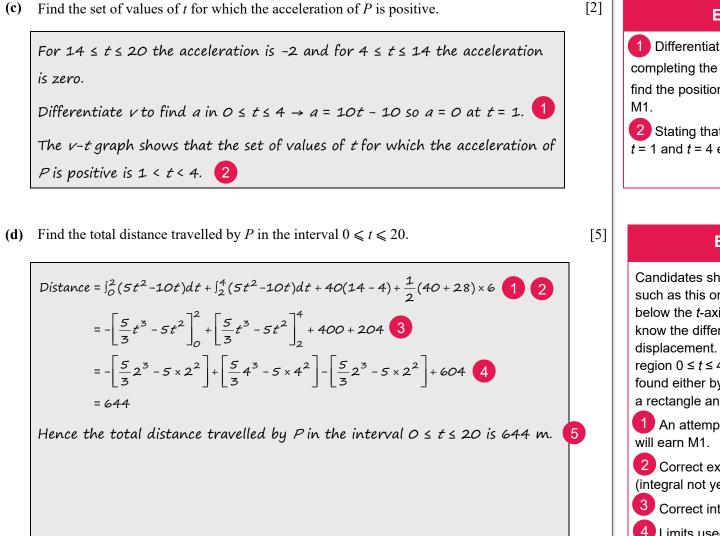
2 Use of v = u + at with u = 30, v = 0 and the candidate's value of a will score M1.

3 Correct evaluation of the time of 25 seconds scores A1.

4 Use of Newton's second law applied either to the car or the trailer with all relevant terms for M1.

5 Correct evaluation of T = -60 for A1. The minus sign is explained as a thrust rather than a tension.





Examiner comment

1 Differentiating *v* or any other method such as completing the square for $v = 5t^2 - 10t$ in order to find the position of the minimum of this curve earns M1.

2 Stating that the acceleration is positive between t = 1 and t = 4 earns A1.

Examiner comment

Candidates should take care with signs in problems such as this one where part of the *v*-*t* graph is below the *t*-axis. Candidates should make sure they know the difference between distance and displacement. Integration must be used for the region $0 \le t \le 4$ but the area for $4 \le t \le 20$ can be found either by integration or by using the areas of a rectangle and a trapezium.

An attempt to integrate v for the region $0 \le t \le 4$ will earn M1.

Correct expression for distance for $0 \le t \le 20$ (integral not yet evaluated) earns A1.

3 Correct integration over $0 \le t \le 4$ will earn B1.

4 Limits used and all correct will earn A1 FT (the FT means this mark is scored even with incorrect 5

Fully evaluated correct final answer for distance = 644 m will earn A1.

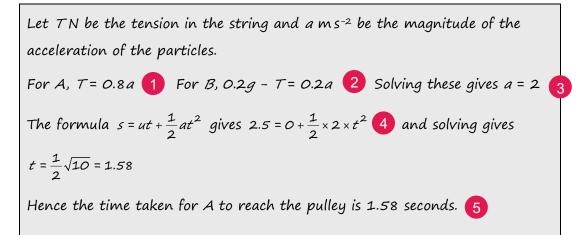
The mark scheme gives three alternative methods for this question, so we have provided a model answer for each.

7 <u>A</u> 2.5 m • B

Two particles A and B, of masses 0.8 kg and 0.2 kg respectively, are connected by a light inextensible string. Particle A is placed on a horizontal surface. The string passes over a small smooth pulley P fixed at the edge of the surface, and B hangs freely. The horizontal section of the string, AP, is of length 2.5 m (see diagram). The particles are released from rest with both sections of the string taut.

(a) Given that the surface is smooth, find the time taken for A to reach the pulley.

Method 1 (Newton's Second Law)



Examiner comment

This problem can be approached either by using Newton's Second law to find the acceleration of the particles and then using the constant acceleration formulae; or the work-energy method could be used. A third option is to consider the effect of the tension on particle *A* only. Any of these methods provides an acceptable answer.

1 Newton's second law applied to A or B earns M1.

2 Newton's second law applied to the other particle earns M1. If the system equation 0.2g = (0.2 + 0.8)a is then applied this would earn both M1 marks.

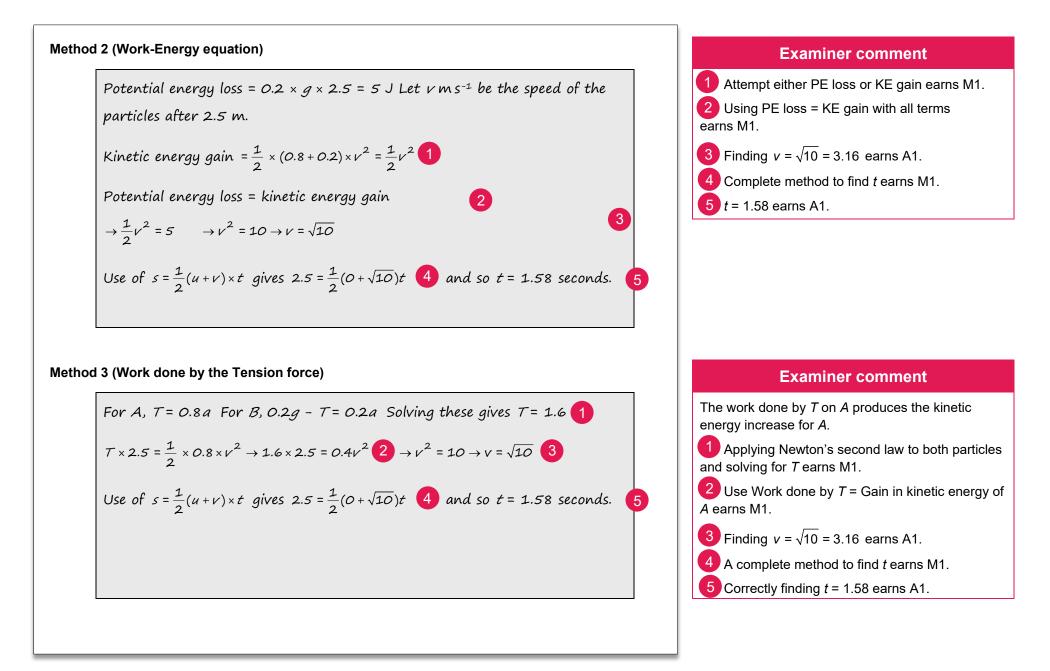
3 Correctly finding that a = 2 earns A1.

4 Complete method to find the required *t* value earns M1.

5 Either *t* = 1.58 seconds or the exact value

 $t = \frac{1}{2}\sqrt{10}$ will earn A1.

[5]



(b) It is given instead that the surface is rough and that the speed of A immediately before it reaches the pulley is $v \text{ m s}^{-1}$. The work done against friction as A moves from rest to the pulley is 2 J.

Use an energy method to find *v*.

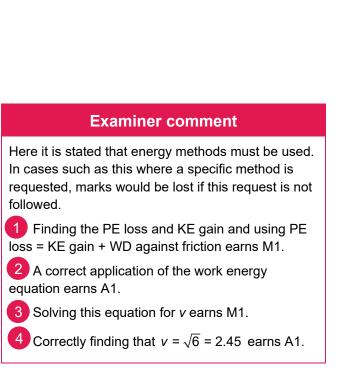
Potential energy loss = $0.2 \times g \times 2.5 = 5 J$

Kinetic energy gain =
$$\frac{1}{2} \times (0.8 + 0.2) \times v^2 = \frac{1}{2} v^2$$

Use the work energy equation in the form

PE loss = KE gain + Work Done against friction 1

$$0.2 \times g \times 2.5 = \frac{1}{2}v^2 + 2$$
 which gives $v^2 = 6$ and so $v = \sqrt{6} = 2.45$ to 3sf. 34



[4]

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