Cambridge Assessment International Education

Specimen Paper Answers – Paper 2

Cambridge International AS & A Level Mathematics 9709

For examination from 2020

Version 2

Cambridge Pathway

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Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709, and to show examples of model answers to the 2020 Specimen Paper 2. Paper 2 assesses the syllabus content for Pure Mathematics 2. We have provided answers for each question in the specimen paper, along with examiner comments explaining where and why marks were awarded. Candidates need to demonstrate the appropriate techniques, as well as applying their knowledge when solving problems.

Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component. The main focus of examination questions will be the AS & A Level Mathematics subject content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of the syllabus.

There are six to eight structured questions in Paper 2; candidates must answer **all** questions. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Candidates are expected to answer directly on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

The mark schemes for the Specimen Papers are available to download from the School Support Hub at www.cambridgeinternational.org/support

2020 Specimen Mark Scheme 2

Past exam resources and other teacher support materials are available on the School Support Hub (www.cambridgeinternational.org/support).

Assessment overview

There are three routes for Cambridge International AS & A Level Mathematics. Candidates may combine components as shown below.

Route 1 **AS Level only** (Candidates take the AS components in the same series)	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either	✓				✓	
Or	✓		Not available for AS Level	✓		Not available for AS Level
Or Note this option in Route 1 cannot count towards A Level	✓	✓				

Route 2 **A Level (staged over two years)**	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either Year 1 AS Level	✓			✓		
Year 2 Complete the A Level			✓		✓	
Or Year 1 AS Level	✓	Not available for A Level			✓	
Year 2 Complete the A Level			✓			✓
Or Year 1 AS Level	✓				✓	
Year 2 Complete the A Level			✓	✓		

Route 3 **A Level** (Candidates take the A Level components in the same series)	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either	✓	Not available for A Level	✓	✓	✓	
Or	✓		✓		✓	✓

Paper 2 – Pure Mathematics 2

- Written examination, 1 hour 15 minutes, 50 marks
- 6 to 8 structured questions based on the Pure Mathematics 2 subject content
- Candidates answer all questions
- Externally assessed by Cambridge International
- 40% of the AS Level

Offered only as part of AS Level.

Assessment objectives

The assessment objectives (AOs) are the same for all papers:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weightings for assessment objectives

The approximate weightings (± 5%) allocated to each of the AOs are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

Question 1

1 (a) The polynomial $2x^3 + ax^2 - ax - 12$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$.

Find the value of a. [2]

Using the factor theorem $p(-1): -2 + a + a - 12 = 0$

$a = 7$

Examiner comment

The first line of the working gains M1 as the candidate has substituted in $x = -1$ and equated to zero. Each term has been shown, which is a good approach as arithmetic errors are less likely to occur. The final answer gains A1.

Candidates often make an error with the sign of the value of x to be substituted in, for this example they use a substitution of $x = 1$. If in doubt, the candidate should equate the factor to zero and solve. Another common error in this type of question is to use algebraic long division rather than the factor theorem; it is a lot more time consuming and arithmetic errors very often occur in the process. Arithmetic errors in simplification are also common; it is better to write out each term separately, as shown here, and then simplify.

(b) When a has this value, find the remainder when p(x) is divided by (x + 3). [2]

$p(x) = 2x^3 + 7x^2 - 7x - 12$

Using the remainder theorem $p(-3) = 2(-3)^3 + 7(-3)^2 - 7(-3) - 12$

$= -54 + 63 + 21 - 12$

Remainder $= 18$

Examiner comment

The candidate gains M1 for the use of the remainder theorem, the substitution of $x = -3$ and the evaluation of the resulting equation. The final answer is awarded A1.

Candidates often make an error with the sign of the value of x to be substituted in, for this example they would use a substitution of $x = 3$. If in doubt, the candidate should equate the factor to zero and solve. Another common error is the use of algebraic long division rather than the remainder theorem, which is a lot more time consuming and arithmetic errors very often occur in the process. Arithmetic errors during simplification are another common error; it is better to write out each term separately, as shown here, and then simplify.

Question 2

2 Solve the equation $3\sin 2\theta \tan\theta = 2$ for $0° < \theta < 180°$. [4]

$3(2\sin\theta\cos\theta) \times \dfrac{\sin\theta}{\cos\theta} = 2$

$6\sin^2\theta = 2$

$\sin\theta = \pm\dfrac{1}{\sqrt{3}}$

Ignore solutions for $\sin\theta = -\dfrac{1}{\sqrt{3}}$ as these will be out of range.

$\theta = 35.3^o,\ 144.7^o$

Examiner comment

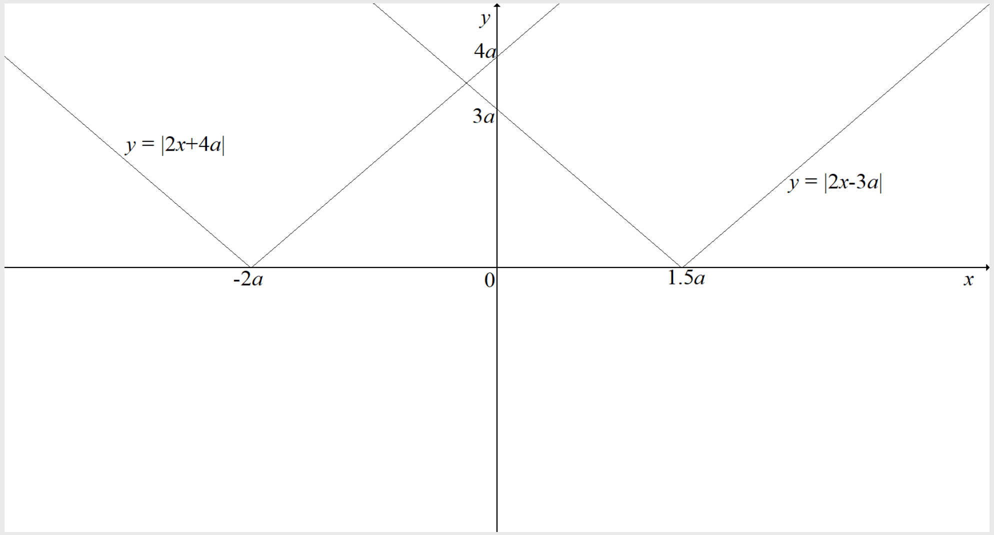
B1 is awarded for the use of the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$. Just stating the formula is not sufficient, there has to be evidence of its application. This is implied by the result in the second line. M1 is awarded for the process of writing $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and simplifying with the double angle formula to obtain an equation of the form $c_1\sin^2\theta = c_2$. The equation in the form $c_1\sin^2\theta = c_2$ has been rearranged to an equation of the form $\sin\theta = k$, to find one value of θ from this equation. This gains another M1. Both solutions obtained are correct and there are no other solutions in the range $0^o < \theta < 180^o$. This gains the final A1. Note that a response including the correct answers but without **supporting evidence** would **not** gain any marks.

Common errors for this kind of question include: not recognising that it is necessary to obtain each of the trigonometric ratios in terms of $\sin\theta$ and $\cos\theta$; arithmetic errors in the simplification; and incorrect use of the calculator – learners should always ensure that it is in the correct mode for the question being answered, in this example the calculator should be in degree mode. Another common error is for answers not to be given to the required level of accuracy – solutions in degrees need to be given to 1 decimal place. Answers in radians, should be given to 3 significant figures.

Question 3

3 It is given that a is a positive constant.

(a)(i) Sketch on a single diagram the graphs of $y = |2x - 3a|$ and $y = |2x + 4a|$. [2]



Examiner comment

Candidates are expected to be able to sketch the graph of $y = |ax + b|$. Each graph has to have the correct V shape, with one vertex on the positive *x*-axis and the other vertex on the negative *x*-axis. This gains M1. As the graphs are in the correct relation to each other, with the appropriate lines being approximately parallel to each other, this will be awarded the A1. At this stage, the coordinates of the intercepts on the axes have not been required, but it is good practice to mark them in as well as labelling each of the graphs. It is also acceptable to draw in 'construction lines' which are then reflected in the *x*-axis, so long as it is clear which lines the final graphs are meant to be.

A common error for questions of this type include not realising that the appropriate 'arms' of each graph have the same gradient and must therefore appear to be parallel to each other.

(a)(ii) State the coordinates of each of the points where each graph meets an axis. [1]

$(-2a, 0)$, $\left(\frac{3}{2}a, 0\right)$, $(0, 4a)$, $(0, 3a)$

Examiner comment

It is intended that the candidate read off the coordinates from their sketch in part **(i)**, which is why it is always necessary to mark in the intercepts with the axes (if possible) on any sketches. The amount of space required for this is reflected in the space available for this part on the examination paper. All the coordinates are correct, so the response is awarded B1. If the candidate had marked the points on the axes as in the sketch for part **(i)**, without stating the actual coordinates, B1 would have still been awarded if they were all correct.

Common errors in this kind of question include confusing the position of the zeroes in the coordinates, e.g. $(4a, 0)$ rather than $(0, 4a)$, and omitting the 'a' from each set of coordinates, e.g. $(0, 4)$ rather than $(0, 4a)$.

(b) Solve the inequality $|2x-3a| < |2x+4a|$. [3]

From the sketch, it can be seen that the graphs intersect at one point, this is the solution to

$2x+4a=-(2x-3a)$

Graphs intersect at $x=-\frac{1}{4}a$

By considering the sketch, the required region is $x>-\frac{1}{4}a$

Examiner comment

The method of considering two linear equations has been used. There is only one solution as indicated by the sketch done in the part **(a)(i)**. M1 is awarded for a valid method to find the point of intersection of the two graphs. Other valid methods include squaring both sides of the inequality in order to obtain a linear inequality, or use of symmetry in the sketch. The correct critical value of $x=-\frac{1}{4}a$ is awarded A1. This may be implied if the candidate has worked with inequalities throughout. The correct solution of $x>-\frac{1}{4}a$ is awarded A1.

Common errors in this kind of question include the candidate not realising that they need to make use of their sketch from part **(a)(i)**. The fact that the same expressions and equations are used throughout, should alert them to this. Arithmetic and algebraic errors in simplification are also common. Other errors include not making use of the sketch to determine the correct region, or choosing the wrong region.

Question 4

4 (a) Solve the equation $5^{2x} + 5^x = 12$, giving your answer correct to 3 significant figures. [4]

$$(5^x)^2 + 5^x - 12 = 0$$

$$(5^x - 3)(5^x + 4) = 0$$

$$5^x = 3$$

$$x \ln 5 = \ln 3$$

$$x = \frac{\ln 3}{\ln 5}$$

$$x = 0.683$$

Examiner comment

It is important to equate the 3 terms in the equation to zero to make it more clear that this is a quadratic equation in terms of 5^x. There is M1 for recognising this fact and for the attempt to solve it. Showing a solution by factorisation (in this case) or by use of formula or completing the square, is preferred to solution by calculator.
A1 is given for the solution $5^x = 3$ only, as an exponential function of this type can never be negative. Using a valid method with logarithms to solve $5^x = 3$ gains M1. It would be acceptable to state $x = \log_5 3$ and use the appropriate calculator function. The final answer, correct to 3 significant figures is given A1.

If the candidate does not recognise that the given equation is a quadratic then they are unable to start the question. Incorrect factorisation leading to an incorrect solution is a common error in this kind of question – it is important to show solution of a quadratic equation by either factorisation, formula, or completing the square. If it can be seen that a valid method is being used, then a method mark is more likely to be awarded. If an incorrect result is written down from use of a calculator with no supporting evidence, then this will be unlikely to gain any credit. Another common error is the incorrect solution of equations of the type $5^x = k$, $k > 0$. It is fine to make use of a calculator provided it is used correctly. It is important that candidates give their answer to the required level of accuracy, which is 3 significant figures. Too many marks are missed due to premature rounding and just not considering the accuracy of the final answer.

(b) It is given that $\ln(y+5) - \ln y = 2\ln x$.

Express y in terms of x, in a form not involving logarithms. [4]

$\ln(y+5) - \ln y = \ln x^2$

$\ln \dfrac{y+5}{y} = \ln x^2$

$\dfrac{y+5}{y} = x^2$

$y + 5 = x^2 y$

$x^2 y - y = 5$

$y(x^2 - 1) = 5$

$y = \dfrac{5}{x^2 - 1}$

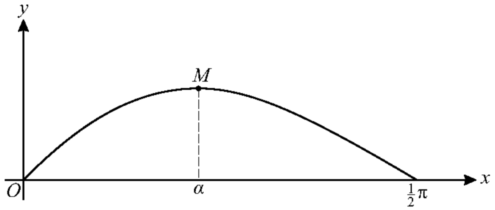
Examiner comment

The use of the logarithm law involving powers gains B1. This should really be the first step as all terms are now in terms of single logarithms. Use of the logarithm law involving subtraction of logarithms gains M1. Equivalent correct statements making use of the addition law for logarithms is also acceptable. As each side of the equation is a single logarithm, the terms in y and x can be equated and rearranged to obtain y in terms of x. This gains M1. This mark would not be available if the previous M1 had not been gained. A correct final answer gains A1.

Common errors in this type of question include the misuse of the laws of logarithms – it is important that the original equation is expressed in terms of single logarithms first. It is quite common to see errors of the type, e.g. $\ln(y+5) - \ln y = \ln x^2$, so $y + 5 - y = x^2$ This type of error results in there being no y terms, so should alert the candidate to an error in their work. Poor algebraic skills are another issue where a correct equation is obtained, e.g. $\dfrac{y+5}{y} = x^2$ but the candidate does not have the necessary skills to rearrange the equation so that y is in terms of x. Not showing all the steps in a calculation is also an issue – candidates are less likely to make a simple error if they take the time to write down each step in a calculation rather than trying to combine steps.

Question 5

5



The diagram shows the curve $y = \dfrac{\sin 2x}{x+2} =$ for $0 \leqslant x \leqslant \dfrac{1}{2}\pi$. The x-coordinate of the maximum point M is denoted by α.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that α satisfies the equation $\tan 2x = 2x + 4$. [4]

$$\frac{dy}{dx} = \frac{(x+2)\times 2\cos 2x - \sin 2x}{(x+2)^2}$$

Maximum point when $x = \alpha$, so $\dfrac{dy}{dx} = 0$ when $x = \alpha$

$$0 = \frac{2(\alpha+2)\cos 2\alpha - \sin 2\alpha}{(\alpha+2)^2}$$

Leading to $0 = 2(\alpha+2)\cos 2\alpha - \sin 2\alpha$

$\sin 2\alpha = 2(\alpha+2)\cos 2\alpha$

$\dfrac{\sin 2\alpha}{\cos 2\alpha} = 2(\alpha+2)$

$\tan 2\alpha = 2\alpha + 4$ as required.

Examiner comment

Correct use of the quotient rule gains M1; an equivalent use of the product rule is also acceptable. The formulae used must be correct. Using the product rule gives a correct answer and gains A1. It is important that each step of the working is shown as the candidate has been asked to show a given result. Substituting $x = \alpha$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ into the result obtained from differentiation, together with a clear attempt at rearrangement, gains M1. All the relevant steps have been seen to obtain the given answer correctly, so A1 is gained.

Common errors in this type of question include: the terms in the numerator of the quotient rule are reversed; the functions denoted by u and v in the quotient rule are reversed. It is essential that candidates use these rules correctly. Incorrect differentiation of the trigonometric function is also a problem – this is usually omission of the coefficient or an incorrect sign. Incorrect simplification of $0 = \dfrac{2(\alpha+2)\cos 2\alpha - \sin 2\alpha}{(\alpha+2)^2}$ to $(\alpha+2)^2 = 2(\alpha+2)\cos 2\alpha - \sin 2\alpha$ is also commonly seen. The zero term has been treated as 1. Not checking working when the given answer has not been reached is an issue. Very often a candidate will contrive to obtain the given answer, rather than go back and check each step of their working. Another common issue is the failure to realise that $\dfrac{\sin 2\alpha}{\cos 2\alpha}$ can be obtained in the simplification and this can then be written as $\tan 2\alpha$. It is important that candidates show sufficient steps of their working to obtain the given result; the key words are ‘show that’.

(b) Show by calculation that α lies between 0.6 and 0.7

[2]

Let $f(x) = \tan 2x - 2x - 4$

$f(0.6) = \tan(1.2) - 1.2 - 4$

$= -2.63$

$f(0.7) = \tan(1.4) - 1.4 - 4$

$= 0.40$

There is a change of sign between $f(0.6)$ and $f(0.7)$ so α lies between 0.6 and 0.7.

Examiner comment

Consideration of the sign of $\tan 2x - 2x - 4$ at $x = 0.6$ and $x = 0.7$, gains M1. Use of $2x + 4 - \tan 2x$ is also acceptable. Correctly evaluating the function at these values and stating a correct conclusion gains A1. Alternative methods are acceptable but often fall short of enough working to justify the conclusion. The method shown here is by far the quickest and the best.

Common mistakes in this kind of question include using an alternative approach involving $\tan 2x$ and $2x + 4$ as separate functions and using the substitutions of $x = 0.6$ and $x = 0.7$ with little explanation or enough working to justify the required result. It is also common for the candidate to not have their calculator in radian mode. The domain in the introduction to the question is in radians, so the candidate should be guided by that as well as the fact that, as calculus is being used, angles are in radians unless otherwise specified. Another common error is that the candidate forgets to come to a conclusion after correct evaluation of their f(0.6) and f(0.7).

(c) Use the iterative formula $x_{n+1} = \frac{1}{2}\tan^{-1}(2x_n + 4)$ to find the value of α correct to 3 decimal places.

Give the result of each iteration to 5 decimal places. [3]

As $0.6 < \alpha < 0.7$, a good starting point for the iterations is 0.65

x_n		$\frac{1}{2}\tan^{-1}(2x_n + 4)$
x_0	0.65	0.69215
x_1	0.69215	0.69358
x_2	0.69358	0.69363

The last two iterations both round to 0.694 correct to 3 decimal places so $\alpha = 0.694$ correct to 3 decimal places.

Examiner comment

The iteration process has been used correctly at least once, so M1 is awarded. The correct final answer is obtained so A1 is gained. There are sufficient iterations to 5 decimal places to justify the correct final answer, so A1 is awarded.

Common errors in this kind of question include: the iterations are not given to the required level of accuracy; the final answer is not given to the required level of accuracy; the candidate's calculator is not in radian mode; the candidate does not use the [ANS] function on their calculator, but substitutes in the value of each iteration separately – while a perfectly acceptable method, it is time consuming and prone to errors as it can very often result in errors in copying or substitution into the formula; the candidate does not realise that part **(b)** has been included to give them the idea of where to start their iterations – either end point is an acceptable starting point as well as the midpoint of the given interval; the candidate chooses a starting point outside the given interval and the iterations fail to converge to the required value.

Question 6

6 The parametric equations of a curve are

$$x = e^{2t},\ y = 4te^{t}.$$

(a) Show that $\frac{dy}{dx} = \frac{2(t+1)}{e^{t}}$. [4]

$$\frac{dx}{dt} = 2e^{2t}$$

$$\frac{dy}{dt} = 4te^{t} + 4e^{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{4te^{t} + 4e^{t}}{2e^{2t}}$$

$$\frac{dy}{dx} = \frac{4e^{t}(t+1)}{2e^{2t}}$$

$$\frac{dy}{dx} = \frac{2(t+1)}{e^{t}} \quad \text{as required}$$

Examiner comment

The product rule has been used to differentiate y with respect to t so M1 is gained. The result $\frac{dy}{dt} = 4te^{t} + 4e^{t}$ is correct and gains A1. The correct relationship $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ is stated and used, gaining M1. There are sufficient steps shown to justify the given answer, so A1 is gained.

Common errors in this type of question include: failure to identify a function that is a product; failure to differentiate exponential functions correctly; the result $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ is used incorrectly; candidates do not go back and check their work when they are unable to obtain the given answer; insufficient working is shown to justify the given answer – it is essential that candidates show all the relevant steps, e.g. factorisation, when attempting a question of this type.

(b) Find the equation of the normal to the curve at the point where $t = 0$. [4]

When $t = 0$, $\frac{dy}{dx} = 2$ and the point has coordinates $(1, 0)$

Equation of the normal $y - 0 = -\frac{1}{2}(x - 1)$

leading to $2y + x - 1 = 0$

Examiner comment

A substitution of $t = 0$ into $\frac{dy}{dx}$ and into each of the parametric equations for x and for y gains B1. The correct derivative and the correct coordinates of the point are obtained, gaining B1. The substitution of $t = 0$ into the appropriate equations is relatively straightforward and the answers may be written down without any additional work being seen. A correct attempt at the normal is made, with the correct use of the property of perpendicular lines being used, to gain M1. A correct equation is obtained, gaining A1. No specified form of this equation is given, so any equivalent form is acceptable.

Common errors in this type of question include: candidates forget that $e^0 = 1$; the equation of the tangent is found rather than that of the normal; an incorrect relationship between the gradients of perpendicular lines is used, e.g. a common error is to use the reciprocal rather than the negative reciprocal of the gradient of the line in question; no attempt is made on part **(b)** as the candidate has not been able to do part **(a)** or has made an error in part **(a)** – the main idea of having 'show that' questions, is that candidates are able to start a question at a later point if they have not been able to get started or have made an error in the first part or parts; the coordinates of the point are used incorrectly in the equation, e.g. $y - 1 = -\frac{1}{2}(x - 0)$.

Question 7

7 (a) Show that $\tan^2 x + \cos^2 x \equiv \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$ and hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} (\tan^2 x + \cos^2 x)\,dx.$$ [7]

$LHS = \tan^2 x + \cos^2 x$

$= \sec^2 x - 1 + \cos^2 x$

$= \sec^2 x - 1 + \frac{1}{2}(\cos 2x + 1)$

$= \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$ as required.

$\int_0^{\frac{\pi}{4}} (\tan^2 x + \cos^2 x)\,dx = \int_0^{\frac{\pi}{4}} \left(\sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}\right) dx$

$= \left[\tan x + \frac{1}{4}\sin 2x - \frac{x}{2}\right]_0^{\frac{\pi}{4}}$

$= \left(\tan\frac{\pi}{4} + \frac{1}{4}\sin\frac{\pi}{2} - \frac{\pi}{8}\right) - (0)$

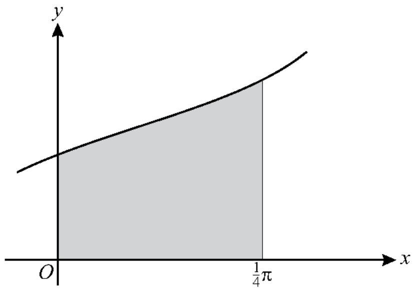
$= \frac{5}{4} - \frac{\pi}{8}$

Examiner comment

The use of the correct identity $\tan^2 x + 1 = \sec^2 x$ gains B1. An attempt to use the double angle formula for $\cos^2 x$ is in a required form, and gains M1. The correct double angle formula has been used, and together with the correct identity, sufficient steps have been shown in order to obtain the given result. This gains A1. The use of the given result in the integral together with an attempt to integrate this expression gains M1. The correct integral is obtained, gaining A1. A correct attempt at using the limits in the square bracket notation gains M1. An exact answer is given as required, and gains A1 as it is correct.

Common errors in this type of question include: the trigonometric identity used has a sign error; candidates often fail to identify the occasions when a double angle formula needs to be used – looking at the given answer gives a clue to what is required; if a double angle formula is used, it very often has a sign error, errors in the coefficients of the terms or not in the required trigonometric ratios; not enough steps are seen to justify the given result – it is essential that the substitution of the limits is seen and this will help the candidate evaluate an exact answer; a calculator is used to produce a decimal answer when evaluating the square brackets – this would probably lose the last A1 mark; a calculator is used to evaluate the definite integral so no appropriate steps are seen and therefore, none of the final 4 marks will be available; the candidate does not appreciate the use of the word 'hence' and so does not make use of the given result – the integration can be attempted even if the candidate has been unable to obtain the given result, but often the candidate does not make use of the given answer.

(b) [4]



The region enclosed by the curve $y = \tan x + \cos x$ and the lines $x = 0$, $x = \dfrac{1}{4}\pi$ and $y = 0$ is shown in the diagram.

Find the exact volume of the solid produced when this region is rotated completely about the x-axis.

$$\text{Volume} = \int_0^{\frac{\pi}{4}} \pi(\tan x + \cos x)^2\,dx$$

$$= \int_0^{\frac{\pi}{4}} \pi(\tan^2 x + \cos^2 x + 2\sin x)\,dx$$

$$= \pi\int_0^{\frac{\pi}{4}} (\tan^2 x + \cos^2 x)\,dx + \pi\int_0^{\frac{\pi}{4}} (2\sin x)\,dx$$

$$\text{Using part (a), volume} = \pi\left(\frac{5}{4} - \frac{\pi}{8}\right) + \pi\left[-2\cos x\right]_0^{\frac{\pi}{4}}$$

$$= \pi\left(\left(\frac{5}{4} - \frac{\pi}{8}\right) - 2\cos\frac{\pi}{4} + 2\right)$$

$$= \pi\left(\frac{5}{4} - \frac{\pi}{8} - \sqrt{2} + 2\right)$$

$$= \pi\left(\frac{13}{4} - \frac{\pi}{8} - \sqrt{2}\right)$$

Examiner comment

A correct statement for the volume is made, gaining B1. At this stage, the presence of the limits is not necessary and the omission of π is not penalised if it appears later. A correct expansion of the integrand is seen, gaining M1. It has been recognised that the integral is partially the integral in part **(a)** and that the result from part **(a)** can be made use of. There is only one additional term to integrate and this has been obtained in the correct form. This gains M1. The correct use of the square bracket notation together with the correct answer to part **(a)** have been combined to produce a correct exact answer. Alternative exact forms are acceptable. This correct final answer gains A1.

Common errors in this type of question include: the candidate either forgets the formula for the volume of revolution or squares the bracket incorrectly; omission of π throughout, or the loss of π in later stages of the work; the candidate integrates all the terms making use of the given result in part **(a)** as they do not realise that they already have done most of the work in part **(a)** and that there is only one additional integration needed – they are not penalised if they do this, but they should be aware that the question is only worth 4 marks, which should be a guide to the amount of work needed; errors in the integration of the additional trigonometric term; inappropriate use of a calculator as mentioned in part **(a)** – a decimal answer will not gain full marks – the mark allocation will depend on what steps of working have been shown; giving a decimal answer rather than an exact answer.

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