Cambridge Assessment International Education

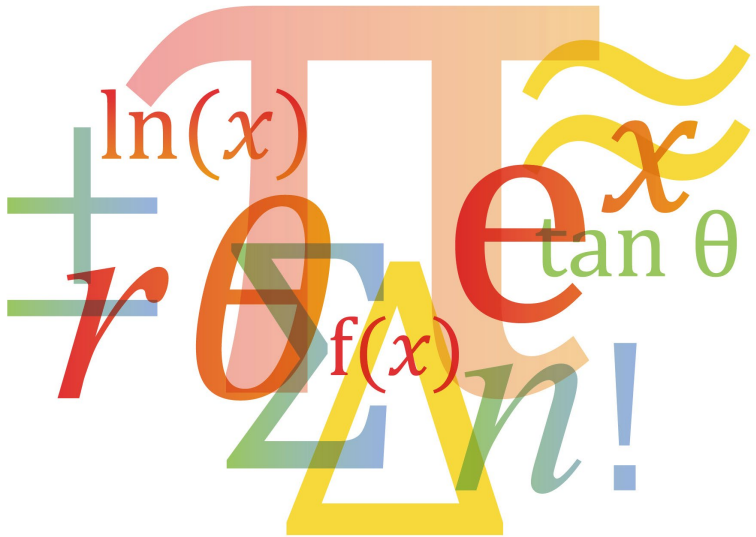
Scheme of Work – Paper 6

Cambridge International AS & A Level

Mathematics 9709

Probability & Statistics 2

For examination from 2020



Version 2

Cambridge Pathway

Contents

Introduction

The Cambridge International AS & A Level Mathematics 9709 scheme of work has been designed to support you in your teaching and lesson planning. The Scheme of Work has been separated into six documents, one for each content section: Pure Mathematics 1, Pure Mathematics 2, Pure Mathematics 3, Mechanics, Probability & Statistics 1 and Probability & Statistics 2. This document relates only to **Probability & Statistics 2**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

Suggestions for independent study **(I)** and formative assessment **(F)** are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that mathematics involves the application of logical methodologies, problem solving and the recognition of patterns as well as the application of these approaches to mathematical modelling. The key concepts are highlighted as a separate item in the new syllabus and you should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. The key concepts for Cambridge International AS & A Level Mathematics are:

Key Concept – Problem solving

Key Concept – Communication

Key Concept – Mathematical modelling

See the syllabus for detailed descriptions of each Key Concept.

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below gives some guidance about how many hours are recommended for each topic.

Topic	Suggested teaching time (hours)	Suggested teaching order
6.1 The Poisson distribution	It is recommended that this should take about 8 hours.	1
6.2 Linear combinations of random variables	It is recommended that this should take about 12 hours.	2
6.3 Continuous random variables	It is recommended that this should take about 6 hours.	3
6.4 Sampling and estimation	It is recommended that this should take about 18 hours.	4
6.5 Hypothesis tests	It is recommended that this should take about 16 hours.	5

Prior knowledge

Knowledge of the content of Paper 5: Probability & Statistics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of calculus within the content for Paper 3: Pure Mathematics 3 will also be assumed.

Resources

You can find the endorsed resources to support Cambridge International AS & A Level Mathematics on the Published resources tab of the syllabus page on our public website here.

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. All textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the specimen assessment materials.

School Support Hub

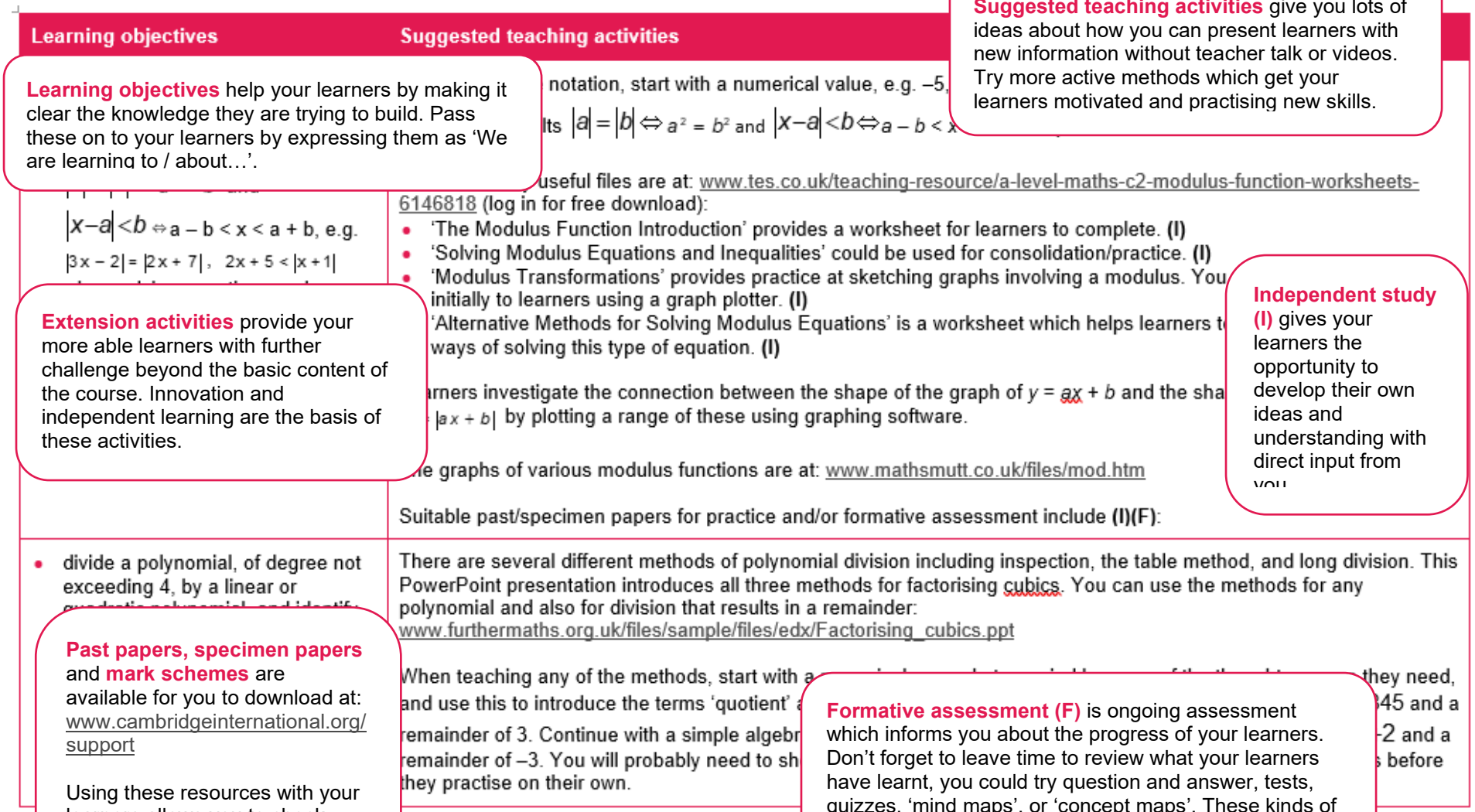
The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services). The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Mathematics 9709 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.



Learning objectives	Suggested teaching activities
	notation, start with a numerical value, e.g. –5,
	lts $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x-a\| < b \Leftrightarrow a - b < x$
$\|x-a\| < b \Leftrightarrow a - b < x < a + b$, e.g. $\|3x - 2\| = \|2x + 7\|$, $2x + 5 < \|x + 1\|$	useful files are at: www.tes.co.uk/teaching-resource/a-level-maths-c2-modulus-function-worksheets-6146818 (log in for free download): • 'The Modulus Function Introduction' provides a worksheet for learners to complete. **(I)** • 'Solving Modulus Equations and Inequalities' could be used for consolidation/practice. **(I)** • 'Modulus Transformations' provides practice at sketching graphs involving a modulus. You initially to learners using a graph plotter. **(I)** 'Alternative Methods for Solving Modulus Equations' is a worksheet which helps learners t ways of solving this type of equation. **(I)** rners investigate the connection between the shape of the graph of $y = ax + b$ and the sha $\|ax + b\|$ by plotting a range of these using graphing software. e graphs of various modulus functions are at: www.mathsmutt.co.uk/files/mod.htm Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**:
• divide a polynomial, of degree not exceeding 4, by a linear or	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a they need, and use this to introduce the terms 'quotient' 45 and a remainder of 3. Continue with a simple algebr -2 and a remainder of –3. You will probably need to sh s before they practise on their own.

Learning objectives help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

6.1 The Poisson distribution

Learning objectives	Suggested teaching activities
• use formulae to calculate probabilities for the distribution Po(λ)	Use a dominoes activity using Tarsia software. For a particular Poisson distribution, give learners cards to match; for example, matching P(X = 6) to its answer. **(F)** You can download Tarsia software free of charge to create this type of activity from: www.mmlsoft.com/index.php/products/tarsia Use the following video in class, pausing the video for learners to complete each task. Alternatively, learners could use the video as an independent revision or consolidation resource. An explanation of the Poisson distribution and use of the formula is at: www.youtube.com/watch?v=2zK3KpV3bx4
• use the fact that if $X \sim$ Po(λ) then the mean and variance of X are each equal to λ; proofs are not required	
• understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model	
• use the Poisson distribution as an approximation to the binomial distribution where appropriate; the conditions that n is large and p is small should be known; $n >$ 50 and $np < 5$, approximately	As a lesson starter activity, ask learners to answer on mini-whiteboards a few simple questions evaluating parameters and considering conditions for possible Poisson distributions. **(F)** Show learners the following video in class, pausing the video for learners to carry out some of the calculations themselves, or they could use the video as an independent revision or consolidation resource. An example of the approximation is shown at: www.youtube.com/watch?v=nZgnov87jVM
• use the normal distribution, with continuity correction, as an approximation to the Poisson	As a lesson starter activity, ask learners to answer on mini-whiteboards simple questions evaluating parameters and considering conditions. **(F)**

Learning objectives	Suggested teaching activities
distribution where appropriate; the condition that λ is large should be known; λ > 15, approximately	Show learners the following video in class, pausing the video for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. Prepare in advance a set of key questions to accompany the video. **(F)** The normal approximation to the Poisson distribution, with conditions and explanation of continuity corrections, is illustrated at: www.youtube.com/watch?v=7RuoHKhzs2I (Note that learners should be familiar with the conditions needed to use this approximation. These are only approximate values: the video clip uses $n > 20$, but $n > 15$ is sufficient for this syllabus.)

6.2 Linear combinations of random variables

Learning objectives	Suggested teaching activities
• use, when solving problems, the results that: – $E(aX + b) = aE(X) + b$ and $Var(aX + b) = a^2Var(X)$ – $E(aX + bY) = aE(X) + bE(Y)$ – $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ for independent X and Y – if X has a normal distribution then so does $aX + b$ – if X and Y have independent normal distributions then $aX + bY$ has a normal distribution – if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution proofs of these results are not required	This video covers linear combinations: www.youtube.com/watch?v=rLdoKZ7w0xI You could show this as a whole class activity, pausing the video for learners to complete tasks independently. Alternatively, learners could watch the video individually as a revision or consolidation resource. To help learners understand the difference between the distributions $2X$ and $X_1 + X_2$, (i.e. when to multiply and when to add), carry out an experiment. For example, consider the question: "How does 'rolling a die and doubling' differ from 'rolling two dice and adding'"? like at: www.s253053503.websitehome.co.uk/msv/msv-19.html Learners can obtain further practice using 'Linear Combinations 3' which is a Tarsia domino activity, available from www.mrbartonmaths.com/jigsaw.htm. Scroll down to 'Mr Barton's Tarsia Jigswa files', listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats1', click on this and then '10. Linear combinations'. To demonstrate adding two independent Poisson variables, show the presentation at: www.s253053503.websitehome.co.uk/msv/msv-38.html

6.3 Continuous random variables

Learning objectives	Suggested teaching activities
• understand the concept of a continuous random variable, and recall and use properties of a probability density function; for density functions defined over a single interval only; the domain may be infinite, e.g. $\frac{3}{x^4}$ for $x \geq 1$	
• use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution; including location of the median or other percentiles of a distribution by direct consideration of an area using the density function; explicit knowledge of the cumulative distribution function is not included	Use a card matching activity: make cards for learners to match a given probability density function to the corresponding mean, variance etc. **(F)** You can download Tarsia software free of charge and use it to create a matching activity from: www.mmlsoft.com/index.php/products/tarsia

6.4 Sampling and estimation

Learning objectives	Suggested teaching activities
• understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples	
• explain in simple terms why a given sampling method may be unsatisfactory; including an elementary understanding of the use of random numbers in producing random samples; knowledge of particular sampling methods, such as quota or stratified sampling, is not required	When working on questions that require samples to be picked, discuss with learners why particular methods were chosen. Alternatively, if the question does not suggest a particular method, ask them what would be a suitable method. As a class activity, generate a random sample experimentally from a parent population using a suitable sampling method.
• recognise that a sample mean can be regarded as a random variable, and use the facts that $E(\bar{X}) = \mu$ and that $Var(\bar{X}) = \frac{\sigma^2}{n}$	Learners will need to practise calculating parameters for the distribution of the sample means. An engaging activity to use in class is 'Sample means and the central limit theorem' which is a Tarsia domino activity, available from www.mrbartonmaths.com/jigsaw.htm. Scroll down to 'Mr Barton's Tarsia Jigswa files', listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats 1', click on this and then '5. Central Limit Theorem'.
• use the fact that $(\bar{X})$ has a normal distribution if X has a normal distribution	Demonstrate this experimentally using an underlying normal distribution: take samples of the same size from the given normally-distributed population and plot the distribution of these sample means. Then take samples of a larger size from the population and carry out the experiment again. **Extension activity**: The following video would be interesting for some learners: http://onlinestatbook.com/2/normal_distribution/history_normalM.html
• use the Central Limit Theorem where appropriate; only an informal understanding of the Central Limit Theorem (CLT) is	Carry out the same experiment as above with a non-normally distributed parent population. This illustrates that the sample means follow a normal distribution regardless of the underlying population distribution as long as the sample size is large enough. Learners need to be aware of when it is appropriate to use the Central Limit Theorem. Note that n > 30 is considered 'large' for the purpose of this syllabus.

Learning objectives	Suggested teaching activities
required; for large sample sizes, the distribution of a sample mean is approximately normal	The experiment can also be run as a simulation using: http://onlinestatbook.com/2/sampling_distributions/SampDist_v1.html#video or the distribution maker here: http://nrich.maths.org/content/id/5932/stage_distributions.swf Show learners the following videos in class, pausing each video for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. Prepare in advance a set of key questions to accompany the video clips. **(F)** An introduction to sampling distributions is at: http://onlinestatbook.com/2/sampling_distributions/intro_samp_distM.html The Central Limit Theorem is illustrated at: www.youtube.com/watch?v=JNm3M9cqWyc and www.youtube.com/watch?v=FXZ2O1Lv-KE. This video discusses the variance: www.youtube.com/watch?v=NYd6wzYkQIM. Learners practise calculating parameters for the distribution of the sample means using a Tarsia jigsaw 'Sample means and central limit theorem'. You can find this at www.mrbartonmaths.com/jigsaw.htm: scroll down to 'Mr Barton's Tarsia Jigswa Files' listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats 1', click on this and then '5. Central Limit Theorem'.
• calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data; only a simple understanding of the term 'unbiased' is required, e.g. that although individual estimates will vary the process gives an accurate result 'on average'	Use a card matching activity here: give learners cards that each show a different set of raw data or summarised data to match with corresponding cards showing biased and unbiased estimates. **(F)** You can download Tarsia software free of charge and use it to create a matching activity: www.mmlsoft.com/index.php/products/tarsia
• determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used	A video that discusses constructing and interpreting a confidence interval for the population mean is at: www.youtube.com/watch?v=KG921rfbTDw. Show learners the video in class, pausing it for learners to carry out some of the calculations themselves, or they could use it as an independent revision or consolidation resource. Learners practise finding the *z* value when constructing a confidence interval using the Tarsia' jigsaw 'Confidence Intervals', available from http://www.mrbartonmaths.com/jigsaw.htm: scroll down 'Mr Barton's Tarsia Jigswa Files' listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats 1', click on this and then on '4. Confidence intervals'.

Learning objectives	Suggested teaching activities
• determine, from a large sample, an approximate confidence interval for a population proportion	Create a 'spot the error' activity sheet. Give some incorrect solutions to learners and ask them to 'mark' the work then discuss the errors. (Incorporate errors deliberately into the solutions, based on common misconceptions.) Show learners the following video in class, pausing the video for learners to complete tasks independently or to discuss key points. Alternatively, they could use it as an independent revision or consolidation resource. This video shows how to construct a confidence interval for a population proportion: www.youtube.com/watch?v=3ReWri_jh3M

6.5 Hypothesis tests

Learning objectives	Suggested teaching activities
• understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic. Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set.	Show learners the following videos in class, pausing them for questioning or for learners to complete tasks independently. Alternatively, learners could use it as an independent revision or consolidation resource. An explanation of null and alternative hypotheses and one and two-tail test is at: www.youtube.com/watch?v=W6Wn_J5qWVw An explanation of type I and type II errors is at: www.youtube.com/watch?v=7mE-K_w1v90 (Note only the first part of this clip is relevant – the power of the test is not required for this syllabus.) A card matching activity could also be used: give learners cards that each show a given scenario and ask them to match them with corresponding cards showing null and alternative hypotheses. **(F)**
• formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using – direct evaluation of probabilities – a normal approximation to the binomial or the Poisson distribution, where appropriate	Show learners the following videos in class, pausing them for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. Prepare in advance a set of key questions to accompany the video clips **(F)** Hypothesis testing for a Poisson Distribution is explained at: www.youtube.com/watch?v=48gjjjaCVYw Hypothesis testing for a Binomial Distribution is explained at: www.youtube.com/watch?v=61Wi04SqF34
• formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used	No specific recommendations.

Learning objectives	Suggested teaching activities
• understand the terms Type I error and Type II error in relation to hypothesis tests.	An interesting article, “Which type of error would you prefer?”, to help learners distinguish and remember the two types of error is at: https://learnandteachstatistics.wordpress.com/2012/10/29/which-type-of-error-do-you-prefer
• calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.	

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: info@cambridgeinternational.org www.cambridgeinternational.org