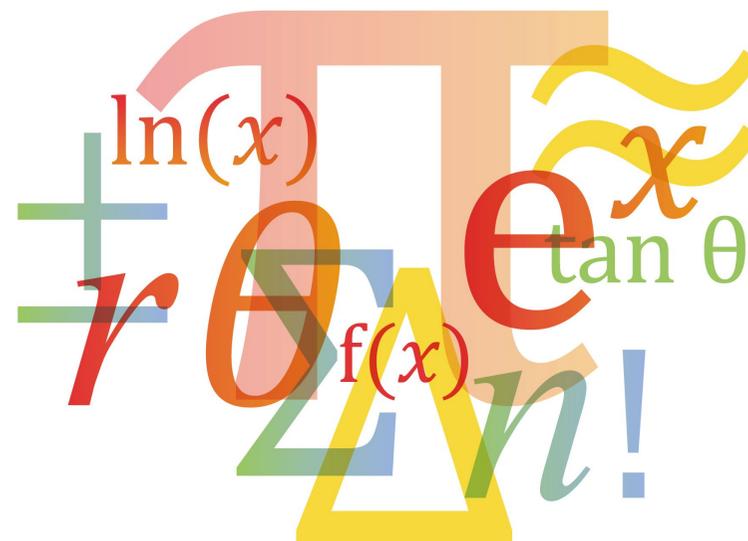


Scheme of Work – Paper 5

Cambridge International AS & A Level Mathematics 9709 Probability & Statistics 1

For examination from 2020



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Introduction

The Cambridge International AS & A Level Mathematics 9709 scheme of work has been designed to support you in your teaching and lesson planning. The Scheme of Work has been separated into six documents, one for each content section: Pure Mathematics 1, Pure Mathematics 2, Pure Mathematics 3, Mechanics, Probability & Statistics 1 and Probability & Statistics 2. This document relates only to **Probability & Statistics 1**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

Suggestions for independent study (**I**) and formative assessment (**F**) are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that mathematics involves the application of logical methodologies, problem solving and the recognition of patterns as well as the application of these approaches to mathematical modelling. The key concepts are highlighted as a separate item in the new syllabus and you should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. The key concepts for Cambridge International AS & A Level Mathematics are:

Key Concept – Problem solving

Key Concept – Communication

Key Concept – Mathematical modelling

See the syllabus for detailed descriptions of each Key Concept.

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below gives some guidance about how many hours are recommended for each topic.

Topic	Suggested teaching time (hours)	Suggested teaching order
5.1 Representation of data	It is recommended that this should take about 14 hours.	1
5.2 Permutations and combinations	It is recommended that this should take about 4 hours.	2
5.3 Probability	It is recommended that this should take about 12 hours.	3
5.4 Discrete random variables	It is recommended that this should take about 8 hours.	4
5.5 The normal distribution	It is recommended that this should take about 12 hours.	5

Prior knowledge

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1. Knowledge of the following probability notation is also assumed:

$P(A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A|B)$ and the use of A' to denote the complement of A .

Resources

You can find the endorsed resources to support Cambridge International AS & A Level Mathematics on the Published resources tab of the syllabus page on our public website [here](#).

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. All textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the specimen assessment materials.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services). The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Mathematics 9709 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Subject content help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as ‘We are learning to / about...’.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Subject content

- select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have

Suggested teaching activities

When working with learners on questions that require them to draw statistical diagrams, it would be good practice for you to discuss with them why that particular diagram was chosen. For example, in the following past papers:

Jun 14 Paper 63 Q1: Why were back-to-back stem and leaf diagrams chosen?
Jun 14 Paper 62 Q6: Why was a histogram chosen?

Extension activity: Learners read the following interesting article from the University of Leicester on ways to present raw data, particularly focusing on how to present data when writing reports.
www2.le.ac.uk/offices/ld/resources/numerical-data/numerical-data

Show learners the following videos in class, pausing the video for learners to complete each of the videos as an independent revision or consolidation resource.

- Constructing and interpreting stem-and-leaf diagrams from [ExamSolutions](http://ExamSolutions.com):
www.youtube.com/watch?v=RPEJjL_Jkp4
- Introduction to histograms from [ExamSolutions](http://ExamSolutions.com):
www.youtube.com/watch?v=CD4iaqtZTcA
- Box and whisker plots explained (Marty Brandl):
www.youtube.com/watch?v=635ErzR9Xzc
- Cumulative frequency – plotting:
www.youtube.com/watch?v=X9ajjBamRPc

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Past papers, specimen papers and **mark schemes** are available for you to download at:
www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Test/specimen papers for practice and/or formative assessment include **(I)(F)**:
Paper 62 Q4 (Reading a histogram)
Paper 62 Q6 (ii) (Constructing a histogram)
Paper 61 Q4 (Constructing a stem-and-leaf diagram)
Paper 63 Q5 (Cumulative frequency)

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

5.1 Representation of data

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none"> select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have. 	<p>When working with learners on questions that require them to draw statistical diagrams, it would be good practice for you to discuss with them why that particular diagram was chosen.</p>
<ul style="list-style-type: none"> draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs; including back-to-back stem-and-leaf diagrams. 	<p>Show learners the following videos in class, pausing the video for learners to complete each task; or they could use the videos as an independent revision or consolidation resource.</p> <ul style="list-style-type: none"> Constructing and interpreting stem-and-leaf diagrams from ExamSolutions: www.youtube.com/watch?v=RPEJjL_Jkp4 Introduction to histograms from ExamSolutions: www.youtube.com/watch?v=CD4iagtZTcA Box and whisker plots explained (Marty Brandl): www.youtube.com/watch?v=635ErzR9Xzc Cumulative frequency – plotting: www.youtube.com/watch?v=X9ajjBamRPc
<ul style="list-style-type: none"> understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation), e.g. in comparing and contrasting sets of data. 	<p>Show learners the following videos in class, pausing the video for learners to complete each task; or they could use the videos as an independent revision or consolidation resource.</p> <ul style="list-style-type: none"> A general introduction to measures of central tendency: www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/statistics-intro-mean-median-and-mode Comparing different measures of central tendency with a discussion of which may be the best to use in certain situations: www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/comparing-distribution-means General introduction to measures of dispersion: www.youtube.com/watch?v=E4HAYd0QnRc
<ul style="list-style-type: none"> use a cumulative frequency graph, e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two 	<p>Make a card matching activity for this topic involving cards with cumulative frequency diagrams on them for learners to match to the corresponding box-and-whisker plots. (F)</p>

Learning objectives	Suggested teaching activities
values.	
<ul style="list-style-type: none">calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x - a)$ and $\Sigma(x - a)^2$, and use such totals in solving problems which may involve up to two data sets.	Textbooks will have many examples of questions that learners can use for calculation practice.

5.2 Permutations and combinations

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none"> understand the terms permutation and combination, and solve simple problems involving selections. 	<p>To help learners understand the terms permutation and combination, use these two videos as a whole class activity, pausing the video at particular points to ask direct questions. (F) Alternatively, learners could watch the videos individually for revision or consolidation. (I)</p> <p>This video explains permutations, including notations used and the formula: www.youtube.com/watch?v=XqQTXW7XfYA</p> <p>This video explains combinations and includes an explanation of the difference between permutations and combinations: www.youtube.com/watch?v=bCxMhncR7PU</p>
<ul style="list-style-type: none"> solve problems about arrangements of objects in a line, including those involving <ul style="list-style-type: none"> repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other); <p>questions may include cases such as people sitting in two (or more) rows; questions about objects arranged in a circle will not be included.</p>	<p>'Combinations and permutations' at www.mathsisfun.com/combinatorics/combinations-permutations.html provides some useful questions and worked examples.</p>

5.3 Probability

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none"> evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events or by calculation using permutations or combinations, e.g. the total score when two fair dice are thrown, or drawing balls at random from a bag containing balls of different colours <p>(Knowledge of the following probability notation may also be required: $P(A)$, $P(A \cup B)$, $P(A \cap B)$ $P(A B)$ and the use of A' to denote the complement of A)</p>	<p>Perform experiments in class enabling learners to work out experimental probabilities and compare them with theoretical probabilities (For example, roll a die 120 times and see how many times it lands on each face. Repeat the experiment by rolling the die 600 times. Repeat with a 'biased' die; can learners work out how it is biased?)</p> <p>Probability 'kits' are widely available from educational suppliers, and often have biased dice and sample bottles in them that can make experiments quicker to perform.</p> <p>You could ask simple probability questions as a starter activity, with learners writing their answers on mini-whiteboards. (F)</p> <p>Learners will need to be familiar with probability notation. A useful activity to test recall of the notation (once introduced) is available at: www.tes.com. Search for 'Probability dominoes' and look for the resource from SRWhitehouse. (I)(F)</p> <p>Some examples of practical activities and examples of evaluating probabilities in simple cases are at www.cimt.org.uk/projects/mepres/alevel/stats_ch1.pdf</p> <p>Extension activity: Balls in a box, involving probabilities and tree diagrams is at: www.s253053503.websitehome.co.uk/msv/msv-23.html</p> <p>The following videos provide interactive resources you can use in class, pausing the videos for direct questioning and for learners to complete tasks independently. Alternatively, learners can use the video independently as a revision or consolidation resource.</p> <p>Basic probability, equally probable events, and experimental probability are explained at: www.youtube.com/watch?v=uzkc-qNVoOk</p> <p>Examples of calculating probabilities using combinations are at: www.youtube.com/watch?v=Xqfcy1rqMbl</p> <p>A useful collection of 'Probability trees' Tarsia jigsaws is available for free download from www.mrbartonmaths.com/jigsaw.htm. Scroll down to 'Mr Barton's Tarsia Jigswa files', and listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats1', click on this and then '1. Probability'. These will help learners practise calculating probabilities from tree diagrams. (You may need to download the free Tarsia software first).</p>
<ul style="list-style-type: none"> use addition and multiplication of probabilities, as appropriate, in 	<p>An explanation of the addition rule for probability is at: www.youtube.com/watch?v=QE2uR6Z-NcU</p>

Learning objectives	Suggested teaching activities
<p>simple cases; explicit use of the general formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.</p>	<p>As a lesson starter activity, ask learners simple questions involving addition and multiplication of probabilities and they answer on mini-whiteboards. (F)</p>
<ul style="list-style-type: none"> understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$ calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram; the use of $P(A B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases. 	<p>Show learners the following video clips as a whole class, or they could watch them independently. Produce a set of questions covering key points from the videos, for learners to answer either while watching or afterwards. (F)</p> <p>An explanation of how to use tree diagrams to work out probabilities is at: www.youtube.com/watch?v=6E_NVnboMB8</p> <p>The formula for conditional probabilities is explained, using both Venn diagrams and tree diagrams at: www.youtube.com/watch?v=h05VK1XjVEY</p> <p>Extension activity: An interesting problem involving independent events and real data is at: http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/950/original/illustrative_mathematics_950.pdf?1390751089</p> <p>Ask learners simple probability questions involving conditional probabilities as a lesson starter activity, with learners writing their answers on mini-whiteboards. (F)</p>

5.4 Discrete random variables

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none"> draw up a probability distribution table relating to a given situation involving a discrete random variable X, and calculate $E(X)$ and $\text{Var}(X)$. 	<p>Make a matching card activity consisting of a set of cards showing probability distribution tables for learners to match with the corresponding $E(X)$ and $\text{Var}(X)$. (F)</p>
<ul style="list-style-type: none"> use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models, including the notations $B(n, p)$ and $\text{Geo}(p)$; $\text{Geo}(p)$ denotes the distribution in which $p_r = p(1 - p)^{r-1}$ for $r = 1, 2, 3, \dots$ 	<p>A Binostat is very useful for calculating binomial probabilities and deriving the formula. Balls are dropped into the Binostat and learners can calculate the probability of a ball getting into each slot. A demonstration of this is at: www.youtube.com/watch?v=ZQZy5834I2s</p> <p>Use the following videos as interactive resources in class, pausing the videos for direct questioning and for learners to complete tasks independently. Alternatively, learners could use the video independently as a revision or consolidation resource.</p> <p>An explanation of the properties of a binomial distribution and the notation used is at: www.youtube.com/watch?v=NaDZ0zVTyXQ</p> <p>The formula is explained from a tree diagram at: www.youtube.com/watch?v=-U2cR-ErRVc</p> <p>A range of resources which can be used to introduce the geometric distribution is at: www.tes.com. Search for 'Geometric distribution':</p> <ul style="list-style-type: none"> 'Geometric distributions' by CK-12 is a video with two example calculations using a geometric distribution. This could be set as preparatory work for a lesson or could be used to support learners who need simple examples. 'Geometric Distribution Simulation. Statistics. 16+' by GrahamIwi is an Excel file which simulates rolling a die until a score of 6 is obtained and then has a sheet for large-scale simulation and a probability distribution sheet. This might be used as an introductory task / example for a lesson on geometric distributions. 'Geometric distribution' by stewarty is a PowerPoint presentation that provides a step-by-step introduction to the geometric distribution and includes example calculations. <p>An example of a question using the geometric distribution is shown at: www.youtube.com/watch?v=LRchV_OrMzw (I)</p>
<ul style="list-style-type: none"> use formulae for the expectation and variance of the binomial distribution and for the 	<p>Use the following videos as interactive resources in class, pausing the video for direct questioning and for learners to complete tasks independently. Alternatively, learners could use the video independently as a revision or consolidation resource.</p>

Learning objectives	Suggested teaching activities
<p>expectation of the geometric distribution; proofs of formulae are not required.</p>	<p>The formulae for the binomial distribution are given and explained, as well as examples using them, at: www.youtube.com/watch?v=zEyLaS2t8FI</p> <p>Extension activity: For challenge, learners consider the proof of the formulae for the binomial distribution, given at: www.s253053503.websitehome.co.uk/msv/msv-40.html (I)</p> <p>The formula for the expectation of the geometric distribution is given and explained, as well as examples using it, at: https://www.youtube.com/watch?v=1cO5KwIFQpI</p> <p>Extension activity: For challenge, learners consider the proof of the formula for the expectation of geometric distribution, given at: www.youtube.com/watch?v=AiQuXEsZCyU. (I)</p> <p>A summary of the geometric distribution and the formulae for expectation and variance of the geometric distribution are at: www.s-cool.co.uk. Search for 'Geometric distribution' for a resource 'The Geometric distribution' with a full summary of the information together with examples. (I)</p>
Past and specimen papers	
<p>Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support (F)</p> <p>9709 Mathematics 2020 Specimen Paper 5, question 5 (c)</p>	

5.5 The normal distribution

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none"> understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables; sketches of normal curves to illustrate distributions or probabilities may be required. 	<p>Use the following videos as interactive resources in class, pausing the videos for direct questioning and for learners to complete tasks independently, or make a set of key questions to accompany the videos. (F) Alternatively, learners could use the videos independently as a revision or consolidation resource.</p> <p>The 'Standard Normal Distribution' is explained at: www.youtube.com/watch?v=c11d3vVM5v8 and at: www.youtube.com/watch?v=xgQhefFOXrM</p> <p>This video explains how to find probabilities using tables: www.youtube.com/watch?v=uxwkx4s7U18</p> <p>(Note that statistical tables can vary. Learners should be familiar with the tables given in the Cambridge International Examinations MF19 List of formulae and tables of the normal distribution.)</p> <p>Some useful 'Tarsia' jigsaws learners can use to practise various calculations connected with the normal distribution can be found at: www.mrbartonmaths.com/jigsaw.htm. Scroll down to 'Mr Barton's Tarsia Jigsaw files', listed under 'Additional Tarsia jigsaw bundles' you will find the link for 'Stats1', click on this and then '3. Normal'. (You may need to download the Tarsia software from www.mmlsoft.com/index.php/products/tarsia.)</p>
<ul style="list-style-type: none"> solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$ including: <ul style="list-style-type: none"> finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ finding a relationship between x_1, μ, and σ given the value of $P(X > x_1)$ or a related probability <p>for calculations involving standardisation, full details of the working should be shown, e.g.</p> $Z = \frac{(X - \mu)}{\sigma}$	<p>This video presents a method for solving a problem involving finding the mean and standard deviation: www.youtube.com/watch?v=CsuNZIQ-fsU</p> <p>Some examples, with support and hints, are available at: http://onlinestatbook.com/2/normal_distribution/ch6_exercises.html</p>

Learning objectives	Suggested teaching activities
<ul style="list-style-type: none">recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems; n sufficiently large to ensure that both $np > 5$ and $nq > 5$.	<p>Use the following videos as interactive resources in class, pausing the videos for direct questioning and for learners to complete tasks independently; or make a set of key questions to accompany the videos. (F) Alternatively, learners could use the videos independently as a revision or consolidation resource.</p> <p>An introduction to the normal approximation to the binomial is at: http://onlinestatbook.com/2/normal_distribution/normal_approxM.html</p> <p>Conditions for this approximation and the use of a continuity correction are discussed in the video: www.youtube.com/watch?v=SmjepW2Mb28</p>

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