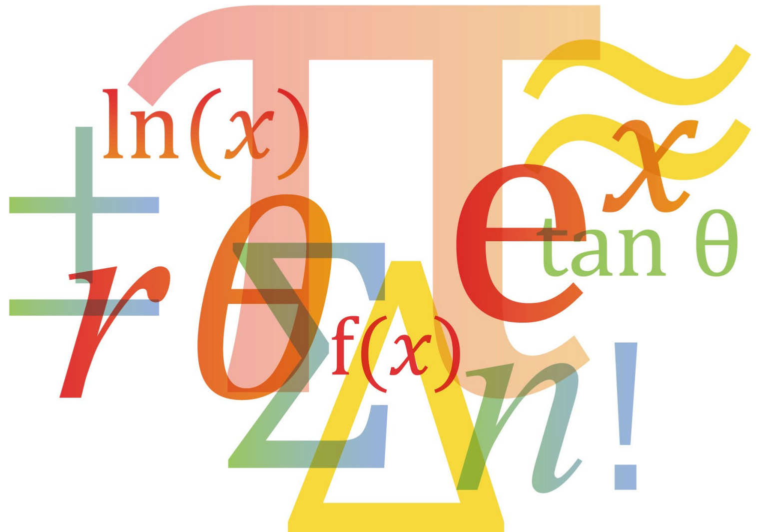
Cambridge Assessment International Education

Scheme of Work – Paper 2

Cambridge International AS & A Level

Mathematics 9709
Pure Mathematics 2

For examination from 2020



Version 2

Cambridge Pathway

Contents

Introduction

The Cambridge International AS & A Level Mathematics 9709 scheme of work has been designed to support you in your teaching and lesson planning. The Scheme of Work has been separated into six documents, one for each content section: Pure Mathematics 1, Pure Mathematics 2, Pure Mathematics 3, Mechanics, Probability & Statistics 1 and Probability & Statistics 2. This document relates only to **Pure Mathematics 2**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

Suggestions for independent study **(I)** and formative assessment **(F)** are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that mathematics involves the application of logical methodologies, problem solving and the recognition of patterns, as well as the application of these approaches to mathematical modelling. The key concepts are highlighted as a separate item in the new syllabus and you should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. The key concepts for Cambridge International AS & A Level Mathematics are:

Key Concept – Problem solving

Key Concept – Communication

Key Concept – Mathematical modelling

See the syllabus for detailed descriptions of each Key Concept.

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below gives some guidance about how many hours are recommended for each topic.

It is recommended that you spend about 80 hours altogether teaching the content of Pure Mathematics 2, covering both the AS and A Level course.

Topic	Suggested teaching time (hours)	Suggested teaching order
2.1 Algebra	It is recommended that this should take about 10 hours.	1
2.2 Logarithmic and exponential functions	It is recommended that this should take about 14 hours.	2
2.3 Trigonometry	It is recommended that this should take about 12 hours.	3
2.4 Differentiation	It is recommended that this should take about 18 hours.	4
2.5 Integration	It is recommended that this should take about 16 hours.	5
2.6 Numerical solution of equations	It is recommended that this should take about 10 hours.	6

Resources

You can find the endorsed resources to support Cambridge International AS & A Level Mathematics on the Published resources tab of the syllabus page on our public website here.

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. All textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the specimen assessment materials.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Mathematics 9709 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Learning objectives helps your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Learning objectives	Suggested teaching activities
• understand the meaning of $\|x\|$, sketch the graph of y = |ax + b| and use relations such as $\|a\|=\|b\| \Leftrightarrow a^2 = b^2$ and $\|x-a\|<b \Leftrightarrow a-b<x<a+b$, e.g. … x |) … t	To introduce the notation, start with a numerical value, e.g. –5, and discuss the meaning of $\|-5\|$. Help learners to deduce the results $\|a\|=\|b\| \Leftrightarrow a^2 = b^2$ and $\|x-a\|<b \Leftrightarrow a-b<x<a+b$ as part of a class discussion. Four extremely useful files are at: www.tes.co.uk/teaching-resource/a-level-maths-c2-modu… 6146818 (log in for free download): • 'The Modulus Function Introduction' provides a worksheet for learners to complete. **(I)** • 'Solving Modulus Equations and Inequalities' could be used for consolidation/practice. (… • 'Modulus Transformations' provides practice at sketching graphs involving a modulus. … ome initially to learners using a graph plotter. **(I)** • 'Alternative Methods for Solving Modulus Equations' is a worksheet which helps learne… ways of solving this type of equation. **(I)** Learners investigate the connection between the shape of the graph of $y = ax + b$ and the … $y = \|ax+b\|$ by plotting a range of these using graphing software. The graphs of various modulus functions are at: www.mathsmutt.co.uk/files/mod.htm Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**:
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a numerical example to remind learners of the thought process they need, and use this to introduce the terms 'quotient' and 're… …ient of 6845 and a … with a simple algebraic e… …t of $x+2$ and a … probably need to show le… …omials before

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or

2.1 Algebra

Learning objectives	Suggested teaching activities
• understand the meaning of $\|x\|$, sketch the graph of y = |ax + b| and use relations such as $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$, e.g. $\|3x - 2\| = \|2x + 7\|$, $2x + 5 < \|x + 1\|$ when solving equations and inequalities; graphs of $y = \|\mathrm{f}(x)\|$ and $y = \mathrm{f}(\|x\|)$ for non-linear functions f are not included	To introduce the notation, start with a numerical value, e.g. –5, and discuss the meaning of $\|-5\|$. Help learners to deduce the results $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ as part of a class discussion. Four extremely useful files are at: www.tes.co.uk/teaching-resource/a-level-maths-c2-modulus-function-worksheets-6146818 (log in for free download): • ‘The Modulus Function Introduction’ provides a worksheet for learners to complete. **(I)** • ‘Solving Modulus Equations and Inequalities’ could be used for consolidation/practice. **(I)** • ‘Modulus Transformations’ provides practice at sketching graphs involving a modulus. You could demonstrate some initially to learners using a graph plotter. **(I)** • ‘Alternative Methods for Solving Modulus Equations’ is a worksheet which helps learners to explore the different ways of solving this type of equation. **(I)** Learners investigate the connection between the shape of the graph of $y = ax + b$ and the shape of the graph of $y = \|ax + b\|$ by plotting a range of these using graphing software. The graphs of various modulus functions are at: www.mathsmutt.co.uk/files/mod.htm
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a numerical example to remind learners of the thought process they need, and use this to introduce the terms ‘quotient’ and ‘remainder’. For example, $54763 \div 8$ leads to a quotient of 6845 and a remainder of 3. Continue with a simple algebraic example $(x^2 + 4x + 1) \div (x + 2)$ which leads to a quotient of $x + 2$ and a remainder of –3. You will probably need to show learners further examples involving more complex polynomials before they practise on their own.

Learning objectives	Suggested teaching activities
	Ideas on possible approaches you can take for long division are at: www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/dividing-polynomials-with-remainders and www.mathsisfun.com/algebra/polynomials-division-long.html A worksheet of examples for practising any of the methods for division is at: www.mathworksheetsgo.com/sheets/algebra-2/polynomials/dividing-polynomials-worksheet.php **(I)** There is another approach 'synthetic division' but learners have to be careful when using it, especially when factorising. Textbooks will have many useful questions for learners to practise.
• use the factor theorem and the remainder theorem, e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients; includes factors of the form ($ax + b$) in which the coefficient of x is not unity, and including calculation of remainders	Summarise the work already done on polynomial division to show that $p(x)$ = (divisor $\times$ quotient) + remainder. Show that algebraic division can often be avoided in questions by substituting into $p(x)$ the value of x that makes the divisor zero (e.g. substituting 3 if the divisor is $x - 3$ and calculating p(3) to find the remainder). Show that the factor theorem is a special case of the remainder theorem when the remainder is zero. There is a good approach of this type which you could use with a whole class at: www.mathsisfun.com/algebra/polynomials-remainder-factor.html Show examples involving finding factors, solving polynomial equations and evaluating unknown coefficients to the whole class, questioning learners individually throughout. Remind learners that they should show all their working as the use of a calculator for finding solutions to polynomial equations will not be accepted in an exam. A useful worksheet which covers basic use of the remainder theorem and evaluating unknown coefficients (log in for free download) is at: www.tes.co.uk/teaching-resource/worksheet-on-the-remainder-theorem-6140286 **(I)** More examples on the remainder theorem and on solving polynomial equations are at: www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)**

9709 Mathematics 2020 Specimen Paper 2, Question 3

2.2 Logarithmic and exponential functions

Learning objectives	Suggested teaching activities
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	Start by defining the terms ‘logarithm’ and ‘exponential’, linking to the concept of indices. To help learners understand a statement such as $\log_a x = b$, describe it to them in words such as “What power of *a* is *x*? Answer: *b*” An introduction with animation showing the relationship between logarithms and exponentials is at: www.purplemath.com/modules/logs.htm . Learners should practise converting expressions from logarithmic to exponential form and from exponential form to logarithmic. Most textbooks will have plenty of examples of this type. A useful worksheet (includes the laws of logarithms) is at: maths.mq.edu.au/numeracy/web_mums/module2/Worksheet27/module2.pdf **(I)** To introduce the laws of logarithms, start with statements $\log_a x = b$ and $\log_a y = c$. Use targeted questioning to encourage learners to write the exponential forms of these statements and reach the conclusion that $a^{b+c} = xy$, rewriting this in logarithmic form to obtain $\log_a xy = \log_a x + \log_a y$. Ask learners to obtain the other two laws in a similar way. Learners will then need to practise applying these laws. Eight files of notes, worksheets and revision (log in for free download) are at: www.tes.co.uk/teaching-resource/a-level-maths-logarithms-worksheets-and-revision-6146791 **(I)** An additional resource which demonstrates the above approach is at: www.mathsisfun.com/algebra/exponents-logarithms.html
• understand the definition and properties of e^x and ln *x*, including their relationship as inverse functions and their graphs; including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of *k*	You can introduce the exponential function e^x in various ways. One approach is to use a graph plotter to show learners the graphs of various exponential functions, e.g. $y = 2^x$, $y = 3^x$, $y = 5^x$. Develop the idea of a particular exponential function that lies between $y = 2^x$ and $y = 3^x$, such that its gradient function is the same as itself. With a suitable graph plotter you can demonstrate that the gradient function of $y = e^x$ is e^x . **Extension activity:** There are other, formal, approaches that you could use with more capable learners. For example

Learning objectives	Suggested teaching activities
	you could consider compound interest and the limit of the series $\left(1+\frac{1}{n}\right)^n$ as shown at: www.mathsisfun.com/numbers/e-eulers-number.html Encourage learners to obtain the logarithmic form of the statement $e^x = a$ and so introduce them to natural logarithms. Building on the work done in Pure Mathematics 1.2 'Functions', develop this into the inverse relationship between e^x and $\ln x$ and demonstrate the inverses on a graph plotter. An interactive exercise covering this relationship is at: http://hotmath.com/help/gt/genericalg2/section_8_5.html **(I)**
• use logarithms to solve equations and inequalities in which the unknown appears in indices, e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$	As a whole class exercise, work through some examples of increasing difficulty, using carefully directed questioning to work through the solutions. Textbooks will include many examples of this type of question and the interactive exercise at the link above includes some too. Demonstrate examples using inequalities, with learners finding critical values first and then deducing the set of solutions. It is helpful to highlight to learners the sign of ln x for $0 < x \le 1$, perhaps through an example where the inequality reverses.
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept, e.g. $y = kx^n$ gives $\ln y = \ln k + n\ln x$ which is linear in ln x and ln y $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and ln y.	If you relate this technique to practical situations, this will help learners when they need to use it in their scientific subjects. Common forms of equation are $y = Ab^x$ and $y = Ax^b$. Learners will need to be able to write these equations in logarithmic form and hence relate them to the equation of a straight line. Sometimes the variables will be letters other than x and y so learners need to spot the form of the equation in order to distinguish the variables from the constants. A useful summary for dealing with situations involving $y = Ax^b$ is at: http://mathbench.umd.edu/modules/misc_scaling/page11.htm Either work through this with learners in class or they could study it independently. **(I)** Use a similar approach for equations of the type $y = Ab^x$. Work through such an example in class, making use of a graph plotter to demonstrate the straight line obtained**.** Textbooks will provide learners with many useful practice questions. For variety, try to choose examples which involve variables other than x and y. Often, learners are asked to work from a given graph in straight line form. Common errors involve learners considering y values rather than ln y values, so practising questions will help to avoid such errors. The Paper 2 past exam papers have examples of this type. To help reinforce this point, split learners into groups or pairs and ask each of them to prepare a question. A simple way

Learning objectives	Suggested teaching activities
	to do this is for learners to 'work backwards' from a logarithmic relationship e.g. $P = At^b$. Each group chooses values for A and b, works out the coordinates of two pairs of coordinates and draws an appropriate straight line graph. Learners circulate their graphs around the other groups who then identify the logarithmic equations used to draw the graphs.

2.3 Trigonometry

Learning objectives	Suggested teaching activities
• understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude.	Start by defining the secant, cosecant and cotangent functions. Learners should know the graphs of the sine, cosine and tangent functions so, as a group or individual task, ask them to think what the graphs of secant, cosecant and cotangent functions look like. For example, give them the graph of $y = \sin x$ (from -360° to 720°) and ask them to sketch $y = \operatorname{cosec} x$ on the same axes. They then check using a graph plotter. Use a similar graphical approach for $y = \sec x$ and $y = \cot x$.
• use trigonometrical identities for the simplification and exact evaluation of expressions, e.g. simplifying $\cos(x – 30°) – 3\sin(x – 60°)$, and in the course of solving equations, e.g. solving $\tan\theta + \cot\theta = 4$, $2\sec^2\theta – \tan\theta = 5$, $3\cos\theta + 2\sin\theta = 1$, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of - $\sec^2\theta \equiv 1+\tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1+\cot^2\theta$ - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ - the expression of $a\sin\theta + b\cos\theta$ in the forms	Start with the identity $\sin^2\theta+\cos^2\theta \equiv 1$ (which learners know already) and ask what they find when: (a) they divide each term in this identity by $\cos^2\theta$ and (b) they divide each term in the original identity by $\sin^2\theta$. A matching exercise and a worksheet for learners to complete as consolidation and practice are at: www.tes.com/teaching-resource/a-level-maths-reciprocal-trig-functions-worksheet-6146865 **(I)** Learners will need plenty of practice at simplifying trigonometric expressions and using the identities, particularly questions of the 'Show that' or 'Prove that' type. The best strategy is to start with one side of the expression (usually the left hand side) and manipulate it using the identities covered so far. Textbooks will include some practice questions. An exercise on simplification is at: http://worksheets.tutorvista.com/proving-trigonometric-identities-worksheet.html **(I)** An exercise on proof is at: https://people.math.osu.edu/maharry.1/150Au2011/TrigIdentities.pdf **(I)** Learners will need to be able to use the identities to solve equations in degrees or radians, and textbooks will contain useful exercises on this. Learners will also need to practise manipulating expressions to obtain an equation (usually quadratic) in terms of one trigonometric ratio e.g. $2\sec^2\theta-3+\tan\theta=0$ will simplify to $2\tan^2\theta+\tan\theta-1=0$ which factorises. For the compound angle (addition) formulae, work through an example of how one formula is derived, perhaps as a whole class exercise. A video proof is at: www.youtube.com/watch?v=a0LvqflQMx4

Learning objectives	Suggested teaching activities
$R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$	The proof of one formula is covered in a similar way at: www.trans4mind.com/personal_development/mathematics/trigonometry/compoundAngleProofs.htm#mozTocId169602 **Extension activity:** Ask learners to work out the proofs of some of the other formulae. Alternatively, start by giving learners the challenge of deriving the compound angle formulae graphically using this interesting investigation: www.tes.co.uk/teaching-resource/the-compound-angle-formulae-lesson-worksheet-6056103 Proving the formulae may come more easily to learners once they are more familiar with them. When learners are competent with the compound angle formulae, ask them to derive the double-angle formulae. They will need to find all possible variants of the formula for $\cos 2\theta$ as well as rearranging them to $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ for use in other applications such as integration. Textbooks include many useful practice exercises on solving equations using the compound and double-angle formulae. Make sure that learners are proficient at using radians as well as degrees. **(I)** A clear summary of how to deal with expressions of the type $a\sin\theta + b\cos\theta$ is at: www.intmath.com/analytic-trigonometry/6-express-sin-sum-angles.php Start with an example e.g. $3\sin\theta + 4\cos\theta$ and show that it may be written in the form $5\sin(\theta + 53.13°)$. This can also be verified using a graph plotter: show learners the graph of $y = 3\sin\theta + 4\cos\theta$ and, with a discussion on transformations, encourage learners to write this expression in a different way. They can check the result by plotting the equivalent expression and seeing that it gives the same graph. Next ask learners to find the maximum and minimum values of the expression and the values of θ at which they occur. (Discourage the use of calculus for questions of this type.) Textbooks include many examples of writing equivalent expressions, solving equations and finding maximum and minimum values. Learners need to be proficient at using radians as well as degrees. **(I)**

2.4 Differentiation

Learning objectives	Suggested teaching activities
• use the derivatives of e^x, ln x, sin x, cos x, tan x, together with constant multiples, sums, differences and composites	A good approach to teaching this section is to use a whole class approach and targeted questioning of learners. For the function $y = e^x$, learners already know that the gradient function is e^x. Build on this by differentiating other functions such as $y = e^{mx}$, $y = e^{f(x)}$, making use of the chain rule where appropriate. To differentiate $y = \ln x$, write $x = e^y$, so $\frac{dx}{dy} = e^y$ and you can obtain the result $\frac{dy}{dx} = \frac{1}{x}$. Using the chain rule, you can generalise to expressions of the form $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$. Textbooks will have exercises for learners to practice. **(I)** To obtain the derivatives of sin x and $\cos x$, consider the gradient of a chord from the origin to a point (h, sin h) on the curve y = sin x. Ask learners to calculate the gradient sin h / h (where h is 0.1 then 0.01 then 0.001) and use this to deduce the gradient at x = 0. They then deduce the gradient at other key points on the graph, for example x = 0, $\pi/2$, π, $3\pi/2$, 2π, use their values to plot the gradient function on a graph of y = sin x and name the graph obtained. Show them that a similar approach will give them the gradient function for y = cos x. You can find this method in many textbooks. It is also covered at:www.mathcentre.ac.uk/resources/uploaded/mc-ty-sincos-2009-1.pdf **Extension activity:** The resource above also covers differentiation from first principles which is suitable as an extension for the more capable learner. Encourage learners to obtain results for the derivatives of sin mx, cos mx, sin f(x) and cos f(x) during a class discussion, making use of the chain rule. Leave the differentiation of y = tan x until the quotient rule has been covered. Many textbooks will have exercises for learners to practice. **(I)**
• differentiate products and	Derive the product and quotient rules as a whole class exercise so that learners (especially the more able) can

Learning objectives	Suggested teaching activities
quotients, e.g. $\dfrac{2x-4}{3x+2}$, $x^2\ln x$, xe^{1-x^2}	understand the formulae more thoroughly. There is a proof using function notation at: http://nrich.maths.org/10086. Alternatively, write the product as *uv* (where *u* and *v* are functions of *x*) then consider increasing the area of a rectangle *uv* to $(u + \delta u)(v + \delta v)$. Expanding the brackets, writing every term over δx and considering the limit as δx -> 0 leads to the product rule. Three files of examples and worksheets on differentiation of products are at: www.tes.co.uk/teaching-resource/product-and-quotient-rules-6146838 **(I)** Appropriate textbooks will have further examples. Try to introduce a variety of different types of functions (such as those in the previous section) and encourage learners to simplify their answers. Set learners the task of deriving the quotient rule by differentiating $y=\dfrac{u}{v}$, where *u* and *v* are functions of *x*, as a product $y=uv^{-1}$, using the product rule. Ask learners to differentiate $y=\tan x$ using the quotient rule. Three files which include examples/worksheets on differentiation of quotients (log in for free download) are at: www.tes.co.uk/teaching-resource/product-and-quotient-rules-6146838 **(I)** Appropriate textbooks will have further examples. Try to introduce a variety of different types of functions (such as those in the previous section) and encourage learners to simplify their answers.
• find and use the first derivative of a function which is defined parametrically or implicitly; e.g. $x = t - e^{2t}$, $y = t + e^{2t}$, e.g. $x^2 + y^2 = xy + 7$, including use in problems involving tangents and normals	Introduce the idea of parametric equations to learners by asking them to imagine two cars moving towards each other along different straight lines on the *x*-*y* plane. You know their lines will intersect but how do you know if the cars will collide or miss each other? You need to consider a third parameter (e.g. time), and express both *x* and *y* in terms of this parameter, in order to say whether or not there will be a collision. Then show learners some simple examples, e.g. $x=2t$, $y=3t^2+5$ and eliminate *t* to obtain the Cartesian form of the curve. A graph plotter may be useful. Show that the gradient function may be obtained using the derivatives $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ together with the chain rule. Extend the work to include parametric equations involving trigonometric functions e.g. $y=3\cos 2\theta$, $x=4\sin\theta$ to help learners to consolidate their knowledge of trigonometric identities and differentiation of trigonometric functions.

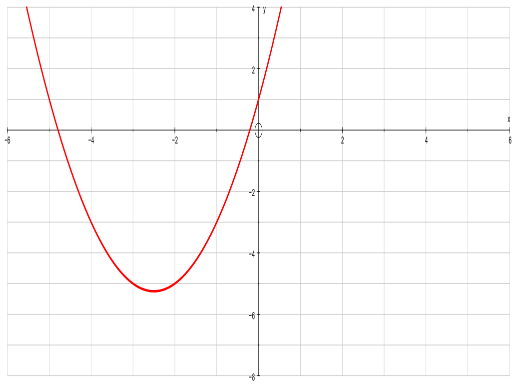
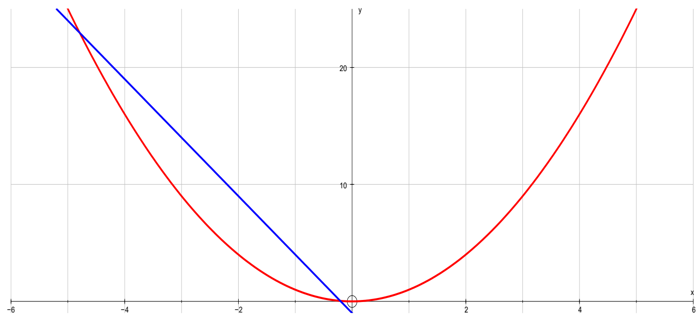
Learning objectives	Suggested teaching activities
	A clear and thorough treatment of the topic with worked examples (see 17.1 Cartesian and parametric equations of a curve and 17.4 Parametric differentiation) is at: www.cimt.org.uk/projects/mepres/alevel/pure_ch17.pdf A good overview of the topic (second derivatives are not required) is at: www.mathcentre.ac.uk/resources/uploaded/mc-ty-parametric-2009-1.pdf **Extension activity:** Learners investigate interesting curves expressed in parametric form using a graph plotter. There are many websites with good examples, for instance this one gives a selection of equations: https://cims.nyu.edu/~kiryl/Calculus/Section_9.1--Parametric_Curves/Parametric_Curves.pdf For implicit differentiation, start with the definition of implicit and explicit functions. Ask learners to consider e.g. $y^2 = x$, rewrite it as $y = x^{\frac{1}{2}}$ then differentiate to obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$. They can rewrite this as $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2y}$ leading to the statement $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 1$. Repeat this exercise, as a whole class or in groups, with several similar examples (powers of *y*) so that learners identify a pattern. Show learners terms of various types: they now know how to differentiate powers of *x* or *y* with respect to *x*. Introduce the idea of a product term by asking them to differentiate equations such as $xy = x$ and $x^2y^3 = 4$ implicitly using the product rule and by rearranging them and differentiating *y* with respect to *x*. Now ask learners to work through an equation from left to right and differentiate it implicitly without rearranging it first. (Give them equations which cannot be rearranged to prevent them doing this.) Useful examples or worksheets are at: www.khanacademy.org/math/differential-calculus/taking-derivatives/implicit_differentiation/v/implicit-derivative-of-x-y-2-x-y-1 www.intmath.com/differentiation/8-derivative-implicit-function.php http://cdn.kutasoftware.com/Worksheets/Calc/03%20-%20Implicit%20Differentiation.pdf

2.5 Integration

Subject content	Suggested teaching activities
• extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$; knowledge of the general method of integration by substitution is not required	Start with a quick review of integration from Pure Mathematics 1.8 'Integration', as a question and answer session with learners writing on mini whiteboards and holding up their responses. This will enable you to assess all learners' understanding before moving on to examples in this section. Divide learners into groups and give them sets of expressions to integrate. Ask them to consider what would need to be differentiated to obtain the given expression, then to work out some general principles. A good approach for integration involving logarithmic functions is at: www.mathcentre.ac.uk/resources/uploaded/mc-ty-inttologs-2009-1.pdf **(I)** Some of the examples may be beyond the range of this syllabus. Textbooks will include exercises on integrating all of these types of function, including finding areas. **(I)**
• use trigonometrical relationships in carrying out integration, e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$	Ask learners to recall the three forms of the trigonometric identity for $\cos 2x$ and then to use them to rewrite $\cos^2 x$ and $\sin^2 x$ in terms of $\cos 2x$. Introduce learners to integrals of the type $\int 2\sin x\cos x\,\mathrm{d}x$, $\int \cos^2 2x\,\mathrm{d}x$ and $\int \tan^2 3x+1\,\mathrm{d}x$. Appropriate textbooks will have examples of these. Try to relate them to areas and also to simple first order differential equations, for example: find the equation of the curve, with gradient function $\frac{\mathrm{d}y}{\mathrm{d}x}=2\sec^2 x+1$ for $0\le x<\frac{\pi}{2}$, which passes through the point $x=\frac{\pi}{4}$ **(I)**
• understand and use the trapezium rule to estimate the value of a definite integral; including use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate	Start by sketching on the board part of a curve with an unknown equation. Ask learners to consider the area under this curve, enclosed by the *x*-axis, split into a number of strips of equal width. How could they work out the area? A PowerPoint presentation that uses this approach and gives some examples (log in for free download) is at: www.tes.co.uk/teaching-resource/trapezium-rule-powerpoint-c2-maths-lesson-3009786 Encourage learners to determine, from a sketch of the curve, whether or not the area they calculate will be an

Subject content	Suggested teaching activities
	overestimate or underestimate. It is important for them to be able to explain their reasoning clearly. Three files which enable learners to work through examples on the trapezium rule and understand its limitations, and which include clear explanations of overestimates and underestimates are at: www.tes.co.uk/teaching-resource/trapezium-rule-6146799 **(I)**

2.6 Numerical solution of equations

Learning objectives	Suggested teaching activities
• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change, e.g. finding a pair of consecutive integers between which a root lies	Introduce this topic by using a graph plotter to demonstrate both sign changes and graphical considerations e.g. $y = x^2 + 5x + 1$: Change of sign  You can see clearly that there are solutions to the equation $x^2 + 5x + 1 = 0$ in the intervals $-5 < x < -4$ and $-1 < x < 0$. Learners consider the sign of y either side of the points of intersection of the curve with the x-axis i.e. using the boundaries above. Demonstrate also that the same result may be obtained by plotting $y = x^2$ against $y = -5x - 1$: 

Learning objectives	Suggested teaching activities
	Learners will need to practise examples of both types. Encourage them to set out their work clearly and accurately. For example, to show that the equation $x^2 = -5x - 1$ has a solution in the interval $-5 < x < -4$, learners should state 'Let $f(x) = x^2 + 5x + 1$' then write the equation as $f(x) = 0$. By calculating and writing down the values of $f(-5)$ and $f(-4)$, they can demonstrate that there is a sign change and state their conclusion e.g. ' There is a change of sign, so a solution lies in the interval $-5 < x < -4$'. A useful overview of the topic, with examples is at: www.cimt.org.uk/projects/mepres/alevel/pure_ch19.pdf
• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	The second part of this chapter deals with convergence to a root of an equation: www.cimt.org.uk/projects/mepres/alevel/pure_ch19.pdf **Extension activity:** The first part of this chapter demonstrates a formal approach to the idea of a sequence of approximations converging to a root of an equation: www-solar.mcs.st-andrews.ac.uk/~clare/Lectures/num-analysis/Numan_chap2.pdf You could use it with able learners or perhaps with a whole class. It explains how an iterative formula generates the sequence; this is the next learning objective.
• understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy; knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected	A video tutorial which learners could watch independently or as a whole class is at: www.tes.com/teaching-resource/iteration-6201516 Iterative formulae are covered in this chapter, which includes examples and activities for learners to try: www.cimt.org.uk/projects/mepres/alevel/pure_ch19.pdf It is a good idea for learners to make full use of their calculator for the iteration process. Using the ANS (answer) key will save them time in finding a root of an equation. For example: Using the iterative formula $x_{n+1} = 3 - \frac{1}{x_n}$ with $x_0 = 3$, show successive iterations to five decimal places and a final answer to three decimal places. • Start by entering the value of x_0 into the calculator: press '3' then '=' (or 'enter', depending on the calculator), so 3 appears as an answer. • Key in the right hand side of the iterative formula, replacing x_n with ANS (or the key that displays a previous answer) i.e. $3-(1\div \text{ANS})$. The calculator will display 2.666666667. Write this down to five decimal places. • Keep pressing the '=' key and successive iterations will appear. Write down as many as the question requires, all

Learning objectives	Suggested teaching activities
	correct to 5 decimal places: 2.62500 2.61905 2.61818 2.61806 2.61804 2.61803 • You have now done enough iterations to show that an answer of 2.618 is correct to three decimal places. Learners will need practice at entering the correct formula into their calculator, using brackets where necessary.

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