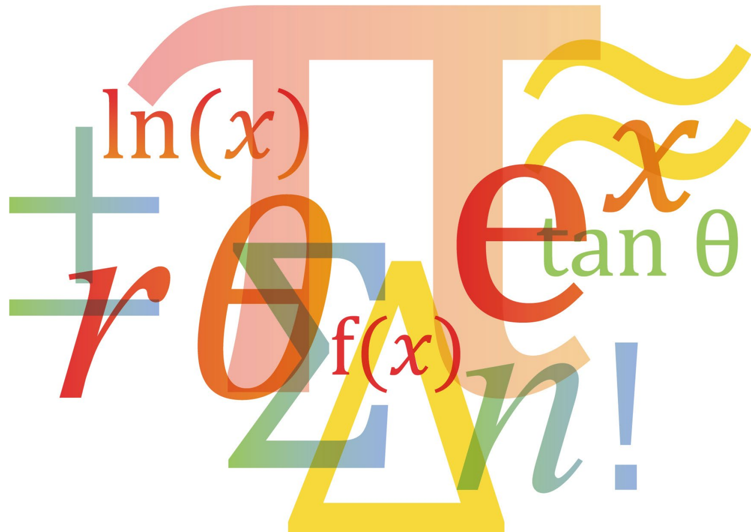
Cambridge Assessment
International Education

Scheme of Work – Paper 1

Cambridge International AS & A Level
Mathematics 9709
Pure Mathematics 1

For examination from 2020



Version 2

Cambridge Pathway

Contents

Introduction

The Cambridge International AS & A Level Mathematics 9709 scheme of work has been designed to support you in your teaching and lesson planning. The Scheme of Work has been separated into six documents, one for each content section: Pure Mathematics 1, Pure Mathematics 2, Pure Mathematics 3, Mechanics, Probability & Statistics 1 and Probability & Statistics 2. This document relates only to **Pure Mathematics 1**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

Suggestions for independent study **(I)** and formative assessment **(F)** are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that mathematics involves the application of logical methodologies, problem solving and the recognition of patterns, as well as the application of these approaches to mathematical modelling. The key concepts are highlighted as a separate item in the new syllabus and you should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. The key concepts for Cambridge International AS & A Level Mathematics are:

Key Concept – Problem solving

Key Concept – Communication

Key Concept – Mathematical modelling

See the syllabus for detailed descriptions of each Key Concept.

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below gives some guidance about how many hours are recommended for each topic.

It is recommended that you spend about 90 hours altogether teaching the content of Pure Mathematics 1, covering both the AS and A Level course.

Topic	Suggested teaching time (hours)	Suggested teaching order
1.1 Quadratics	It is recommended that this should take about 10 hours.	1
1.2 Functions	It is recommended that this should take about 12 hours.	2
1.3 Coordinate geometry	It is recommended that this should take about 10 hours.	3
1.4 Circular measure	It is recommended that this should take about 4 hours.	4
1.5 Trigonometry	It is recommended that this should take about 14 hours.	5
1.6 Series	It is recommended that this should take about 10 hours.	6
1.7 Differentiation	It is recommended that this should take about 16 hours.	7
1.8 Integration	It is recommended that this should take about 16 hours.	8

Prior knowledge

Knowledge of the content of the Cambridge IGCSE® Mathematics 0580 (Extended curriculum), or Cambridge International O Level (4024/4029), is assumed. Candidates should be familiar with scientific notation for compound units, e.g. $5\,\text{ms}^{-1}$ for 5 metres per second.

In addition, candidates should:

- be able to carry out simple manipulation of surds (e.g. expressing $\sqrt{12}$ as $2\sqrt{3}$ and $\frac{6}{\sqrt{2}}$ as $3\sqrt{2}$)
- know the shapes of graphs of the form $y = kx^n$, where k is a constant and n is an integer (positive or negative) or $\pm\frac{1}{2}$.

Resources

You can find the endorsed resources to support Cambridge International AS & A Level Mathematics on the Published resources tab of the syllabus page on our public website here. Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. All textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the specimen assessment materials.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Mathematics 9709 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Syllabus content help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Syllabus content	Suggested teaching activities
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant, e.g. to determine the number of real roots of the equation	The website www.khanacademy.org provides a helpful video covering the three cases – 'Using the quadratic formula: number of solutions' – which has the potential to be used for flipped learning. **(I)** The Khan Academy website also has a helpful summary of the meaning o link to the quadratic formula and the three different cases – 'Discriminant r includes examples and a practice exercise to test understanding. **(I)(F)** The website www.mathshelper.co.uk has a useful worksheet on use of the quadratic polynomial to determine the number of real roots 'C1 discrimina also find this listed on the 'Worksheets Page'.) **Extension activity:** The activity 'Discriminating' on the website https://undergroundmathematics.org is a set of cards for learners to consi statement about the quadratic equation must be true, may be true or can't search facility, or navigate from the 'Quadratics' station on the map.) **(I)** Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: 2020 Specimen Paper 1 Q11(c)

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

1.1 Quadratics

Learning objectives	Suggested teaching activities
• carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form, e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph	Some of this content should be already familiar to learners, so consider using activities that review work done in the Cambridge IGCSE® Mathematics 0580 course, or equivalent. A good starting point would be a quick summary of solving quadratic equations by factorisation and using the quadratic formula. A useful set of differentiated cards that you can use to revise the basic method of completing the square is on the TES website www.tes.com/teaching-resource/completing-the-square-6409449 (or from www.tes.com search for 'Completing the square by headofslytherin'). **(I)(F)** The task 'C1 – Linking the properties and forms of quadratic functions' at www.mrbartonmaths.com/resources/standard_unit_pdfs/Calculus is an excellent way to get learners to explore the different forms of quadratic equations that they are already familiar with, and to link these to the shapes of the associated quadratic graph. The cards from set A are matched with the graphs from set B to make seven sets – learners should discuss their reasoning as they work through this task. You could encourage the use of graphical calculators to support some learners. TES (www.tes.com) has a range of excellent resources for a wide range of topics, which can be found by searching for 'Sketching quadratics'. In particular look for: • 'Sketching quadratic graphs from an equation' by 'thenatsalisbury' www.tes.com/teaching-resource/sketching-quadratic-graphs-from-an-equation-11312156 which gives a set of resources that includes a PowerPoint presentation with a recap of the basic techniques (factorising, completing the square) and determining the key features of the graph; a summary sheet of the key steps; and two different levels of worksheet. **(I)** • 'Sketching quadratic graphs' by Salters www.tes.com/teaching-resource/sketching-quadratic-graphs-6442934 which is a worksheet providing a structured table containing increasingly difficult questions to complete. **(I)** The website https://undergroundmathematics.org has a number of resources for teaching quadratics. (Use the search facility, or navigate from the 'Quadratics' station). In particular, 'Pick a card…' is a task that can be used to build fluency with this topic by starting with different sets of information and asking learners to work forwards and backwards through the problem while considering which is the most challenging start point. **Extension activity:** Another task from the quadratics section of the underground mathematics website, 'GeoGebra constructions… quadratic equation' has a set of differentiated problem-solving questions. **(I) (F)**

Learning objectives	Suggested teaching activities
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant, e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$; knowledge of the term 'repeated root' is included	Introduce the idea of the discriminant of a quadratic expression, $b^2 - 4ac$, and show that it comes from the quadratic formula for solving $ax^2 + bx + c = 0$. By displaying the quadratic formula on a board, you can encourage learners to identify the three different values that the discriminant can take and to relate those to the number of roots of a particular quadratic equation. The website www.khanacademy.org provides a helpful video covering the three cases – 'Using the quadratic formula: number of solutions' – which has the potential to be used for flipped learning. **(I)** The Khan Academy website also has a helpful summary of the meaning of the discriminant, the link to the quadratic formula and the three different cases – 'Discriminant review'. This also includes examples and a practice exercise to test understanding. **(I)(F)** The website www.mathshelper.co.uk has a useful worksheet on use of the discriminant of a quadratic polynomial to determine the number of real roots 'C1 discriminant.pdf'. **(I)** (You can also find this listed on the 'Worksheets Page'.) **Extension activity:** The activity 'Discriminating' on the website https://undergroundmathematics.org is a set of cards for learners to consider whether a given statement about the quadratic equation must be true, may be true or can't be true. (Use the search facility, or navigate from the 'Quadratics' station on the map.) **(I)**
• solve quadratic equations, and quadratic inequalities, in one unknown, by factorising, completing the square and using the formula	The website www.khanacademy.org has a range of videos and activities that can be used to revise how to solve quadratics by factorising, completing the square and use of the formula (this is useful support for less confident learners). (Search for 'solving quadratics'.) **(I)** Another video and worksheets can be found at https://corbettmaths.com/contents. (Under 'Additional Topics' look for 'Inequalities: quadratic'.) **(I)** Cliff notes has descriptions of how to solve quadratic equations using different methods at www.cliffsnotes.com/study-guides/algebra/algebra-i/quadratic-equations/solving-quadratic-equations **(I)** The website also has two quizzes that can be used to test understanding: 'Quiz: Solving equations in quadratic form' at www.cliffsnotes.com/study-guides/algebra/algebra-ii/quadratics-in-one-variable/quiz-solving-equations-in-quadratic-form and 'Quiz: Solving quadratic equations' at www.cliffsnotes.com/study-guides/algebra/algebra-i/quadratic-equations/quiz-solving-quadratic-equations. **(I)(F)** Sets of questions covering the common techniques for solving quadratic equations are available from the TES website www.tes.com: search for 'Solving quadratic equations by climbingjack' or 'Solving quadratics differentiated by labrown20'.

Learning objectives	Suggested teaching activities
	(I)(F) Further practice questions can be found at www.cimt.org.uk/projects/mepres/allgcse/pr10-es.pdf. **(I)** **Extension activity:** To get learners thinking about the different possible methods for solving quadratics, use the 'Quadratic solving sorter' resource on the website https://undergroundmathematics.org. This activity requires learners to group quadratic equations by the possible solution methods. (Use the search facility, or navigate from the 'Quadratics' station on the map.) **(I)** **Extension activity:** Ask learners to work through the scaffolded task 'Proving the quadratic formula' from the 'Quadratics' section of https://undergroundmathematics.org The resource 'Solving Inequalities: An overview' www.purplemath.com/modules/ineqsolv.htm provides an explanation of solving inequalities and makes it clear that you can treat linear inequalities in a similar way to linear equations. It emphasises that multiplying or dividing throughout by a negative number reverses the inequality. (You can instead consider addition or subtraction of terms to move them to the other side of the inequality.) The interactive activity 'Solving Linear Inequalities' www.geogebra.org/m/p3xmmJV7 allows learners to practise the basic solution of linear inequalities and check their answers. (From the Geogebra website, select 'materials' and search for 'solving linear inequalities by KK Ip'.) **(I)(F)** Practice questions on solving linear and quadratic inequalities can be found on the CIMT website www.cimt.org.uk/projects/mepres/allgcse/pr16-es.pdf **(I)** When solving a quadratic inequality, it is important to obtain the critical values first and then to determine the correct set of values for *x* that satisfies the inequality. Finding the critical values by solving the equation gives learners a chance to consolidate work done in the previous sections. Encourage learners to sketch the graph of the quadratic function to help them solve the inequality. The video Quadratic inequalities: graphical approach from www.khanacademy.org/math demonstrates this approach. **(I)** There is an interesting puzzle resource, 'Tarsia - Quadratic Inequalities 2' on the TES website at www.tes.com/teaching-resource/tarsia-quadratic-inequalities-2-6110158. **(I)(F)** (Note: you will need the Tarsia software to open the file, but this is available for free download from www.mmlsoft.com/index.php/products/tarsia). A useful worksheet of questions can be found by searching the TES website for 'Solving quadratic inequalities by marcopront'. **(I)**
• solve by substitution a pair of	You can start with fairly straightforward examples with an obvious substitution e.g. $2x^2 - 3x - y = 1$ and $y = 3x + 7$.

Learning objectives	Suggested teaching activities
simultaneous equations of which one is linear and one is quadratic e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$	The website www.mathsisfun.com has examples of the solution of simultaneous equations where one is linear and one is quadratic. There are also interactive questions which learners can use to test themselves. (Search for 'systems of linear and quadratic equations'.) **(I)** A video and worksheets can be found on the Corbettmaths website https://corbettmaths.com/contents, under 'Simultaneous equations (advanced)'. **(I)** The TES website www.tes.com has some excellent resources on solving simultaneous equations where one is linear and one is quadratic. (Search for 'Simultaneous equations one linear, one quadratic'.) Particularly good examples include 'Simultaneous equations' by SRWhitehouse (a structured worksheet starting from two linear equations) and 'Puzzle for one linear, one quadratic' by SRWhitehouse. You can generate a worksheet of questions on the https://onemathsquestions.com website by selecting 'Generate Worksheets' then 'Simultaneous Eqn Linear Quad' and selecting a difficulty level. Worked solutions are also produced. **(I)** Practice questions on solving simultaneous equations (one linear and one quadratic) can be found on the CIMT website www.cimt.org.uk/projects/mepres/allgcse/add_10.12a.pdf. (Or follow the links from www.cimt.org.uk to Resources, MEP, GCSE, section 'Additional Pupil Text Material for Unit 10'. 'New section 10.12A Algebraic solution of linear/quadratic simultaneous equations'.) **(I)**
• recognise and solve equations in x which are quadratic in some function of x, e.g. $x^4 – 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.	The first two equations given in this learning objective provide a good starting point for this section ($x^4 – 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$). Throughout the course, you could introduce quadratic equations of different types when you reach the appropriate part of the syllabus, e.g. $e^{2x} - 7e^x + 12 = 0$, $2\left(3^{2x+1}\right) - 5\left(3^x\right) + 1 = 0$, $3\tan^2 x - \tan x - 2 = 0$. A worksheet on quadratics in a function of x, 'C1 quadratics in disguise', is available from the website www.mathshelper.co.uk. (You can also find this listed on the 'Worksheets Page'.) **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)**

9709 Mathematics 2020 Specimen Paper 1, Question 11(c)

1.2 Functions

Learning objectives	Suggested teaching activities
• understand the terms function, domain, range, one-one function, inverse function and composition of functions	Start by defining the terms ‘function’, ‘domain’ and ‘range’. There is an informal introduction to these terms at www.coolmath.com/algebra/15-functions/01-whats-a-function-domain-range-01. **(I)** You will cover definitions of the terms ‘one-one function’, ‘inverse function’ and ‘composition of functions’ in later sections, together with the appropriate notation.
• identify the range of a given function in simple cases, and find the composition of two given functions, e.g. range of $\mathrm{f}: x \mapsto \frac{1}{x}$ for $x \geq 1$ and range of $\mathrm{g}: x \mapsto x^2 + 1$ for $x \in \mathbb{R}$, including the condition that a composite function gf can only be formed when the range of f is within the domain of g	Start with some simple functions with straightforward domains and ask learners to identify the ranges for each function, e.g. $\mathrm{f}(x) = 5x + 1,\ x \in \mathbb{R}$, $\mathrm{f}(x) = 5x + 1,\ x \geq 2$. Encourage learners to sketch graphs to help them. Move on to examples where learners could be required to choose a suitable domain and hence find the corresponding range, e.g. $y = \sqrt{x+3}$, $\mathrm{f}(x) = \frac{1}{\sqrt{2x-5}}$ The webpage www.intmath.com/functions-and-graphs/2a-domain-and-range.php has examples and an interactive applet that demonstrate identifying the domain and range for different functions**(I)** There are matching activities on the TES website **(I)(F)**: • ‘Functions, domain and range’ at www.tes.com/teaching-resource/functions-domain-and-range-6334518 which involves matching the graph of the function with the function shown, the domain and range (includes one modulus) • ‘Tarsia - Quadratic Inequalities 2’ at www.tes.com/teaching-resource/tarsia-quadratic-inequalities-2-6110158 Introduce the idea of composite functions by considering two separate functions, f and g, as rules for converting input into output, e.g. $\mathrm{f}(x) = 2x - 3,\ x \in \mathbb{R}$ and $\mathrm{g}(x) = 1 - 2x,\ x \in \mathbb{R}$. Ask learners to consider an input of $x = 2$, for example, and to apply the rule for f and then the rule for g to obtain the result. Repeat the process but with the rule for g applied first, followed by the rule for f. Introduce the notation used and emphasise the importance of the order. Go to www.mathwarehouse.com/sheets then search for ‘Composition of Functions’ to find a link to an interactive lesson showing the importance of the order of carrying out functions. There is another link to a worksheet of suitable questions. **(I)(F)** As you work through the syllabus, you may wish to return to this topic when considering trigonometric functions,

Learning objectives	Suggested teaching activities
	exponential functions and logarithmic functions.
• determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases, e.g. finding the inverse of $\mathrm{h}: x \mapsto (2x+3)^2 - 4$ for $x < -\frac{3}{2}$	Learners need to classify different types of functions. Start with some straight lines, quadratic curves and cubic curves. Go to www.mathwarehouse.com/sheets then search for 'One to one functions' to find a resource on how to identify one-one functions. There is also a worksheet called '1 to 1 function worksheet' for practice. **(I)(F)** This will lead conveniently to introducing the inverse of a one-one function. Start with some simple one-one functions and ask learners to state the inverse functions, e.g. $y = x + 2$, $y = 3x$, $y = x - 7$ and $y = \frac{x}{7}$ (all with a domain $x \in \mathbb{R}$). Move on to some more complicated one-one functions and see if learners can deduce a quick way of obtaining the inverses. Introduce the idea of restricting the domain of a function so that a many-one function becomes a one-one function over a certain domain. Examples of finding the inverse of different functions can be found at www.purplemath.com by searching for 'Finding the Inverse of a Function'. **(I)** Go to www.mathwarehouse.com/sheets then search for 'Inverse functions' to locate a resource on finding the inverse and a worksheet on inverse functions. Another worksheet ('Composite function and inverse function worksheets' by joezhou) is available from the TES website, at www.tes.com/teaching-resource/composite-function-and-inverse-function-worksheets-11633641 **(I)(F)**
• illustrate in graphical terms the relation between a one-one function and its inverse; sketches should include an indication of the mirror line $y = x$	Start with a straightforward function that learners could easily sketch e.g. $y = 2x + 1,\ x \in \mathbb{R}$. Ask them to work out the inverse of the function and also to sketch this. Then ask them to consider the result of $\mathrm{f} \circ \mathrm{f}^{-1}$ and $\mathrm{f}^{-1} \circ \mathrm{f}$. Repeat with a more complicated function e.g. $\mathrm{f}(x) = \sqrt{x-5},\ x \geq 5$. Ask learners to present their results and conclusions. The 'Inverse functions' resource at www.mathsisfun.com/sets/function-inverse.html has a demonstration of the important points which are followed by some questions.**(I)** Go to www.geogebra.org/m/BUuRknns for an interactive resource, 'Graphic Relationship of a Function and Inverse Function' which enables the relationship between a one-to-one function and its inverse to be explored. **(I)**
• understand and use the transformations of the graph of $y = \mathrm{f}(x)$ given by $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(ax)$ and simple combinations of these; including	Go to www.tes.com and search for 'Transformations of trigonometric graphs worksheet by vorrico', for a structured investigation using graphing software for sine and cosine transformations (can be extended to other transformations). **(I)** A card sort with single step transformations for a range of graphs is also available from the TES website – search for 'Transformation of Graphs by ianmckenzie'. This would be a good resource to support learners who have found the topic more difficult. **(I)(F)**

Learning objectives	Suggested teaching activities
use of the terms 'translation', 'reflection' and 'stretch' in describing transformations; questions may involve algebraic or trigonometric functions, or other graphs with given features	Geogebra has a number of interactive graphs which allow learners to explore these transformations: • Go to www.geogebra.org/m/HJvZSUna for an interactive resource that allows learners to explore these transformations and combinations of them. (Or from www.geogebra.org, select 'Materials' and search for 'function transformations by tim brzezinski'.) There are four different graphs available to transform. **(I)** • Go to www.geogebra.org/m/Hknxbnjb for an interactive resource which allows exploration of transforming the sine graph. (Or search for 'sine graph transformations by jackgladas'.) **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)**

9709 Mathematics 2020 Specimen Paper 1, Question 5
9709 Mathematics 2020 Specimen Paper 1, Question 2
9709 Mathematics 2020 Specimen Paper 1, Question 11(a)

1.3 Coordinate geometry

Learning objectives	Suggested teaching activities
• find the equation of a straight line given sufficient information e.g. given two points, or one point and the gradient	Learners will have met the equation of a straight line before, so again this should be a revision exercise. The ‘Equation of a straight line’ resource at www.mathsisfun.com/equation_of_line.html has examples of equations of a straight line and graphs illustrating them. There is an interactive part which allows learners to explore different gradients and intercepts. This is followed by a selection of questions for consolidation. **(I)** ‘Two-variable linear equations intro’ at www.khanacademy.org/math/algebra/two-var-linear-equations provides a series of videos and activities that cover writing the equation of a line from different pieces of information. Note: this website uses US terminology – ‘slope’ instead of ‘gradient’. **(I)**
• interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems; including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines	**Midpoint/length/gradient** This should be consolidation/revision of work done previously. For a quick review of this work, give learners the coordinates of the end points of different line segments and ask them to find the length, gradient and midpoint of each line segment. **(I)** Tasks available from https://undergroundmathematics.org are a rich task, ‘Distance between two points’ as well as ‘In-betweens’, which relates to finding gradient and points on lines. (Use the search facility, or navigate from the ‘Thinking about geometry’ station on the map.) **(I)** **Gradients of parallel and perpendicular lines** A good activity on the website https://undergroundmathematics.org to check prior knowledge of the equations of lines including parallel lines, perpendicular lines, and intercepts is the card sort activity ‘Lots of lines!’ **(I)(F)** Another excellent resource is ‘The equation of a straight line’ which looks at many different ways of finding the equation of a straight line. **(I)** (Use the search facility, or navigate from the ‘Geometry of equations’ station on the map.) For more able learners, you could demonstrate the proof that the product of the gradients of perpendicular lines is -1. Basic practice in working on parallel and perpendicular lines can be found on the CIMT website www.cimt.org.uk/projects/mepres/allgcse/allgcse.htm, (or from www.cimt.org.uk, select ‘Resources’, then ‘Mathematics Enhancement Programme (MEP)’, then ‘GCSE course material’) then look for ‘Additional Pupil Text Material for Units 13 and 14’ for several resources related to this topic area including problem-solving questions. **(I)** Learners should be able to use both forms of the straight line equation. A worksheet of mixed questions which can be used to consolidate the learning in this topic can be found at www.tes.com/teaching-resource/a-level-maths-c1-coordinate-

Learning objectives	Suggested teaching activities
	geometry-worksheet-6135231. (Or from www.tes.com, search for 'A level maths C1 coordinate geometry worksheet by phildb'.) **(I)(F)**
• understand that the equation $(x-a)^2+(y-b)^2=r^2$ represents the circle with centre (a, b) and radius r ; including use of the expanded form $x^2+y^2+2gx+2fy+c=0$	The 'Circle equations' resource at www.mathsisfun.com/algebra/circle-equations.html shows the different forms of the equation of a circle. The explanation is accompanied by some self-marking questions related to the topic. **(I)(F)** The resource ('Equation of a circle' by loopy188) at www.tes.com/teaching-resource/equation-of-a-circle-6317138 is a notebook that can be used to introduce the topic to a class.
• use algebraic methods to solve problems involving lines and circles; including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry; implicit differentiation is not included	A useful set of worksheets ('A level maths C1: Circles resources – worksheets' by SRWhitehouse) to summarise the key points relating to equations of circles and moving on to consider the tangent to a circle can be found at www.tes.com/teaching-resource/a-level-maths-c1-circles-resources-worksheets-6146676. **(I)** Many other useful resources for this topic can be found on the TES website. Go to www.tes.com and search for 'Circle equations' to find worksheets that can be used to test understanding and to extend: • 'Equations of circles' by LeonD06 at www.tes.com/teaching-resource/equations-of-circles-6422649 • 'Equations of Tangents to Circles' by mrsmorgan1 at www.tes.com/teaching-resource/equations-of-tangents-to-circles-11080079 to practice finding the equation of the tangent to a circle. **(I)(F)** **Extension activity:** circles thoughts and crosses at www.tes.com/teaching-resource/solving-equations-thoughts-and-crosses-11554989 **(I)(F)**
• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations, e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a	Most learners will already be familiar with points of intersection of straight lines. As a revision exercise, the webpage at www.mathopenref.com/coordintersection.html provides an interactive resource allowing you to change the equations of the straight line graphs. (Or to find this resource from www.mathopenref.com, in the 'Coordinate geometry' section select 'Points, lines and planes' and look for 'intersecting lines'.) **(I)** Give learners an example to investigate e.g. the intersection of the line $y = 4x - 9$ and the curve $y = 4x(x - 2)$. They could use this to explore the link between a line as a tangent to a curve, the repeated root of an equation and the discriminant. Other examples could be considered, for example: given that the line $y = mx - 3$ is a tangent to the curve $y = x^2 - 5x + 6$, find the value of m. **(I)**

Learning objectives	Suggested teaching activities
quadratic curve	The link www.geogebra.org/m/uJ2j9pBq has an interactive file which allows investigation of the intersection of a quadratic graph and a line. (Or from the Geogebra website, select 'Materials' and search for 'Simultaneous Equations Involving a Quadratic Equation by Andrew Shaw'.) **(I)**

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)** 9709 Mathematics 2020 Specimen Paper 1, Question 10 9709 Mathematics 2020 Specimen Paper 1, Question 1(a)

1.4 Circular measure

Syllabus content	Suggested teaching activities
• understand the definition of a radian, and use the relationship between radians and degrees	Ask learners to calculate the angle, in degrees, subtended by an arc, of length 1 unit, of a circle which has radius 1 unit. This will illustrate the definition of a radian and give learners an idea of the approximate size of a radian in degrees. You can then present more formally the relationship between radians and degrees in terms of π and learners can practise converting degrees to radians by using the resource at www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/we-converting-degrees-to-radians Searching for 'degrees to radians' on the site will also provide questions to practice, e.g. www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/e/degrees_to_radians **(I)**
• use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle; including calculation of lengths and angles in triangles and areas of triangles	To obtain the formula for length of arc, consider a circle of radius *r* and an arc of length *s* that subtends an angle θ radians at the centre of the circle of circumference $2\pi r$. Ask learners to find the length of the arc using the ratio of the angles, $\frac{\theta}{2\pi}$, and to derive the formula for the sector area of a circle using a similar method. The 'Circle Sector and Segment' resource at www.mathsisfun.com/geometry/circle-sector-segment.html is a tutorial and practice questions on arc lengths and areas of sector. Various different geometrical approaches may be used in questions. Learners need to practise as many questions as possible so that they become as familiar with radians as with degrees. You could also usefully remind them about the formula for the area of a triangle.

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)** 9709 Mathematics 2020 Specimen Paper 1, Question 9

1.5 Trigonometry

Learning objectives	Suggested teaching activities
• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians); including e.g. $y = 3\sin x$, $y = 1 - \cos 2x$, $y = \tan\left(x + \frac{1}{4}\pi\right)$	An interactive graph plotter is very helpful for showing different trigonometric curves. All of these are free resources: www.desmos.com/calculator (register to get free examples such as Trigonometry – unit circle), www.geogebra.org (free to download) http://rechneronline.de/function-graphs, http://graph-plotter.cours-de-math.eu Go to the resource 'Sine problem' at https://nrich.maths.org/436 to find an interesting pattern generated by transformations of sin *x*. The problem involves naming the functions used to generate the pattern and provides useful practice on visualising the graphs. **(I)** **Extension activity:** The 'Catching waves' task at https://undergroundmathematics.org relates to the link between sine and cosine and waveforms in the world around us. (Use the search facility, or navigate from the 'Trigonometry: triangles to functions' station on the map.) **(I)**
• use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles, e.g. $\cos 150° = -\frac{1}{2}\sqrt{3}$ $\sin\frac{3}{4}\pi = \frac{1}{\sqrt{2}}$	Most scientific calculators will now give exact values of sin, cos and tan of the given angles. Use an equilateral triangle of side 2 units to find the exact values of sin, cos and tan of 30° and 60°, and a right-angled isosceles triangle of sides 1, 1, $\sqrt{2}$ to find the exact values of sin, cos and tan of 45°. For the trigonometric ratios of angles in other quadrants, consider projections onto the *x*-axis as shown in the booklet 'Trigonometric ratios of an angle of any size' at www.mathcentre.ac.uk/resources/uploaded/mc-ty-trigratiosanysize-2009-1.pdf. Alternatively, you could deduce the results from the trigonometric graphs. It is a great advantage for learners to be able to understand and use the graphs, for example in solving equations. The structured resource 'From stars to waves' at https://undergroundmathematics.org supports this. (Use the search facility, or navigate from the 'Trigonometry: triangles to functions' station on the map.) **(I)**
• use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations; no	Learners should already be familiar with the notation used, but it is worth reinforcing the principal values for each trigonometric ratio. An interesting activity is to ask learners to look for real-life applications of trigonometry and to create their own questions

Learning objectives	Suggested teaching activities
specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected	based on these contexts. This could involve questions on finding lengths (related to earlier sections) and finding angles.
• use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$, e.g. in proving identities, simplifying expressions and solving equations	Introduce the identities using a right-angled triangle. Alternatively, a structured worksheet and matching activity (which also involves consideration of transformations of trigonometric graphs) can be found at www.tes.com/teaching-resource/a-level-maths-trigonometry-identities-worksheets-6146808. (Or from www.tes.com search for 'A level Maths: Trigonometry Identities worksheets' and look for the resource by SRWhitehouse.) **(I)** Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Jun 2015 Paper 11 Q1 (See also questions listed below.)
• find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included), e.g. solve 3 sin 2x + 1 = 0 for $-\pi < x < \pi$, 3 $\sin^2\theta$ – 5 $\cos\theta$ – 1 =0 for $0^\circ \le \theta \le 360^\circ$.	To obtain all the solutions for a given equation, encourage learners to use the graphs of the trigonometric functions. A good start point for this is to ask learners to sketch the sine graph from -360° to +360° and ask them to mark all of the points where sin θ = 0.7 and then to try to identify the corresponding angles. This can be extended to other trigonometric graphs with other ranges. **(I)** Once learners are confident with these examples then extend to examples such as 3 sin 2x + 1 = 0 for $-\pi < x < \pi$, 3 $\sin^2\theta$ – 5 $\cos\theta$ – 1 = 0 for $0^\circ \le \theta \le 360^\circ$. **(I)** The pdf file 'Trigonometric equations' at www.mathcentre.ac.uk/resources/uploaded/mc-ty-trigeqn-2009-1.pdf takes learners through examples of these types of question and provides some practice questions. (Note: one of the trigonometric identities used in this document is beyond what is required for Pure 1). **(I)** Another approach is to make use of the unit circle. Resources to support this can be found on the TES website. The resource 'CAST diagram for solving trigonometric equations' at www.tes.com/teaching-resource/cast-diagram-for-solving-trigonometric-equations-6332281 has a series of worked examples and some practice questions. **(I)** The resource 'Core 2 – Trigonometry. Powerpoint lesson' at www.tes.com/teaching-resource/core-2-trigonometry-powerpoint-lesson-6030080 is a presentation covering converting between degrees and radians, use of radians in finding arc length and area of sector, finding exact values of the sine, cosine and tangent of 30°, 45°, 60°. It then progresses to look at finding all solutions to trigonometrical equations using sketch graphs and CAST diagrams. These resources provide examples, and most A Level maths textbooks will have many questions. Set learners a mixture of

Learning objectives	Suggested teaching activities
	questions in radians and degrees and remind them to check that their calculator is in the correct mode. Many exam questions involve a proof or 'show that' exercise, the result of which is then used in solving a trigonometric equation. Most of the questions listed below are of this type.

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)** 9709 Mathematics 2020 Specimen Paper 1, Question 7

1.6 Series

Learning objectives	Suggested teaching activities
• use the expansion of $(a + b)^n$, where n is a positive integer; including the notations $\binom{n}{r}$ and n!; knowledge of the greatest term and properties of the coefficients are not required	Start by asking learners to expand $(a+b)^2$, $(a+b)^3$ and $(a+b)^4$, setting out their work logically with powers of *a* decreasing and powers of *b* increasing, and to spot the pattern from Pascal's triangle. (Learners could do this in groups or individually.) They can then deduce the results for $(a+b)^5$ and $(a+b)^6$ without having to work through any expansions. **(I)** Using the same pattern and Pascal's triangle coefficients, learners should be able to expand an expression such as $(2x-3y)^4$. A very common error is to write $(2x)^4$ as $2x^4$ rather than $16x^4$. Most textbooks will provide many examples for learners to practise. **(I)** The resource 'Binomial theorem' at www.mathsisfun.com/algebra/binomial-theorem.html shows the above process and relates it to the binomial theorem showing correct notation. A useful PowerPoint demonstration of Pascal's triangle and the formula for the binomial expansion using combinations is available at www.tes.com/teaching-resource/binomial-expansion-powerpoint-6071493. (Or from www.tes.com search for 'Binomial Expansion Powerpoint by Nerys.packwood'.) Alternatively, introduce learners to the formula and explain the notation: $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + ... b^n$ where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$. Show learners that their calculator will work out these coefficients using the ${}^nC_r$ button and draw their attention to the patterns formed by these and the powers of *a* and *b*. If you prefer, you could consider combinations of terms to derive the result more formally. Most A Level textbooks have practice exercises on using the formula both to expand fully and to identify specific terms. Further development includes using an expansion to solve a problem such as: • Find the term in x^2 in the expansion of $(2+x)(3+4x)^5$ • Find the term independent of x in the expansion of $\left(2x^3 - \frac{1}{x}\right)^8$ Using such questions, you can encourage learners to consider the patterns formed when they expand binomial expressions.

Learning objectives	Suggested teaching activities
	A set of worksheets to practise these types of expansion is available at www.tes.com/teaching-resource/a-level-maths-c2-binomial-expansion-worksheets-6146793 (Or from www.tes.com search for 'A Level Maths C2: Binomial Expansion worksheets', and select the resource by SRWhitehouse. **(I)** **Extension activity:** There are some questions on https://undergroundmathematics.org which are aimed at efficiently determining terms in a binomial expansion (main content) but also includes use of binomial expansions to make estimates of calculations. (Select the 'Counting and Binomials' station and look for the Review questions aimed at their Key question 3. **(I)**
• recognise arithmetic and geometric progressions	Start by giving learners a selection of sequences and asking them to identify any patterns. There are many examples in textbooks and on the internet (try searching for 'sequences + worksheet'). Help them to identify the main features of arithmetic and geometric progressions. It is a good idea to explain the meaning of the terms 'sequence', 'series' and 'progression'.
• use the formulae for the *n*th term and for the sum of the first *n* terms to solve problems involving arithmetic or geometric progressions; including knowledge that numbers *a*, *b*, *c* are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent); questions may involve more than one progression	For arithmetic progressions, introduce the notation that is commonly used for the first term, the common difference and the sum of *n* terms. The resource 'Arithmetic Sequences and Sums' at www.mathsisfun.com/algebra/sequences-sums-arithmetic.html is a good example of the approach you could take with the whole class or with individuals wanting extra practice or revision. You can encourage learners to work out the formula for the *n*th term and then to derive the formula for the sum of *n* terms. **(I)** **Extension activity:** On the website https://undergroundmathematics.org select 'Sequences' and then the task 'Change one thing' or 'Connect three?' for interesting tasks relating to arithmetic progressions. **(I)** For geometric progressions, introduce the notation that is commonly used for the first term, the common ratio and the sum of *n* terms. Encourage learners to work out the formula for the *n*th term themselves. You could give them a hint and encourage them to work out the formula for the sum of a geometric progression. The resource 'Geometric Sequences and Sums' at www.mathsisfun.com/algebra/sequences-sums-geometric.html is a good example of the approach that you could use, together with some examples. You could use this with the whole class or with individual learners wanting extra practice, revision or consolidation. **(I)** **Extension activity:** On the website https://undergroundmathematics.org select 'Sequences' and then the task 'Sort it out' for an interesting activity which gets learners to think about the properties of sequences. There are different possible groupings for the sequences depending upon the classifications used (see possible groupings). One option is to ask learners to group the sequences, another would be to present the different suggested groupings and ask learners to identify the method of grouping (could be used once learners have met the concepts of convergent and divergent). **(I)**

Learning objectives	Suggested teaching activities
	Extension activity: On the website https://undergroundmathematics.org select 'Sequences'. The 'Common terms' problem gets learners to consider the properties of arithmetic and geometric progressions. **(I)** Show learners that there are two possible versions of the formula for the sum of *n* terms of a geometric progression; the value of the common ratio can make one version easier to work with than the other. To encourage independent learning, give learners the link to https://nrich.maths.org and ask them to search for 'Summing Geometric Progressions'. **(I)** The website www.khanacademy.org/math has some worded problems related to geometric series. Search for 'Finite geometric series word problems'. **(I)** Most textbooks will have exercises for learners to practise using all the formulae, together with general problems. **(I)**
• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression	Start by considering a simple convergent geometric progression e.g. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots..$ Find the sum of *n* terms using the formula, i.e. $S_n = \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}$ which leads to $S_n = 2\left(1-\left(\frac{1}{2}\right)^n\right)$. Illustrate this with two identical pieces of coloured paper. Stick one piece on the board (area 1), then cut the other piece in half and stick one piece on the board (area 1/2), cut the remaining piece in half and stick it on the board (area 1/4), and so on. By positioning the pieces carefully, learners can see that the sum can never be greater than 2. You can then point out the similarity to $n \to \infty$ and hence the sum to infinity, introducing the common notation used**.** As a group exercise, ask learners to see if they can deduce the general formula for the sum to infinity, together with any restrictions on the value of the common ratio. **Extension activity:** On the website https://undergroundmathematics.org there is an interesting video and explanation relating to Achilles and the tortoise and linked to summing an infinite geometric series. Select 'Sequences' and then 'Achilles and the tortoise'. **(I)** Most textbooks will have practice examples. The website https://undergroundmathematics.org also has a range of Review questions which cover the different objectives within this unit. Select 'Sequences'. **(I)(F)**

Past and specimen papers
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)** 9709 Mathematics 2020 Specimen Paper 1, Question 3 (arithmetic) 9709 Mathematics 2020 Specimen Paper 1, Question 6

1.7 Differentiation

Learning objectives	Suggested teaching activities
• understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for first and second derivatives; only an informal understanding of the idea of a limit is expected; e.g. includes consideration of the gradient of the chord joining the points with *x* coordinates 2 and (2 + *h*) on the curve $y = x^3$; formal use of the general method of differentiation from first principles is not required.	The website https://undergroundmathematics.org has some interesting activities which can be used to introduce this topic – select 'Introducing Calculus' and look for the scaffolded task 'Zooming in' and the task 'A tangent is…'. **(I)** A motivation for the topic can be found in 'Why are gradients important in the real world?' Many of the other resources on this website would also be suitable for use in this topic. **Extension activity:** Differentiation from first principles is not a syllabus requirement, however, learners may find it useful to learn about this, particularly if they are planning to study mathematics at a higher level. A useful resource to support this is 'Differentiation from first principles' at wwwf.imperial.ac.uk/metric/metric_public/differentiation.**(I)** Another resource that could be used for this are 'Differentiation from first principles' by SRWhitehouse at www.tes.com/teaching-resource/differentiation-from-first-principles-6147220. **(I)**
• use the derivative of x^n (for any rational *n*), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule, e.g. find $\frac{dy}{dx}$ given $y = \sqrt{2x^3 + 5}$	The website https://undergroundmathematics.org has a task that provides a structured investigation enabling learners to identify the derivative of x^n (select 'Calculus of Powers' then the task 'Gradient spotting'). **(I)** A set of worksheets which allow learners to practise basic differentiation are available at www.tes.com/teaching-resource/a-level-maths-c1-differentiation-worksheets-6146718. (Or from www.tes.com, search for 'A level maths differentiation worksheet' and look for the resource pack by SRWhitehouse.) **(I)** This set of resources also contains a dominoes game which can be used to check learning. **(I)(F)** First make sure that learners are confident in the differentiation of constant multiples, sums and differences before introducing the chain rule for differentiating a composite function (function of a function). Textbooks will have many exercises for learners to practise. **(I)**

Learning objectives	Suggested teaching activities
	A useful resource 'Chain rule' by SRWhitehouse is available at www.tes.com/teaching-resource/chain-rule-6146849 (or from www.tes.com use the search facility). **(I)** **Extension activity:** On the website https://undergroundmathematics.org, select 'Calculus of Powers' and then the resource 'Binomials are the answer!' for a structured investigation leading to the general result for the gradient of the curve $y = x^n$ which can be extended to constant multiples, sums etc. **(I)**
• apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change; including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.	Remind learners that the gradient function will give the gradient of a **tangent** to the curve at a particular point, so differentiation will be the first step in finding the equation of the tangent. The resources in the previous section may be useful if a recap is needed. Work through a straightforward example on the board or give it out as a group exercise, e.g. Find the equation of the tangent to curve $y = 2x^3 - 4x^2 + 5x - 7$ at the point where $x = 1$. The activity could be extended to finding the equation of the normal at that point and some more examples of increasing difficulty. Learners should be able to progress to textbook examples of different types. **(I)** The website www.tes.com has two resources that can be used to support this: • www.tes.com/teaching-resource/find-the-tangent-normal-matching-cards-commentary-6162092 is a card matching activity which requires learners to order the steps (from www.tes.com search for 'Find the Tangent/Normal Matching Cards Commentary by maidofsteel') • www.tes.com/teaching-resource/a-level-maths-c1-tangents-and-normals-worksheet-6146716 worksheets including structured questions (from www.tes.com search for 'A level Maths C1: Tangents and Normals worksheet' and look for the resource by SRWhitehouse). **(I)(F)** An interactive activity 'Tangents and normal line challenge' to practise the tangents and normals aspect with multiple choice, self-marking questions, is at www.khanacademy.org/math. **(I)(F)** Further questions on finding, and then using, the equations of tangents and normals are available at www.tes.com/teaching-resource/maths-worksheets-tangents-and-normals-6139694 (or from www.tes.com search for 'Maths Worksheets: Tangents and Normals by phildb'). **(I)** Define increasing and decreasing functions by using the example of a cubic function and asking learners to imagine walking along it from left to right. (You could lay a piece of rope or heavy string on the floor in the shape of a cubic curve. Ask one learner to walk slowly along it while the rest of the class describes the gradient at each stage.) A general worksheet and matching game is at www.tes.com/teaching-resource/a-level-maths-c1-worksheet-function-

Learning objectives	Suggested teaching activities
	turning-point-6146765 (or from www.tes.com search for 'A level Maths C1: worksheet Function turning point' and look for the resource by SRWhitehouse). **(I)(F)** The website https://undergroundmathematics.org has a section of resources that are relevant to this section of work. (Select 'Calculus meets Functions' on the map). The resources available include investigations, questions which deepen understanding and review questions. Use the list of key questions at the top of the page to identify which aspects are covered by each resource. **(I)(F)** Introduce rates of change by reminding learners that $\frac{\mathrm{d}y}{\mathrm{d}x}$ represents the rate of change of y with respect to x. Introduce different derivatives e.g. $\frac{\mathrm{d}A}{\mathrm{d}t}$, so this must be the rate of change of A with respect to t. Then introduce the idea that t could represent time. Remind learners of the chain rule from the previous section and show how they can form equations with it, e.g. $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$. Introduce the idea that A could represent the area of a circle of radius r (you could give the example of a circular ink stain) so, if they know the rate of change of the radius with respect to time, they can calculate the corresponding rate of change of the area. Most textbooks will have exercises that learners may use for practice. **(I)(F)** **Extension activity:** The website https://undergroundmathematics.org has some interesting activities that can be used to extend more confident learners. In the section 'Introducing Calculus' the tasks 'Walk-sorting' and 'Gradient match' ask learners to match the graph of a function to the graph of its gradient function. **(I)(F)**
• locate stationary points and determine their nature, and use information about stationary points in sketching graphs; including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified; knowledge	The link www.nuffieldfoundation.org/fsmqs/level-3-calculus provides several resources. In the section 'Maxima and minima' there is a worksheet which you could use to introduce stationary points. The resources in the section 'Stationary points' cover maxima, minima and points of inflexion (beyond P1) together with the use of the second derivative. The material on Points of inflexion could be omitted or used as **extension activity** material for the more able learner. The nature of the stationary points is also covered. There are worked examples and further examples for learners to try themselves. **(I)** Most textbooks have plenty of examples on stationary points for learners to practise. **(I)(F)**

Learning objectives	Suggested teaching activities
of points of inflexion is not included.	Highlight the main features needed when sketching curves: find any intercepts with the coordinate axes, find any stationary points and determine their nature then attempt to sketch a curve. Questions are often structured so that learners identify the main features before they are asked to sketch the graph. A matching activity 'Floppy hair' requiring learners to determine the coordinates of maxima in order to match equations to curves can be found on the website https://undergroundmathematics.org. (Use the search facility, or navigate from the ''Calculus of Powers' station on the map.) **(I)(F)** Extension activity: On https://undergroundmathematics.org, select 'Introducing Calculus' on the map and look for the review question 'When is this parabola's turning point nearest to the origin?' **(I).**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)**

9709 Mathematics 2020 Specimen Paper 1, Question 12(a)(b)
9709 Mathematics 2020 Specimen Paper 1, Question 8(ii)
9709 Mathematics 2020 Specimen Paper 1, Question 8(i)

1.8 Integration

Syllabus content	Suggested teaching activities
• understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except –1), together with constant multiples, sums and differences, e.g. $\int(2x^3 - 5x + 1)\,dx$, $\int \frac{1}{(2x+3)^2}\,dx$	Start with an example of a gradient function e.g. $\frac{dy}{dx} = 3x^2 + 2$ and ask learners to deduce possible functions that could have this gradient. Then introduce the idea of including a constant of integration to cover an infinite number of possible solutions. Graphically, learners could visualise this as a family of curves; knowing the constant of integration will identify which curve they need. Graphing software may be helpful to show a family of curves. Starting with indefinite integration will encourage learners to include a constant of integration in their answers. Introduce notation here. Explain the importance of using it correctly; many learners forget to include ‘d*x*’ if they do not fully understand what it means. Integrate various examples of functions, leaving $(ax+b)^n$ until the end. Ask learners to work out an integral of this type by expanding it first, e.g. $\int(2x+3)^3\,dx$, then see if they can deduce a general formula for integrating this type of expression. They can check it works by differentiating again. Also ask them why this formula will work for any rational n except –1. **(I)** A set of presentations that supports most of the integration topic in Pure 1 is available at www.tes.com/teaching-resource/integration-powerpoint-6402321 (or from www.tes.com search for ‘integration powerpoint by rajuhussain’). The first lesson provides a good approach to the introduction of integration, the terminology and notation, and to the use of a constant of integration. The resource at www.tes.com/teaching-resource/indefinite-integration-6146782 (or from www.tes.com search for ‘Indefinite Integration’ and find the resource by SRWhitehouse) provides a short exercise for learners in the file ‘Introduction to Integration’, and ‘Indefinite integration dominoes’ is a useful assessment tool. **(I)(F)**
• solve problems involving the evaluation of a constant of integration, e.g. to find the equation of the curve through (1, –2) for which $\frac{dy}{dx} = \sqrt{2x+1}$	Point out to learners that they are now going to be evaluating the constant of integration. The two links to the TES website in the previous section contain useful resources for this topic. At www.tes.com/teaching-resource/indefinite-integration-6146782 ‘Finding the Arbitrary Constant’ and ‘Indefinite Integration Problems’ provide good resources for learners to use in or out of the classroom. ‘Indefinite Integration Dominoes’ provides a group activity which you could use to check learning either at the start or the end of a lesson. **(I)(F)** Finding the equation of a curve from information on its tangent and normal is investigated in the ‘Tangent or normal’ task to https://undergroundmathematics.org. (Use the search facility, or navigate from ‘Calculus of Powers’ on the map.) **(I)(F)** Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**:

Syllabus content	Suggested teaching activities
• evaluate definite integrals; including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}}\,dx$ and $\int_1^{\infty} x^{-2}\,dx$	Introduce learners to the notation used for definite integration and demonstrate how to evaluate an integral using limits and square brackets. You could introduce the area of a region enclosed by a curve, the x-axis and lines $x = a$ and $x = b$ at this point. There is a range of files at www.geogebra.org that may be useful (select 'Materials' and search for 'definite integrals'). The presentation at www.tes.com/teaching-resource/integration-powerpoint-6402321 mentioned in the previous two sections is useful here. Part of lesson 3 provides a quick overview on evaluating definite integrals. The task 'Integral chasing' on the website https://undergroundmathematics.org is a really nice set of questions that combine definite integrals with algebra. (Use the search facility, or navigate from 'Calculus of Powers' on the map.) Some learners may be daunted by the idea of an infinite limit, so it will be useful to work through a couple of examples, such as $\int_1^{\infty} x^{-2}\,dx$. Looking at the graph may also help them. The 'A level Maths C2: Definite Integration worksheet' resource at www.tes.com/teaching-resource/a-level-maths-c2-definite-integration-worksheet-6146778 (or go to www.tes.com and use the search facility) provides useful practice here. The file 'Definite Integration Intro' provides good examples of the notation used and evaluation of definite integrals. The file 'Definite Integration Dominoes' provides a group activity which you could use to check learning either at the start or the end of a lesson. **(I)(F)** **Extension activity:** The activity 'Stretch the function' from the website https://undergroundmathematics.org uses the idea of stretching a function to allow learners to practise evaluating definite integrals and to explore the difference between the definite integral and the area under a curve with the functions being chosen to highlight issues. (Use the search facility, or navigate from 'Calculus of Powers' on the map.) **(I)**
• use definite integration to find: – the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves – a volume of revolution about one of the axes; a volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and	You could introduce this topic by considering the area under a curve divided into a series of strips. There are various approaches to this e.g. looking at (1) the area of a set of strips containing the area required and (2) the area of a set of strips which is part of the area required. The value of the area required must therefore lie between the values of areas (1) and (2). Make a comparison with the actual value obtained by integration. There are lots of files available at www.geogebra.org which would support this (select 'Materials' and search for 'area under a curve'). Alternatively, demonstrate a formal proof to those learners who would appreciate it, or go directly to the statement that the area of the region enclosed by a curve, the x-axis and lines $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$, where $y = f(x)$ is the equation of the curve. One possible approach can be found in the 'Definite Integrals' resource at www.mathsisfun.com/calculus/integration-definite.html. Make sure you include an example of a curve that passes below the x-axis so that learners realise the effect it has on the

Syllabus content	Suggested teaching activities
$y = 5$ rotated about the x-axis.	value of their integral e.g. the area enclosed by the curve $y=(x-3)^2(x+1)$ and the x-axis. A useful resource is the 'A level Maths C2: Definite Integration worksheet' by SRWhitehouse at www.tes.com/teaching-resource/a-level-maths-c2-definite-integration-worksheet-6146778. The files 'Using Integration to Find Areas' and 'Definite Integration to find Areas' provide a good approach to the introduction of the topic together with examples which may be used as practice. **(I)** Next, introduce finding the area between two curves. Some of the files available at www.geogebra.org may be helpful to illustrate this (select 'Materials' and search for 'area between two curves'). To find the area enclosed by two curves, show learners that they can integrate the difference between the two functions $\int_a^b (y_1 - y_2)\,dx$ where $x = a$ and $x = b$ are the x-coordinates of the points of intersection of the two curves. A video 'Area between curves' to illustrate this is available at www.khanacademy.org/math. **Extension activity:** The task 'Meaningful areas' at https://undergroundmathematics.org gets learners to think in more depth about finding the area between two curves. (Use the search facility, or navigate from the 'Calculus of Powers' station.) **(I)** For volumes of revolution, use an approach like the one shown in the video 'disk method around x axis' from www.khanacademy.org/math. It involves splitting the area under the curve into strips which, when rotated, form discs. This leads to the formula for rotation about the ***x*-axis**. Challenge learners to deduce the formula required when the area is rotated about the ***y*-axis** instead. There are also lots of helpful animations to illustrate volume of revolution available at www.geogebra.org (select 'Materials' and search for 'volume of revolution'). The resource 'Volume of Revolution' by SurriyaMughal at www.tes.com/teaching-resource/c4-integration-volume-of-revolution-6340006 provides a PowerPoint presentation on the disc method, together with a worksheet of examples which learners can use for practice (or go to www.tes.com and use the search facility). **(I)**

Past and specimen papers

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(F)**

9709 Mathematics 2020 Specimen Paper 1, Question 12(c)
9709 Mathematics 2020 Specimen Paper 1, Question 4

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