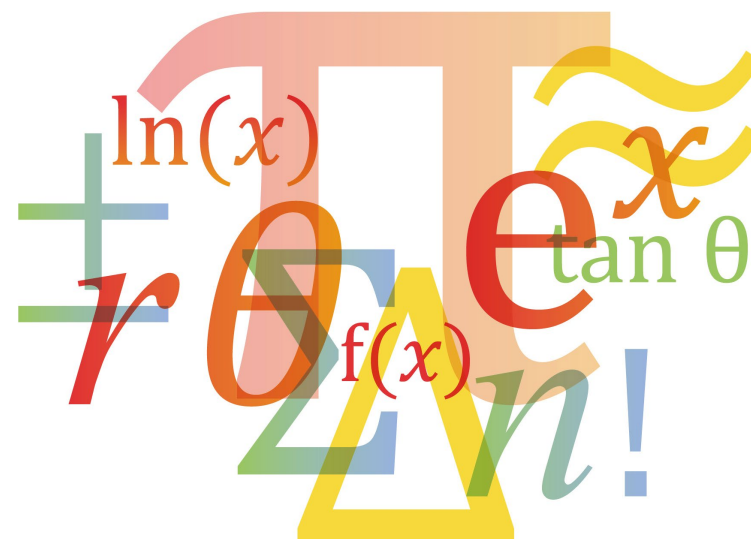# Scheme of Work – Paper 4

Cambridge International AS & A Level
Mathematics 9709
Mechanics

For examination from 2020

Contents

Introduction

The Cambridge International AS & A Level Mathematics 9709 scheme of work has been designed to support you in your teaching and lesson planning. The Scheme of Work has been separated into six documents, one for each content section: Pure Mathematics 1, Pure Mathematics 2, Pure Mathematics 3, Mechanics, Probability & Statistics 1 and Probability & Statistics 2. This document relates only to **Mechanics**.

Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take and you should always check the syllabus for the content of your course.

Suggestions for independent study **(I)** and formative assessment **(F)** are also included. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Key concepts

This scheme of work is underpinned by the assumption that mathematics involves the application of logical methodologies, problem solving and the recognition of patterns as well as the application of these approaches to mathematical modelling. The key concepts are highlighted as a separate item in the new syllabus and you should be aware that learners will be assessed on their direct knowledge and understanding of the same. Learners should be able to describe and explain the key concepts as well as demonstrate their ability to apply them to novel situations and evaluate them. The key concepts for Cambridge International AS & A Level Mathematics are:

Key Concept – Problem solving

Key Concept – Communication

Key Concept – Mathematical modelling

See the syllabus for detailed descriptions of each Key Concept.

Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 180 hours for Cambridge International AS Level, and 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below give some guidance about how many hours are recommended for each topic.

Topic	Suggested teaching time (hours)	Suggested teaching order
4.1 Forces and equilibrium	It is recommended that this should take about 20 hours.	1
4.2 Kinematics of motion in a straight line	It is recommended that this should take about 12 hours.	2
4.3 Momentum	It is recommended that this should take about 6 hours.	3
4.4 Newton's laws of motion	It is recommended that this should take about 10 hours.	4
4.5 Energy, work and power	It is recommended that this should take about 12 hours.	5

Prior knowledge

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90° - \theta) \equiv \cos\theta \ , \ \cos(90° - \theta) \equiv \sin\theta \ , \ \tan\theta \equiv \frac{\sin\theta}{\cos\theta} \ , \ \sin^2\theta + \cos^2\theta \equiv 1$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

Resources

You can find the endorsed resources to support Cambridge International AS & A Level Mathematics on the Published resources tab of the syllabus page on our public website here.

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. All textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. In addition to reading the syllabus, teachers should refer to the specimen assessment materials.

School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge International AS & A Level Mathematics 9709 syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Learning objectives help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Learning objectives	Suggested teaching activities
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an extensible string and thrust in a connecting rod; if any other forces	It is essential that learners are able to identify the direction of motion and understand that forces resolved perpendicular to the motion must be balanced, while the resultant force parallel to the motion is what will cause the acceleration. A good description of all three of Newton's laws of motion, presented in an interesting way and w[ith] questions in quiz form is at: www.khanacademy.org/science/physics/forces-newtons-laws/newton[s] **Extension activity:** Interested and more able learners might find the video at: http://mathcentre.ac.uk/students/topics/mechanics/newton worth watching, although it deals with dimensions, such as orbital motion, and therefore goes beyond the scope of this scheme of work[.] There is good coverage of this topic at www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.p[df] exercises for learners, perhaps for independent study. **(I)** Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Of the past paper questions that explore this objective, many also require other objectives, for ex[ample] mass and weight, or motion on an inclined plane. Here are some which focus largely on horizontal motion[.] [...]8 Q1; Paper 43 Q6; Nov [...]

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Past papers, specimen papers and **mark schemes** are available for you to download at: www.cambridgeinternational.org/support

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Formative assessment (F) is ongoing assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

4.1 Forces and equilibrium

Learning objectives	Suggested teaching activities
• identify the forces acting in a given situation, e.g. by drawing a force diagram.	The idea of forces should already be familiar to learners, but you should discuss them with the class as much as time allows. For example, start with a brainstorm eliciting as many named forces as possible and types of situations in which they might be present, and allowing the opportunity for learners' misconceptions to be brought out and discussed. Where possible, practical activities are also helpful, reinforcing the idea that the study of Mechanics is modelling real, physical situations. For example, use an air track and slider pulled by a mass over a pulley. It is particularly helpful if you can vary the angle of elevation of the pulling force. If an air track is not available, any mass on different types of surface is equally useful, e.g. books pulled by string up a sloping desk. You can use this demonstration to bring out ideas about equilibrium and motion due to unbalanced forces, as well as normal reactions and the variable nature of frictional forces. Learners should be able to analyse situations which involve the following forces: weight, tension, friction, normal reaction, air resistance (make sure they are clear that this will often be neglected in mathematical models that involve particles) and the driving force (e.g. of an engine). Provide learners with a variety of situations or diagrams and ask them to indicate the forces and their directions.
• understand the vector nature of force, and find and use components and resultants; calculations are always required, not approximate solutions by scale drawing.	(The approach to this will depend on whether or not Pure Mathematics 3.7 'Vectors 'has been covered already.) Divide the class into groups then set up a mass on a string passing over a pulley for each group. Learners hold this in place with a Newton meter attached to the free end of the string. They measure the size of the single force required to hold the mass stationary. Then they replace the single supporting Newton meter with two positioned at differing angles. Learners take readings from both meters and also measure the angles involved. They repeat this as many times as possible, allowing them time to investigate any connections they can find between the two forces and the original single force. Encourage learners to represent this geometrically by drawing accurate vector representations of the forces for each set of measurements. Starting with the angle between the pairs of forces as a right angle is useful, particularly for the less mathematically confident learners. Ask for feedback from each group to see what they have found and what ideas they have come up with. This should lead

Learning objectives	Suggested teaching activities
	naturally to the idea that a single force can be replicated by two or more forces (the idea of components). Or, conversely, that two or more forces can be represented by their overall effect (the idea of a resultant). Show learners how to resolve a single force into two perpendicular components $F\cos\theta$ and $F\sin\theta$ and also how to find the resultant of two perpendicular forces. Provide plenty of these for learners to practise, either as an exercise for individual learners to do in class or independently later as this will be an essential skill throughout both Mechanics modules. Most textbooks will contain many useful examples. **(I)**
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero; solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.	Cover both approaches here: (1) the triangle of forces and (2) finding the sum of the components of all the forces in a chosen direction. Learners who are less confident mathematically will probably find the second approach more accessible as the first one is likely to involve the use of the sine and cosine rules. Introduce the triangle of forces gradually, after covering the technique in three force situations where two of the forces are perpendicular, making the trigonometry more straightforward. Provide examples where the given forces are already in equilibrium and ask learners to demonstrate that they are. Also provide examples where the forces are not in equilibrium and ask learners to find the magnitude and direction of the force that is needed for equilibrium. The resource at http://tap.iop.org/mechanics/static/202/page_46254.html has good ideas for practical activities, as well as group discussion ideas and individual questions for learners.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component.	Placing simple objects on a horizontal surface makes a good starting point for class discussion about a contact force balancing the weight of an object. Ask learners to draw a diagram showing the forces and their directions. Repeat this with the surface at different angles of elevation. Explain that the contact force can be represented by two perpendicular components, the normal reaction (ensure that the significance of 'normal' is clear) signifying a 'push' by the surface, and the frictional force resisting movement parallel to the surface.
• use the model of a 'smooth' contact, and understand the limitations of this model.	Explain the significance of the word 'smooth' in modelling situations: it indicates that you can ignore the frictional component of the contact force that resists motion parallel to the surface. Make sure that learners understand the significance of the direction of the contact force.

Learning objectives	Suggested teaching activities
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate; terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.	Discuss why the object does not slide until a 'critical' angle is achieved. Learners investigate this, with different objects and/or different surfaces. A similar investigation includes just a horizontal surface and a measurable pulling force, to see what size force is required to move the object. www.examsolutions.net provide useful video tutorials summarising most of the ideas about friction in mechanics, for example 'What is friction, limiting equilibrium and the coefficient of friction?'
• use Newton's third law, e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.	Give learners a variety of situations in which they identify a force. Define Newton's third law and ask for the equal and opposite force to each one that learners have identified. Some typical cases to consider (or use these after covering section 4.4 on Newton's Laws of Motion): (1) the resultant force experienced by a pulley when two connected masses are hung over it (2) a mass in a lift/elevator supported or raised by a cable (3) a 'towing' situation e.g. car and trailer/caravan **Extension activity:** Adapt case (1) above to create a problem involving strings at different angles. Good notes and a few examples and questions are at: www.mathcentre.ac.uk/resources/uploaded/mc-web-mech2-11-2009.pdf

4.2 Kinematics of motion in a straight line

Learning objectives	Suggested teaching activities
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities; restricted to motion in one dimension only; the term 'deceleration' may sometimes be used in the context of decreasing speed.	Reiterate the difference between scalar and vector quantities. In one dimension, the direction of a vector will determine whether you use a positive or negative sign. Give a few simple examples. Stress that negative speed or distance is incorrect. Also make sure that learners understand that a negative acceleration is equivalent to a positive deceleration.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that: - the area under a velocity–time graph represents displacement - the gradient of a displacement–time graph represents velocity - the gradient of a velocity-time graph represents acceleration.	A summary of the key points for the two types of graphs, with a few questions, is at: www.mathcentre.ac.uk/resources/uploaded/mc-web-mech1-10-2009.pdf A more complete discussion, with extensive examples and questions, is at: www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf (This includes the calculus aspect and the equations of motion for constant acceleration, as well as a treatment of the forces involved in changing the motion of an object.)
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration; calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.	The document mentioned above (www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf) has a good explanation of the use of differentiation and integration in the context of displacement, velocity and acceleration. It is worth spending time on examples that require learners to find constants of integration.
• use appropriate formulae for	The document mentioned above (www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf), demonstrates the

Learning objectives	Suggested teaching activities
motion with constant acceleration in a straight line; questions may involve setting up more than one equation, using information about the motion of different particles.	derivation and use of the formulae, as well as providing questions for learners to try for themselves. **Extension activity:** Challenge learners to derive the formulae, perhaps giving them a few hints. They could start from the basic definition of constant acceleration as the difference in velocities per second, or from their understanding of acceleration as the gradient of a velocity-time graph. A video with a straightforward example demonstrating very clearly the use of the formulae (sometimes known as 'suvat' equations, named after the five variables involved) is at: www.youtube.com/watch?v=jfOSQBB7Bhs. It is particularly important to make clear to learners that these formulae can only be used in cases where the acceleration is constant, not where acceleration varies with time. It is also important that learners have experienced, and discussed the relevance of, cases where the time has to be found from a quadratic equation, giving two solutions. Give learners a wide variety of questions requiring them to use the equations of motion, e.g. from textbooks and at the links above.

4.3 Momentum

Learning objectives	Suggested teaching activities
• use the definition of linear momentum and show understanding of its vector nature; for motion in one dimension only.	Introduce learners to the concept of linear momentum (**p**) as the product of the mass (m) and the velocity (**v**) of a particle. Emphasise that momentum is a vector quantity that acts in the direction of motion of the particle. Examples to illustrate this idea are kicking a football straight on (direct kick) and flicking a marble from behind; in both of these cases the particle moves in a straight line. When learners are secure with the definition of linear momentum, ask questions to test their understanding of the concept **(I)(F)** • If the velocity of a particle is doubled, what can we say about its momentum? • If the two particles with mass 2m and m have equal velocities, what can we say about their momenta? • If particle A has mass m and velocity **v** and particle B has mass 2m and velocity 2**v**, what can we say about their momenta? • If particle A has mass m, particle B has mass 2m and the two particles have equal momenta, what can we say about the velocities?
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies; including direct impact of two bodies where the bodies coalesce on impact; knowledge of impulse and the coefficient of restitution is not required.	As part of a flipped learning task ask learners to research conservation of linear momentum. Websites which may be helpful to learners doing this are www.khanacademy.org and www.s-cool.co.uk. Emphasise to learners that they do not need to look at examples in two dimensions. **(I)** If you prefer to introduce conservation of linear momentum as a whole class activity then these resources are also useful. To find the resources at www.khanacademy.org, search for 'Introduction to momentum' to find 'Introduction to momentum (video)', and 'What are momentum and impulse (article)', then search for 'conservation of momentum', to find 'What is conservation of momentum? (article)', 'Bouncing fruit collision (video)' and 'Momentum: ice skater throws ball (video)'. **(I)** From www.s-cool.co.uk learners should search for 'Principle of the Conservation of Momentum' and look for the page with that title. **(I)** Other useful video resources for introducing conservation of linear momentum are at: www.tes.com/resources/search. Search for 'Conservation of linear momentum' and look for the videos by M4thsVideos. The video 'Conservation of linear momentum (COLM)' demonstrates conservation of linear momentum and looks at example questions. **(I)**

Learning objectives	Suggested teaching activities
	The Centre for Innovation in Mathematics Teaching (CIMT) has a pdf chapter available which explains conservation of linear momentum, gives examples including one where the bodies coalesce and has a section of questions for practice: www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf. The relevant section is 2.8 (pages 44-47 as labelled in the document). **(I)** Many textbooks will also have suitable questions on this topic. **(I)**

4.4 Newton's laws of motion

Learning objectives	Suggested teaching activities
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an extensible string and thrust in a connecting rod; if any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.	It is essential that learners are able to identify the direction of motion and understand that forces resolved perpendicular to the motion must be balanced, while the resultant force parallel to the motion is what will cause the acceleration. A good description of all three of Newton's laws of motion, presented in an interesting way and with thoughtful questions in quiz form is at: www.khanacademy.org/science/physics/forces-newtons-laws/newtons-laws-of-motion **Extension activity:** Interested and more able learners might find the video at: http://mathcentre.ac.uk/students/topics/mechanics/newton worth watching, although it deals with motion in two dimensions, such as orbital motion, and therefore goes beyond the scope of this scheme of work. There is good coverage of this topic at www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf with useful exercises for learners, perhaps for independent study. **(I)**
• use the relationship between mass and weight; $W = mg$; in this component, questions are mainly numerical, and use of the approximate numerical value 10 (ms^{-2}) for g is expected.	Make sure that learners are familiar with the instruction on the front of the exam paper to use 10 ms^{-2} as the acceleration due to gravity. Many learners will come across other values, for example in their study of Physics. Some good ideas for practical work and discussions concerning acceleration due to gravity, and how to obtain the force of weight from a known mass are at: www.cimt.org.uk/projects/mepres/alevel/mechanics_ch2.pdf Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support This objective is covered extensively in questions listed under the vertical motion and motion on an inclined plane objective (see later).
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration; including, for example, motion of a particle on a	This short video clip shows clearly how to analyse the forces on an object moving on an inclined plane: www.youtube.com/watch?v=dA4BvYdw7Xg

Learning objectives	Suggested teaching activities
rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.	
• solve simple problems which may be modelled as the motion of connected particles, e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid towbar.	Discuss the significance of the modelling assumptions here: 'smooth' peg or pulley implies constant tension along the length of the string, and 'light inextensible string' implies that the connected particles have identical acceleration. Helpful notes, examples and questions are at: www.mathcentre.ac.uk/resources/uploaded/mc-web-mech2-12-2009.pdf **(I)** www.examsolutions.net provide a number of video tutorials on vertical strings over a smooth pulley, and inclined planes including when one is not vertical.

4.5 Energy, work and power

Learning objectives	Suggested teaching activities
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force; $W = Fd\cos\theta$; use of the scalar product is not required.	www.examsolutions.net provide useful video tutorials including: • a definition of work done by a force and a tutorial on how to use this definition to solve problems when the motion is horizontal • the same as above but for vertical motion; this also gives an introduction to gravitational potential energy • motion on an inclined plane.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae.	www.examsolutions.net provide useful video tutorials including: • a tutorial that defines gravitational potential energy and demonstrates how to calculate it • the same as above but for kinetic energy; tt also includes a derivation of change in kinetic energy from work done by a force.
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy; including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.	Make sure that learners are clear about which forces are treated as 'external' in each situation. A common error is to include the weight twice, first in calculating work done as 'weight x distance' and again as a change in potential energy. A complete treatment of the theory involved, with worked examples as well as questions for learners to try is at: www.cimt.org.uk/projects/mepres/alevel/mechanics_ch6.pdf **(I)** The video tutorial 'Conservation of energy / work energy principle' at: www.examsolutions.net/tutorials/conservation-of-energy-work-energy-principle is useful.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion; including calculation of (average) power as $\dfrac{\text{work done}}{\text{time taken}}$; $P = Fv$	A good explanation of the concept of power and the mathematical implications of the definition, as well as worked examples and questions for learners is at: www.cimt.org.uk/projects/mepres/alevel/mechanics_ch6.pdf **(I)** Encourage learners to think about ideas such as the maximum velocity for a given power with a given resistance. They need to be able to use the definition of power in the context of analysing forces. Make sure that learners are clear that the power relates to the **driving** force, not to any of the other 'external' forces involved.

Learning objectives	Suggested teaching activities
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	A good, clear force diagram is very important for these problems. Encourage learners to label the driving force as *P/v* wherever appropriate. You will find many useful tutorials at www.examsolutions.net including a video that defines the concept of power, develops its link with force and velocity, and works through an example of a vehicle on an inclined plane, and one that develops this to also consider acceleration.