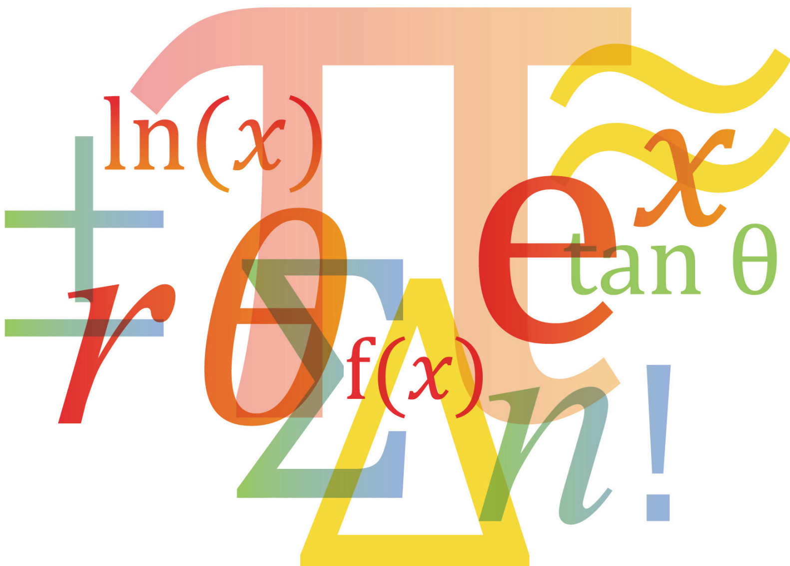
Cambridge Assessment International Education

Example Candidate Responses – Paper 6

Cambridge International AS & A Level Mathematics 9709

For examination from 2020



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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709 and to show how different levels of candidates' performance (high, middle and low) relate to the syllabus requirements.

In this booklet, candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

November 2020 Question Paper 62

November 2020 Paper 62 Mark Scheme

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

Example Candidate Response – high	Examiner comments
$1 - \left[\binom{150000}{0}\left(\frac{1}{50000}\right)^{0}(0.99998)^{150000} + \binom{150000}{1}\left(\frac{1}{50000}\right)^{1}(0.99998)^{149999}\right.$ $\left. + \binom{150000}{2}\left(\frac{1}{50000}\right)^{2}(0.99998)^{149998}\right]$ 1	1 The candidate uses the binomial distribution rather than a suitable approximating distribution.
$1 - [0.04978557476 + 0.1493597115 + 0.2240425545]$ $1 - 0.4231878407$ $= 0.5768121593$ $\approx \underline{0.577}$ 2	2 This is a correct calculation using the binomial distribution so one mark is awarded. **Mark awarded = 1 out of 3**

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

The candidate used the given binomial distribution rather than a suitable appr… obtained the correct answer of 0.577, they did not follow the method required by the question. It is important to read the question and to use a particular method if requested.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- Not using an approximating distribution or choosing an incorrect one (some candidates incorrectly chose a normal distribution).
- Using incorrect values for λ.
- Errors in interpreting the inequality required, either including extra terms in their expression or omitting terms.
- It is good practice to state the reason for choosing a Poisson distribution, even if the question does not ask for this.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high	Examiner comments
1 On average, 1 in 50 000 people have a certain gene. Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3] $X \to$ People with a certain gene $X \sim B(150\,000, 0.00002)$ ① $n > 50$ and $p < 0.1$ $\therefore$ Poisson approximation is used $X \sim Po(np)$ i.e $X \sim Po(3)$ ② $P(X > 2) = 1 - P(X \leq 2)$ $= 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!}\right)$ ③ $= 1 - 0.423190081$ $= 0.577$	① The candidate states clearly the given distribution. ② The candidate states the Poisson approximation correctly and uses the correct parameter, so a method mark is awarded. ③ The candidate writes the correct Poisson expression clearly. **Mark awarded = 3 out of 3**

How the candidate could have improved their answer

It would have been better to give a more accurate answer before rounding to three significant figures (i.e. an extra line in the working showing 0.5768099). This would have ensured the candidate could be awarded the accuracy mark even if they had made an error in rounding to three significant figures.

Example Candidate Response – middle	Examiner comments
1 On average, 1 in 50 000 people have a certain gene. Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3] $X \sim Bin\left(150000 \, ; \, \frac{1}{50000}\right)$ 1 $\cancel{X \sim Bin}$ $X \sim Po(np)$ $X \sim Po(3)$ 2 $P(X > 3) = 1 - P(x \leq 3)$ $= 1 - e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right)$ 3 $= 1 - 0.647231888$ $= 0.352768111$ $= 0.353$	1 The candidate states clearly the given distribution. 2 The candidate identifies the approximating distribution clearly and uses the correct parameter, so a method mark is awarded. 3 The Poisson expression is clear, but contains an extra, incorrect, term. A method mark is awarded but no further marks are available. **Mark awarded = 2 out of 3**

How the candidate could have improved their answer

- The candidate used the correct approximating distribution $Po(3)$. The question did not require justification for this but writing it down would be good practice.
- The candidate misinterpreted the requirements of the question. They needed to calculate the probability of more than two people, i.e. greater than or equal to 3 (not just greater than 3). This led to an incorrect extra term in their expression. It is important to read the question carefully to prevent such errors.

Example Candidate Response – low

1 On average, 1 in 50 000 people have a certain gene.

Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3]

$Po\left(\frac{1}{50000}\right)$

$P(X>2) = 1 - P(X \leq 2)$

$1 - \left[e^{-1/50000} \times \frac{\left(\frac{1}{50000}\right)^0}{0!} + e^{-1/50000} \times \frac{\left(\frac{1}{50000}\right)^1}{1!} + e^{-1/50000} \times \frac{\left(\frac{1}{50000}\right)^2}{2!}\right]$

$1 - [0.9999800002 + 1.99996 \times 10^{-5} + 1.99996 \times 10^{-10}]$

Binomial:

$B \sim X\left(150000, \frac{1}{50000}\right)$

$P(X>2) = 1 - P(X \leq 2)$

$1 - \left[\binom{150000}{0}\left(\frac{1}{50000}\right)^0 (0.99998)^{150000} + \binom{150000}{1}\left(\frac{1}{50000}\right)^1 (0.99998)^{149999} \right.$ 1

$\left. + \binom{150000}{2}\left(\frac{1}{50000}\right)^2 (0.99998)^{149998}\right]$

$1 - [0.04978557476 + 0.1493597115 + 0.2240425545]$

$1 - 0.4231878407$

$= 0.5768121593$

≈ 0.577 2

Examiner comments

1 The candidate uses the binomial distribution rather than a suitable approximating distribution.

2 This is a correct calculation using the binomial distribution so one mark is awarded.

Mark awarded = 1 out of 3

How the candidate could have improved their answer

The candidate used the given binomial distribution rather than a suitable approximating distribution. Although they obtained the correct answer of 0.577, they did not follow the method required by the question. It is important to read the question and to use a particular method if requested.

Common mistakes candidates made in this question

- Not using an approximating distribution or choosing an incorrect one (some candidates incorrectly chose a normal distribution).
- Using incorrect values for λ.
- Errors in interpreting the inequality required, either including extra terms in their expression or omitting terms.
- It is good practice to state the reason for choosing a Poisson distribution, even if the question does not ask for this.

Question 2

Example Candidate Response – high	Examiner comments
2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws. **(a)** Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3] $\frac{1-0.96}{2} = 0.02$ $1 - 0.02 = 0.98$ $\Phi^{-1}0.98 = z = \pm 2.054$ 1 $\cancel{CI}$ $p = \frac{56}{300}$ $CI = \frac{56}{300} \pm 2.054\sqrt{\frac{\frac{56}{300}\left(1-\frac{56}{300}\right)}{300}}$ 2 3 $\therefore$ CI is 0.140 to 0.233	1 This is a correct z value and is shown clearly. 2 The candidate substitutes correct values into an expression of the correct form. 3 The candidate gives the answer as an interval to 3 significant figures accuracy. The candidate states the correct probability. Mark for (a) = 3 out of 3

How the candidate could have improved their answer

(a) This was a very good answer. The candidate showed all steps in their working and clear calculations to find the z value. So that there is no ambiguity, it would have been better to neatly cross out the incorrect value in the denominator and replace it with the correct one.

Example Candidate Response – high	Examiner comments
2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws. **(b)** Maroulla claims that the die is biased. Use your answer to part (a) to comment on this claim. [1] ~~P=~~ 0.167 4 lies in the confidence interval therefore there is no evidence to suggest that the die is biased. 5	4 The candidate states the correct probability. 5 This is a clear statement, with good use of the phrase 'there is no evidence to suggest'. Mark for (b) = 1 out of 1

How the candidate could have improved their answer

(b) To improve clarity, the candidate could have stated that 0.167 came from the probability of $\frac{1}{6}$. Their comment was clear, using appropriate language that indicated a level of uncertainty. Definite phrases should not be used.

Example Candidate Response – middle	Examiner comments
2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws. **(a)** Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3] $p = \frac{56}{300} \Rightarrow 0.187$ $q = 0.813 \quad \alpha = 0.98$ $n = 300 \quad \alpha = 2.054$ 1 $P \pm \alpha \sqrt{\frac{pq}{n}}$ $0.187 - (2.054)\sqrt{\frac{0.19(0.81)}{300}} < M < 0.187 + 2.054\sqrt{\frac{0.19(0.81)}{300}}$ 2 $0.187 - 0.0465 < M < 0.187 + 0.0465$ $0.140 < M < 0.234$ 3	1 This is a correct value of z. 2 The candidate uses an expression of the correct form, so the method mark is awarded. However, they substitute values that are only to 2 significant figures instead of 4 significant figures which would avoid rounding errors. 3 The candidate gives the answer as an interval, but their values are not correct to the required accuracy. No further marks are available. Mark for (a) = 2 out of 3

How the candidate could have improved their answer

(a) The candidate rounded $\frac{56}{300}$ prematurely to 0.19 or 0.187 (rather than 0.1867). This led to 0.234 rather than 0.233 as the final answer. It is advisable to work with decimal values to at least four significant figures to ensure the final answer is accurate to three significant figures.

Example Candidate Response – middle, continued	Examiner comments
2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws. **(b)** Maroulla claims that the die is biased. Use your answer to part **(a)** to comment on this claim. [1] The die is not biased since $p = \frac{56}{300}$ lies between 4 0.140 and 0.233. 5	4 The candidate gives a definite statement instead of using language with a degree of uncertainty. 5 This is an incorrect probability. Mark for (b) = 0 out of 1

How the candidate could have improved their answer

(b) The candidate used the wrong probability; they should have checked $p = \frac{1}{6}$. They used a definite statement ('... the die is not biased ...') instead of a statement that implied a degree of uncertainty (e.g., '... there is evidence that ...').

Example Candidate Response – low

2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws.

(a) Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3]

$p \pm z.\sqrt{\frac{pq}{n}}$

$\cancel{pq = 56/300}$

$\cancel{z =}$ $p = 56/300 = 0.18666$

$q = 1 - 56/300 = 0.81334$

$pq = 0.152$

$\left(56/300 + 0.9772.\sqrt{\frac{\frac{56}{300}\times\left(1-\frac{56}{300}\right)}{300}}\right)$; $\cancel{\left(\frac{56}{300} - 0.9772.\sqrt{\;}\right.}$

$\left(\frac{56}{300} - 0.9772.\sqrt{\frac{\frac{56}{300}\times\left(1-\frac{56}{300}\right)}{300}}\right)$

$= (0.20899 ; 0.16500)$

$= (0.209 , 0.165)$

Examiner comments

1 The candidate does not give the z value.

2 The values the candidate substitutes are correct except for 0.9772. Their expression is not of the correct form as 0.9772 is not a z value, so the method mark is not awarded.

Mark for (a) = 0 out of 3

How the candidate could have improved their answer

(a) The candidate's value of 0.9772 was not a z value, but an area found by looking up a z value of 2.0. It is helpful to show all steps in calculating z to prevent errors.

Example Candidate Response – low, continued	Examiner comments
2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws. **(b)** Maroulla claims that the die is biased. Use your answer to part **(a)** to comment on this claim. [1] $\frac{1}{6} \times 300 = \frac{50}{300} = \frac{1}{6}$ 3 ∴ 0.1667 is ~~as~~ part of the ~~interval~~ confidence interval ~~so the~~ 4	3 This is the correct probability. 4 The candidate's statement is not clear, and they make no comment on the claim. The mark is not awarded. Mark for (b) = 0 out of 1

How the candidate could have improved their answer

(b) The phrase 'part of…' was not clear; a phrase such as 'within the confidence interval' would have been better. The candidate needed to comment on the claim in order to answer the question.

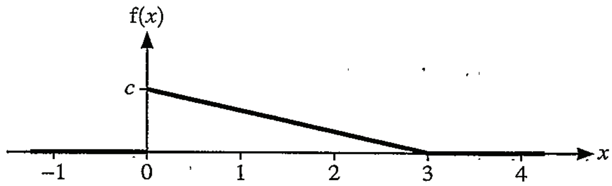
Common mistakes candidates made in this question

- Not using expressions of the correct form for the confidence interval.
- Confusion over when to use $\frac{56}{300}$ and when to use $\frac{1}{6}$.
- Using incorrect values for z in expressions for the confidence interval.
- Giving comments on the claim as definite statements.

Question 3

Example Candidate Response – high

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

$\frac{1}{2} b \times h = 1$

$\frac{1}{2}(3) \times h = 1$ 1

$\frac{3h}{2} = 1$

$h = 1 \times \frac{2}{3}$

$h = \frac{2}{3}$

$c = \frac{2}{3}$ 2

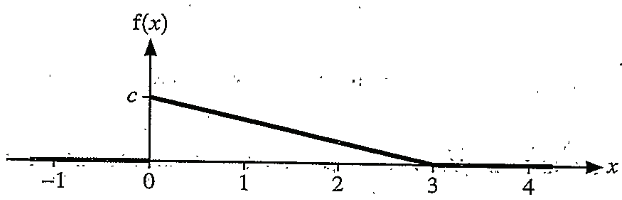
Examiner comments

1 This is a clear statement that the area of the triangle is equal to 1.

2 The candidate shows all necessary working leading to the required result.
Mark for (a) = 1 out of 1

How the candidate could have improved their answer

(a) This was a good answer with all relevant algebraic working shown. The candidate clearly understood that the triangular area below the line was equal to 1 but it would have been better to state this explicitly.

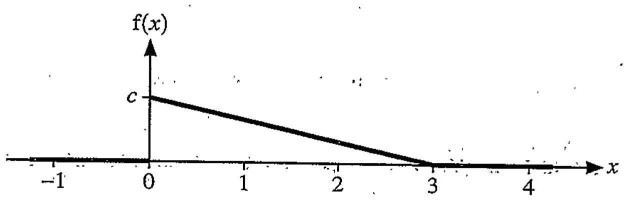
Example Candidate Response – high, continued	Examiner comments
3  A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant. (b) Find $\mathrm{P}(X > 2)$. [2] $P(X>2) = \frac{1}{2}\times 1\times h = \frac{1}{2}\times 1\times\frac{2}{9} = \frac{1}{9} = 0.11$ 4 $h = \frac{1}{3}\times c = \frac{2}{9}$ 3	3 The candidate calculates the height of the triangle correctly. 4 The candidate calculates the area of the triangle correctly using a valid method and obtains the correct answer of $\frac{1}{9}$. Mark for (b) = 2 out of 2

How the candidate could have improved their answer

(b) The candidate used a valid method, but with limited explanation. They found the height of the triangle under the line from two to three using similar triangles, although this was not specifically stated, and their answer would have been substantially improved by explaining this. The answer of $\frac{1}{9}$ was correct. The decimal equivalent 0.11 was only correct to two significant figures and would not have been accepted if this was the only answer seen.

Example Candidate Response – high, continued

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(c) Calculate E(X). [4]

$$E(x) = \int x \cdot f(x)\, dx$$ 5

$$= \int_0^3 x\left(\frac{-2x}{9} + \frac{2}{3}\right) dx$$

$$= \int_0^3 -\frac{2x^2}{9} + \frac{2x}{3}\, dx$$

$$= \left[-\frac{2x^3}{9(3)} + \frac{2x^2}{3(2)}\right]_0^3$$ 6

$$= \left[-\frac{2x^3}{27} + \frac{2x^2}{6}\right]_0^3$$

$$= \left[-\frac{2(3)^3}{27} + \frac{2(3)^2}{6}\right] - \left[-\frac{2(0)}{27} + \frac{2(0)}{6}\right]$$ 7

$$= 1 - 0$$

$$= 1 \leftarrow$$ 8

Examiner comments

5 The candidate uses the correct equation of the line, so a method mark is awarded.

6 The candidate attempts to integrate an expression of the correct form, so a method mark is awarded.

7 The candidate's integration is correct, and they use the correct limits, so a mark is awarded.

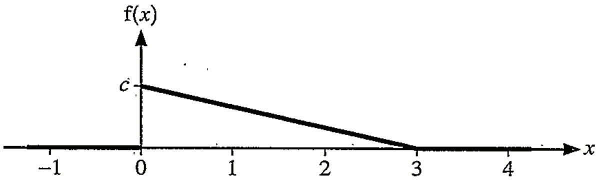
8 This is the correct answer, so the final mark is awarded.
Mark for (c) = 4 out of 4

How the candidate could have improved their answer

(c) The candidate could have shown how they obtained the correct equation of the line (if not stated earlier). Their solution was clearly presented showing the integration and substitution of limits.

Example Candidate Response – middle	Examiner comments

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

$y = mx + q$ $\begin{cases} c = q \\ 0 = 3m + q \; ; \; 3m + c = 0 \; ; \; 3m = -\frac{c}{3} \end{cases}$

$y = -\frac{c}{3}x + c$

$\int_0^3 f(x) = 1 \; ; \; \int_0^3 -\frac{c}{3}x + c = 1 \; ; \; \left[-\frac{c}{6}x^2 + cx\right]_0^3 = 1$

$-\frac{3^2 c}{\cancel{6}2} + 3c = 1 \; ; \; \frac{-3c + 6c}{2} = 1 \; ; \; \frac{3c}{2} = 1 \; ; \; c = \frac{2}{3}$

Examiner comments

1 The candidate uses a correct although rather lengthy method.

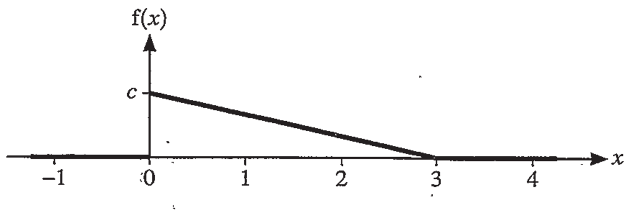
2 The candidate shows all relevant working leading to the required result.
Mark for (a) = 1 out of 1

How the candidate could have improved their answer

(a) The candidate used a valid method, but it would have saved them time to find the area of the triangle using ½ base × height rather than by integration.

Example Candidate Response – middle, continued	Examiner comments

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(b) Find $P(X > 2)$. [2]

$(0, \frac{2}{3})$ $(3, 0)$

$y = f(x)$

$m = \frac{-9}{2}$ (3)

$y - 0 = \frac{-9}{2}(x - 3)$

$f(x) = \frac{-9}{2}(x-3)$ (4)

$\frac{-9}{2}\int_2^3 x - 3\,dx$

$\frac{-9}{2}\left[\frac{x^2}{2} - 3x\right]_2^3$ (5)

$\frac{-9}{2}\left(\frac{-9}{2} + 4\right)$

$P(X>2) = \frac{9}{4}$

Examiner comments

(3) The candidate finds the gradient of the line incorrectly.

(4) The candidate attempts to find the equation of the line (with an incorrect gradient).

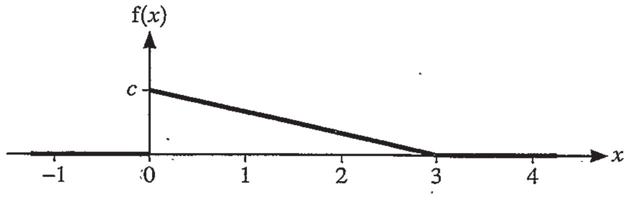
(5) The candidate attempts integration with correct limits using their equation of the line, so a method mark is awarded. No further marks are available.
Mark for (b) = 1 out of 2

How the candidate could have improved their answer

(b) This candidate stated the coordinates $\left(0, \frac{2}{3}\right)$ and $(3,0)$, but used them incorrectly to find the gradient. If they had shown full working or used a diagram, they may have avoided this error.

Example Candidate Response – middle, continued

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(c) Calculate E(X). [4]

$E(X) = \int x\left(\frac{2}{9}x + \frac{2}{3}\right)dx$ 6

$= \int\left(\frac{2}{9}x^2 + \frac{2}{3}x\right)dx$ 7

$= \left[\frac{\frac{2}{9}x^3}{3} + \frac{\frac{2}{3}x^2}{2}\right]_0^3$

$= \left[\frac{2}{27}x^3 + \frac{1}{3}x^2\right]_0^3$ 8

$= (2+3) - (0)$

$E(X) = 5$ 9

Examiner comments

6 Although it is not correct, the candidate uses an equation of the correct form ($y = mx + c$) so a method mark is awarded.

7 The candidate integrates an expression of the correct form, so the second method mark is awarded.

8 The candidate's expression is correct for their equation of the line, and they use correct limits, so a follow-through accuracy mark is awarded.

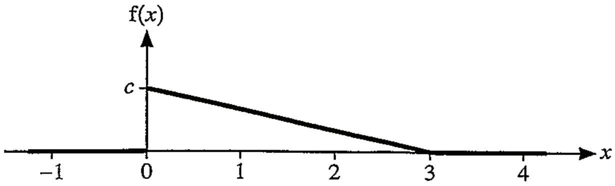
9 Because of the candidate's earlier error, they reach an incorrect final answer.
Mark for (c) = 3 out of 4

How the candidate could have improved their answer

(c) The equation of the line was incorrect: the candidate's gradient was positive rather than negative. They could have checked whether the answer made sense using a diagram. It would have been a good idea to show how they had found the equation if the working was not included in earlier parts of the question.

Example Candidate Response – low

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

$P = (0, c)$ $\quad Q = (3, 0$

$\frac{1}{2}(b) \times h =$ 1

$\frac{1}{2}(3)h$

Examiner comments

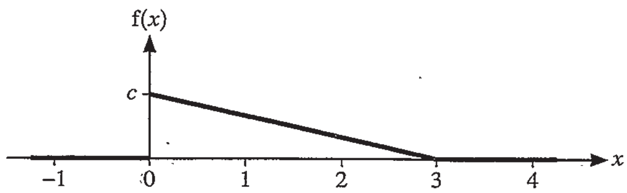
1 The candidate does not equate the area of the triangle to 1.
Mark for (a) = 0 out of 1

How the candidate could have improved their answer

(a) The candidate did not equate the area of the triangle to 1. They needed to use this property of probability density functions.

Example Candidate Response – low, continued

3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(b) Find $P(X > 2)$. [2]

$f(x) = y_1 - 0 = -\frac{2}{9}(x-3)$ $\quad m = \frac{\frac{2}{3} - 0}{0 - 3}$

$y = -\frac{2}{9}x + \frac{6}{9}$ 2 $\quad = -\frac{2}{9}$

$= \int_0^2 f(x)$

$= \left[-\frac{x^2}{9} + \frac{6x}{9} \right]_0^2$ 3

$= \frac{8}{9}$

$= 0.888...$

$= 0.889$

Examiner comments

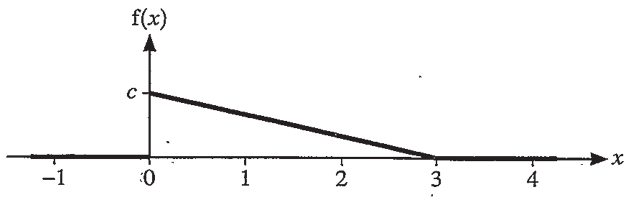
2 The candidate finds the equation of the line correctly.

3 The expression is not of the correct form as the limits are incorrect. The method mark is not awarded.
Mark for (b) = 0 out of 2

How the candidate could have improved their answer

(b) The equation of the line was correct, but the candidate's limits corresponded to a probability that X was less than 2, not greater than 2. They needed to read the question carefully.

Example Candidate Response – low, continued	Examiner comments
3	



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(c) Calculate $\mathrm{E}(X)$. [4]

$$E(X) = \int x f(x)\, dx$$

$$f(x) = -\frac{2}{9}x + \frac{2}{3}$$

$$\int_0^{\frac{2}{3}} -\frac{2}{9}x^2 + \frac{2}{3}x \; dx$$

$$\left[-\frac{2}{27}x^3 + \frac{1}{3}x^2\right]_0^{\frac{2}{3}}$$

$$\left[-\frac{2}{27} \times \left(\frac{2}{3}\right)^3 + \frac{1}{3}\left(\frac{2}{3}\right)^2\right] - \left[-\frac{2}{27}(0)^3 + \frac{1}{3}(0)^2\right]$$

$$\left(-\frac{16}{729} + \frac{4}{27}\right) - 0 = \frac{92}{729}$$

$$\boxed{\text{Ans: } \frac{92}{729} \text{ or } 0.126}$$

Examiner comments

4 The candidate uses a correct equation of the line and is awarded the first method mark.

5 The candidate integrates a correct expression, so the second method mark is awarded.

6 The candidate's limits are incorrect, so no further marks are available.
Mark for (c) = 2 out of 4

How the candidate could have improved their answer

(c) If not shown in a previous part of the question, the candidate needed to show working to find $\mathrm{f}(x)$. The equation of their line was correct, and they multiplied by x and integrated. However, they used limits that were y values, not x values from 0 to 3.

Common mistakes candidates made in this question

- Using incorrect equations for the straight line.
- Omitting to find an equation for the line.
- Using incorrect limits for integration.

Question 4

Example Candidate Response – high

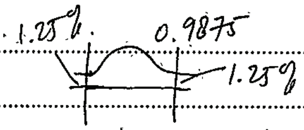
4 The areas, X cm^2, of petals of a certain kind of flower have mean μ cm^2. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

$H_0: \mu = 8.9$

$H_1: \mu \neq 8.9$ 1



$z = 2.24$

$\mu_s = \frac{\Sigma x}{200} = \frac{1850}{200} = 9.25$

$\text{var} = \frac{1}{199}\left(17850 - \frac{(1850)^2}{200}\right) = 3.706 \text{ or } \left(\frac{1475}{398}\right)$ 2

$\frac{9.25 - 8.9}{\sqrt{\frac{3.706}{200}}} = 2.57115$ 3

$2.57115 > 2.24$ 4

There is sufficient evidence to say that the mean is no longer 8.9 following a climate change. 5

Examiner comments

1 These are correct hypotheses, clearly stated.

2 The candidate calculates mean and variance correctly, showing relevant working.

3 The candidate finds the correct z value.

4 The candidate gives a clear statement of comparison leading to the conclusion of the test.

5 The conclusion is clear, worded appropriately (with non-definite language) and refers to the context of the question.

Mark awarded = 8 out of 8

How the candidate could have improved their answer

This was an exemplary answer, with nothing that the candidate could do to improve. They stated their hypotheses clearly and demonstrated correct calculations and a clear comparison with the critical value. Their conclusion used appropriate language with the required level of uncertainty.

Example Candidate Response – middle	Examiner comments

4 The areas, X cm^2, of petals of a certain kind of flower have mean μ cm^2. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

$H_0 ; \mu = 8.9$

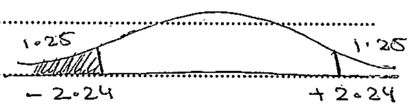
$H_1 ; \mu \neq 8.9$ 1

$\bar{x} \Rightarrow \frac{1850}{200}$ 2 $\quad \sigma^2 \Rightarrow \frac{1}{199}\left(17850 - \frac{1850^2}{200}\right)$ 3

$\bar{x} \Rightarrow 9.25$ $\quad \sigma^2 \Rightarrow 3.71$

$z \Rightarrow \pm 2.24$

$z = \dfrac{8.9 - 9.25}{\sqrt{\dfrac{3.71}{200}}}$ 4



$z = -2.56$

5

No Evidence that claim is justified but the mean is greater than 8.9

Examiner comments

1 These are correct hypotheses.

2 The candidate calculates the mean correctly.

3 The candidate calculates the unbiased variance correctly.

4 The candidate's expression for the z value is correct so the method mark is awarded. However, the value of 3.71 is only to 3 significant figures accuracy and should be to at least 4 significant figures

5 The candidate does not make a clear comparison between 2.24(1) and the z value they calculated, so no method mark is awarded and no further marks are available.
Mark awarded = 5 out of 8

How the candidate could have improved their answer

The candidate used a correct expression to find z but truncated the answer instead of rounding it to three significant figures. Their calculation led to an answer of –2.56978… which rounds to –2.57 not –2.56. They also needed to compare their calculated z value with –2.24, then either write an inequality statement or mark –2.56 on their diagram to show that it was less than –2.24. The test was to look for a change, so 'greater than 8.9' is an incorrect conclusion. Conclusions must be in the context of H_0 and H_1.

Example Candidate Response – low

4 The areas, X cm^2, of petals of a certain kind of flower have mean μ cm^2. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{1850}{200}$

$= 9.25$ 1

$\sigma^2 = \cancel{\dfrac{1850}{200}} \quad \dfrac{17\,850}{200} - (9.25)^2$

$\Rightarrow 3.6875$ 2

$H_0 : \mu = 8.9$

$H_1 : \mu \neq 8.9$ 3

Reject H_0 if $Z_{cal} < 2.5\%$ or $Z_{cal} > 2.5\%$

Reject H_0 if $Z_{cal} < -1.960$ or $Z_{cal} > 1.960$

$Z_{calc} \Rightarrow \dfrac{9.25 - 8.9}{\frac{1.92}{\sqrt{200}}}$

$= 2.577$ 4

5

Since Z_{cal} is greater than 1.960 i.e it is 2.577, we reject H_0 and therefore conclude that μ has changed.

Examiner comments

1 This is a correct value for the mean.

2 The candidate calculates biased variance so is not awarded a mark.

3 The candidate states both correct hypotheses clearly.

4 The candidate calculates a correct z value for their biased variance, so the method and accuracy marks are awarded.

5 The candidate makes an incorrect inequality statement as they are using the z value for a one-tailed test. The method mark and final mark are not awarded.

Mark awarded = 4 out of 8

How the candidate could have improved their answer

The candidate calculated biased variance instead of unbiased variance (the formula is given on the formula sheet). Although they indicated this was a two-tailed test, the z value they used was for a one-tailed test. Their conclusion was a definite statement when they needed to use a statement with a level of uncertainty (for example using a phrase such as 'there is evidence that ...').

Common mistakes candidates made in this question

- Using biased rather than unbiased variance.
- Using an incorrect formula for z (e.g., with variance instead of standard deviation).
- Not showing a clear comparison to justify their conclusion.
- Giving conclusions with definite statements or without any context from the question.

Question 5

Example Candidate Response – high	Examiner comments
5 Customers arrive at a shop at a constant average rate of 2.3 per minute. **(a)** State another condition for the number of customers arriving per minute to have a Poisson distribution. [1] They must arrive randomly and independently 1	1 These two conditions are correct and are expressed in terms of the context given. Mark for (a) = 1 out of 1

How the candidate could have improved their answer

(a) It would have been better to write 'the customers' rather than 'they'. The candidate gave two correct conditions when only one was needed.

Example Candidate Response – high, continued	Examiner comments
It is now given that the number of customers arriving per minute has the distribution Po(2.3). **(b)** Find the probability that exactly 3 customers arrive during a 1-minute period. [2] $P(X=3) = e^{-2.3}\left(\frac{2.3^3}{3!}\right)$ 2 $= 0.203308$ $= 0.203$ 3	2 The candidate provides a correct expression for exactly three customers, so a method mark is awarded. 3 This is the correct final answer. Mark for (b) = 2 out of 2

How the candidate could have improved their answer

(b) This was a good solution. The method was clearly shown and plenty of figures were recorded from the calculator before rounding to three significant figures.

Example Candidate Response – high, continued	Examiner comments
(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3] let Y be the number of customers arriving in a two minute period. $Y \sim \lambda = 2.3 \times 2$ $\therefore Y \sim Po(4.6)$ 4 $P(Y>3) = 1 - P(Y \leq 3) = 1 - e^{-4.6}\left(1 + 4.6 + \frac{4.6^2}{2!} + \frac{4.6^3}{3!}\right)$ 5 $= 0.674$ 6	4 The candidate states clearly that they intend to use the Poisson distribution and their parameter is correct. 5 The candidate states the required terms of the Poisson distribution, and a method mark is awarded. 6 This is the correct answer given to 3 significant figures as required. Mark for (c) = 3 out of 3

How the candidate could have improved their answer

(c) While the candidate's method is clear, it would have been better to show more figures from the calculator before rounding to three significant figures. This would ensure they were awarded accuracy marks even if they made a rounding error subsequently.

Example Candidate Response – high, continued	Examiner comments
(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3] ~~$X \sim B(5,$~~ $X \sim Po(2.3)$ – 1 min $P(X = 0) = e^{-2.3}$ 7 $X \sim B(5, e^{-2.3})$ $\therefore q = 1 - e^{-2.3}$ $P(x = 2)$ 8 $= {}^5C_2 \times (e^{-2.3})^2 \times (1 - e^{-2.3})^3$ 9 $= 0.0732$	7 The candidate states the probability that no customers arrive, then uses it later. 8 The candidate states clearly that they intend to use the binomial distribution with these parameters. 9 This is a correct expression. Mark for (d) = 3 out of 3

How the candidate could have improved their answer

(d) This solution was very well explained, and the candidate stated the parameters for the binomial distribution they used. It would have been better if the candidate had shown more figures from the calculator before rounding to 3 significant figures. This would ensure they were awarded accuracy marks even if they made a subsequent rounding error.

Example Candidate Response – middle	Examiner comments
5 Customers arrive at a shop at a constant average rate of 2.3 per minute. **(a)** State another condition for the number of customers arriving per minute to have a Poisson distribution. [1] The events must be independent 1	1 The candidate's statement does not mention the context of the question and so the mark is not awarded. Mark for (a) = 0 out of 1

How the candidate could have improved their answer

(a) The candidate needed to refer to the context of the question and specify 'arrival of customers' rather than 'the events'. Instead, the candidate gives a generic response which does not have context.

Example Candidate Response – middle, continued	Examiner comments
It is now given that the number of customers arriving per minute has the distribution Po(2.3). **(b)** Find the probability that exactly 3 customers arrive during a 1-minute period. [2] $P(X=3) = \frac{e^{-2.3} \times 2.3^3}{3!}$ 2 $= 0.2033082225$ $= 0.20$ (2d.p) 3	2 The candidate gives a correct expression for exactly 3 customers. 3 The candidate provides a correct answer to more than 3 significant figures accuracy, so the final mark is awarded. Mark for (b) = 2 out of 2

How the candidate could have improved their answer

(b) The candidate gave a final answer to 2 decimal places when 3 significant figures would have been preferable. However, since they gave a more accurate answer on the previous line, they were awarded the final mark. It is good practice to write final answers down from the calculator before rounding them.

Example Candidate Response – middle, continued	Examiner comments
(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3] $\lambda = 4.6$ 4 $P(X > 3)$ $= 1 - P(X \leq 3)$ $1 - P(0, 1, 2, 3)$ $= 1 - 0.325706283$ 5 $P(X > 3) = 0.674$ 6	4 The candidate states 4.6 as the parameter but there is no evidence they are using a Poisson distribution. 5 The candidate gives no indication of where these figures come from and no correct Poisson expression. 6 This is the correct answer, unsupported by working, so the final mark is not awarded. Mark for (c) = 2 out of 3

How the candidate could have improved their answer

(c) The candidate found the correct parameter, 4.6, but used an expression for $P(X \leqslant 3)$ rather than $P(X > 3)$.

Example Candidate Response – middle, continued	Examiner comments
(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3] $P(X=0) = e^{-2.3} = 0.1$ 7 $X \sim B(5\ ;\ 0.1)$ $P(X=2) = {}^5C_2 \times (0.1)^2 \times (0.9)^3$ 8 $\therefore P(X=2) = 0.0729$ 9	7 The candidate finds the correct probability of no customers arriving, then uses a rounded value. 8 The candidate uses the correct expression, so the second method mark is awarded. 9 Due to their premature approximation earlier, the candidate does not reach the correct answer. Mark for (d) = 2 out of 3

How the candidate could have improved their answer

(d) The candidate used the correct method but the (correct) value for $P(0)$ was rounded prematurely. They should have retained the value to at least four significant figures (i.e., 0.1003 or better, rather than 0.1) to ensure that their final answer was accurate to three significant figures.

Common mistakes candidates made in this question

- Not referring to the context when stating a required condition in part **(a)**. It is not enough just to quote a textbook definition.
- Using incorrect values for λ in Poisson expressions.
- Interpreting the requirements of the question incorrectly (for example misinterpreting ‘exactly 3’ and ‘more than 3’)
- Premature approximations leading to a final answer which was not correct to 3 significant figures (particularly in part **(d)**).
- Incorrect binomial expressions in part **(d)**.
- Giving correct but unsupported answers. All relevant working must be shown.

Question 6

Example Candidate Response – high	Examiner comments
6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.	
(a) Carry out the test at the 10% significance level. [5]	
$X \sim B(20; \frac{1}{3})$ $\alpha = 0.1$ $H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$ 1	1 The candidate states both hypotheses clearly; use of 'p' is accepted.
$P(X \leq 2)$ $= P(0) + P(1) + P(2)$ $= \left(20C_0 \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^{20}\right) + \left({}^{20}C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{19}\right) + \left({}^{20}C_2 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{18}\right)$ 2	2 The candidate gives the terms of the binomial expression in the correct form so a method mark is awarded here.
$= \left(3.0073 \times 10^{-4}\right) + \left(3.0073 \times 10^{-3}\right) + 0.014285$ $= 0.0176$ 3	3 The candidate reaches an answer that is correct to 3 significant figures.
~~Accept~~ Reject H_0 since $0.0176 < 0.1$ 4	4 The candidate makes a valid comparison and is awarded a method mark.
There is enough evidence to conclude that the proportion of packets containing the prize offer is less than $\frac{1}{3}$ 5	5 The candidate's conclusion uses the context of the question and the language demonstrates the required level of uncertainty. Mark for (a) = 5 out of 5

How the candidate could have improved their answer

(a) Although 'p' was accepted here, it would have been better to define 'p' as P(contains offer) to relate it to the question. There was evidence that this candidate used 4 significant figures in the working, but they could have given the final answer to more than 3 significant figures before rounding. Their conclusion was correct and demonstrated the required level of uncertainty in the language used.

Example Candidate Response – high, continued

(b) Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer.

Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3]

$P(\text{Type II error}) = P(\text{Accept } H_0 \text{ ~~and~~ when } H_1 \text{ is true})$

if H_1 is true then $X \sim B(20, \frac{1}{7})$ 6

$P(X > 3) = 1 - [P(X=0) + P(X=2) + P(X=3) + P(X=1)]$

$= 1 - [({}^{20}C_0 \times (\frac{1}{7})^0 \times (\frac{6}{7})^{20}) + ({}^{20}C_1 \times (\frac{1}{7})^1 \times (\frac{6}{7})^{19}) +$

$({}^{20}C_2 \times (\frac{1}{7})^2 \times (\frac{6}{7})^{18}) + ({}^{20}C_3 \times (\frac{1}{7})^3 \times (\frac{6}{7})^{17})]$ 7

$= 1 - 0.6822...$

$= 0.31777...$

≈ 0.318 8

Examiner comments

6 The candidate identifies the distribution and parameters to be used.

7 The candidate writes the required binomial expression in full and is awarded the method mark and accuracy mark.

8 This is the correct answer with evidence that 4 significant figures were used in the calculations.
Mark for (b) = 3 out of 3

How the candidate could have improved their answer

(b) This was a particularly good answer. The candidate stated the parameters they used for the binomial distribution and gave a full and correct expression for $P(X > 3)$. No improvements were needed.

Example Candidate Response – high, continued

(c) Explain what is meant by a Type II error in this context. [1]

Type II error is when you conclude that the proportion of packets containing the prize offer is least 1 in 3 when actually it is less than 1 in 3. 9

Examiner comments

9 This is a correct statement written in the context of the question.
Mark for (c) = 1 out of 1

How the candidate could have improved their answer

(c) This response showed good understanding and used the context of the question.

Example Candidate Response – middle

6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.

(a) Carry out the test at the 10% significance level. [5]

$X \sim Bin(20, 1/3)$

$H_0 : \mu = 1/3$

$H_1 : \mu < 1/3$ 1

If H_0 is true $X \sim$ ~~A~~ $Bn(20, 1/3)$ test at 10% significance level in one tailed test.

$P(X < 2)$.

$P(X=0) + P(X=1)$.

$\binom{20}{0} \frac{1}{3}^0 \times \frac{2}{3}^{20} + \binom{20}{1} \frac{1}{3} \times \frac{2}{3}^{19}$ 2

$\approx 0{,}0003007258 + 0{,}003007286$.

$\approx 0{,}0033 < 10\%\ 0{,}1$ 3

∴ we reject H_0 and conclude that at 10% level of significance the ~~pos~~ proportion of packets containing the prize offer is less than 1/3. 4

Examiner comments

1 The hypotheses are incorrect as the test is not for a population mean.

2 The candidate calculates the probability for P(0 or 1) so is not awarded the method mark.

3 This comparison is valid, and the previous error is condoned, so the method mark is awarded.

4 The candidate's conclusion, while in the context of the question, is a definite statement so the final mark is not awarded.
Mark for (a) = 1 out of 5

How the candidate could have improved their answer

(a) The question required a hypothesis test of a population proportion, not a population mean, so their hypotheses should have been $p = \frac{1}{3}$ and so on. There was a term missing in their expression for probability: they needed to calculate P(0, 1 or 2) not P(0 or 1). They could have improved their conclusion by using a phrase such as 'There is evidence to suggest …'.

Example Candidate Response – middle, continued	Examiner comments
6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer. **(b)** Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer. Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3] $P(x \leq 3)$ 0.682 (5) 1 − 0.682 = 0.3128 (6)	(5) The value of 0.682 appears with no supporting calculations. The correct binomial expression must be clearly shown. (6) This is a correct answer but without a supporting method so cannot be awarded full marks. Mark for (b) = 2 out of 3

How the candidate could have improved their answer

(b) The candidate needed to show all relevant working. The answer of 0.318 was correct but unsupported by working; there was no evidence they had used a correct binomial expression.

Example Candidate Response – middle, continued	Examiner comments
6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer. **(c)** Explain what is meant by a Type II error in this context. [1] Biscuit manufacturer claim is accept when it's not ture. it is accepted that it is 1/3 when it's not. (7)	(7) This statement does not fully describe the context since it omits 'when it is actually less than $\frac{1}{3}$'. The mark is not awarded. Mark for (c) = 0 out of 1

How the candidate could have improved their answer

(c) The phrase used by the candidate 'when it's not' did not fully describe the context of the question. It would have been better to write 'when it is actually less than $\frac{1}{3}$'.

Example Candidate Response – low	Examiner comments

6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.

(a) Carry out the test at the 10% significance level. [5]

$\cancel{p = \frac{1}{3} \times 20}$

$H_0 = \frac{1}{3}$

$\cancel{= \frac{20}{3}}$ $q = \frac{2}{3}$

$H_1 < \frac{1}{3}$ 1

$\cancel{q = \frac{40}{9}}$

$\cancel{{}^{20}C_2 \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{18} = 0.0143}$ $\cancel{\mu = \frac{1}{3} \times 20 = \frac{20}{3}}$

$\cancel{\sigma^2 = \frac{1}{3} \times 20 \times \frac{2}{3} = \frac{40}{9}}$

$\cancel{\frac{\frac{1}{3} - \frac{20}{3}}{\sqrt{\frac{40}{9}}} = -3.00}$ $p = \frac{1}{3}$, $q = \frac{2}{3}$

$\cancel{z = -1.282}$ ${}^{20}C_2 \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{10}$

$= 0.0143$ 2

$\cancel{-3.00 < -1.282}$ $-0.0143 > -1.282$

3

There's no evidence that ~~exactly 2 of~~ 1 in 3 packets of biscuits contain a prize offer. ~~H_0 True~~ 4

Examiner comments

1 Both hypotheses are incomplete so the mark is not awarded.

2 The candidate calculates a point probability, so no method mark is awarded.

3 The candidate's comparison is not valid because they compare a point probability with a normal distribution z value.

4 Because the candidate makes no valid comparison earlier, no marks are available for the conclusion.
Mark for (a) = 0 out of 5

How the candidate could have improved their answer

(a) The candidate did not define $\frac{1}{3}$ in their hypotheses. They calculated $P(2)$ rather than the tail probability $P(0,1 \text{ or } 2)$. They needed to compare their probability with 0.1 instead of a normal distribution z value. Their answer could have been presented more clearly.

Example Candidate Response – low, continued	Examiner comments
6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer. **(b)** Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer. $P(x \leq 3)$ Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3] Reject Ho when $P(x \leq 3) \Rightarrow H_0: p = 1/3$ for manufacture $H_1: p < 1/3$ for Maria $H_0: p = 1/7$ $H_1: p < 1/7$ $P(\text{Type II error}) = P(\text{DNR } H_0 \mid H_0 \text{ is false})$ $= P(x > 4 \mid p = 1/7)$ $= 1 - P(x = 0, 1, 2, 3, 4)$ $= 1 - {}^{20}C_0 \times \frac{1}{7}^0 \times \frac{6}{7}^{20} + {}^{20}C_1 \times \frac{1}{7}^1 \times \frac{6}{7}^{19} + {}^{20}C_2 \times \frac{1}{7}^2 \times \frac{6}{7}^{18}$ $+ {}^{20}C_3 \times \frac{1}{7}^3 \times \frac{6}{7}^{17} + {}^{20}C_4 \times \frac{1}{7}^4 \times \frac{6}{7}^{16}$ 5 $= 1 - (P(x \leq 2) + {}^{20}C_3 \times \frac{1}{7}^3 \times \frac{6}{7}^{17} + {}^{20}C_4 \times \frac{1}{7}^4 \times \frac{6}{7}^{16})$ $= 1 - 0.4306$ $= \underline{0.569}$	**5** The candidate uses the correct binomial distribution but their expression contains an extra term. The method mark is awarded condoning the extra term but no further marks are available. Mark for (b) = 1 out of 3

How the candidate could have improved their answer

(b) The candidate should not have included the extra term for $P(X = 4)$ in calculating $P(X > 3) = 1 - (X = 0, 1, 2, 3)$.

Example Candidate Response – low, continued	Examiner comments
6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer. **(c)** Explain what is meant by a Type II error in this context. [1] It means that Ho is accepted when Ho is false 6	6 This explanation does not refer to the context so no marks can be awarded. Mark for (c) = 0 out of 1

How the candidate could have improved their answer

(c) The candidate's answer needed to be put into the context of the question, not just given as a textbook definition.

Common mistakes candidates made in this question

- Omitting hypotheses, or giving incorrect hypotheses, in part **(a)**.
- Not using appropriate language.
- Not referring to the context of the question.
- The comparison required in part **(a)** was occasionally incorrect or not fully shown.
- Calculating $P(X = 2)$ rather than $P(X \leqslant 2)$ in part **(a)**.
- Giving answers unsupported by working.

Question 7

Example Candidate Response – high

7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models.

A book of this type is chosen at random.

(a) Find the probability that the number of minor errors is at least 200 more than the number of major errors. [5]

$380.4 \geqslant 200 +$

$X_1 \sim N(380; 140) \qquad X_2 \sim N(210; 80)$

$\cancel{P(X_1 \geqslant 200)}$

$P(X_1 \geqslant 200 + X_2)$

$P(X_1 - X_2 - 200 \geqslant 0)$

$E(X_1 - X_2 - 200) = -30$

$Var(X) = 140 + 80$ 1

$= 220$

$P(X_1 - X_2 - 200 \geqslant 0) = Z \geqslant \frac{0 - -30}{\sqrt{220}}$ 2

$= 1 - \Phi(2{,}023)$

$= 1 - (0{,}9783 + 0{,}0001)$ 3

$= 0{,}0216$

Examiner comments

1 The candidate gives correct values of –30 and 220 so is awarded both independent marks.

2 The candidate applies the standardising equation correctly using their values and a method mark is awarded.

3 The candidate finds the correct probability and two marks are awarded.
Mark for (a) = 5 out of 5

How the candidate could have improved their answer

(a) The candidate provided a correct solution that was well explained. They showed their method, explaining symbols used, although '*X*' was used rather than $X_1 - X_2 - 200$ at one point.

Example Candidate Response – high, continued	Examiner comments
See below	See below

7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models.

A book of this type is chosen at random.

The costs of correcting a minor error and a major error are 20 cents and 50 cents respectively.

(b) Find the probability that the total cost of correcting the errors in the book is less than $190. [5]

Total C = 20 Min + 50 Maj

$190 = 19000 cents

$E(20\,\text{Min} + 50\,\text{Maj}) = 18100$

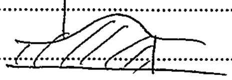
$Var(\text{" "}) = 20^2(140) + 50^2(80)$ 4

$= 256000$

$P(X < 19000)$ $N(18100, 256000)$

5

$\dfrac{19000 - 18100}{\sqrt{256000}} = 1.779$



1.779 $= 0.9624$ 6

Examiner comments

4 The values for E(total cost) and Var(total cost) are correct in cents.

5 The candidate applies the standardising equation correctly and units are consistently given as cents.

6 The candidate uses the correct method to find the probability of less than 19 000 and obtains the correct answer.
Mark for (b) = 5 out of 5

How the candidate could have improved their answer

(b) The candidate gave a correct, well-explained answer with little improvement needed. They could have shown how they calculated 18 100 at the start. Having changed the $190 into cents, they worked consistently with cents throughout the question. Their diagram helped them to find the correct area for less than 19 000.

Example Candidate Response – middle	Examiner comments
7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models. A book of this type is chosen at random. **(a)** Find the probability that the number of minor errors is at least 200 more than the number of major errors. [5] $Min \sim N(380, 140)$ $Maj \sim N(210, 80)$ $P(Min + 200 \geq Maj)$ $P(Min + 200 - Maj \geq 0)$ $E[Min + 200 - Maj] = 380 + 200 - 210$ $= \cancel{370} - 30$ $Var(Min + 200 - Maj) = 140 + 0 + 80$ (1) $= 220$ Let Y denote $(Min + 200 - Maj)$ $\therefore Y \sim N(\overset{-30}{\cancel{370}}, 220)$ $P(Y > 0)$ (2) $z = \dfrac{0 - \cancel{370} - 30}{\sqrt{220}}$ (3) $z = 2.022599...$ $z = 2.023$ $P(z > z) = \Phi^{-1}(2.023)$ (4) $= 0.9783 + 0.0001$ $= 0.9784$	(1) The candidate finds correct values of -30 and 220, supported by working. (2) The candidate clearly states that they are looking for the probability of greater than zero. (3) The candidate uses the standardising equation correctly. (4) The candidate uses an incorrect method so the method mark is not awarded. Mark for (a) = 3 out of 5

How the candidate could have improved their answer

(a) The candidate was looking for $P(Y > 0)$ with a mean of -30, so the answer should have been $1 - 0.9784$. If they had included a sketch of the normal curve with mean -30, they would have seen that the probability should have related to a small area (<0.5).

Example Candidate Response – middle, continued	Examiner comments
7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models. A book of this type is chosen at random. The costs of correcting a minor error and a major error are 20 cents and 50 cents respectively. **(b)** Find the probability that the total cost of correcting the errors in the book is less than \$190. [5]	
$P(\text{Total } C < 190)$ 0.2 Min and 0.5 Maj $P(0.2\,\text{Min} + 0.5\,\text{Maj} < 190)$ let $R = 0.2\,\text{Min} + 0.5\,\text{Maj}$ $E(R) = (0.2 \times 380) + (0.5 \times 210)$ $= 181$ 5	5 The candidate clearly shows calculations for E(total cost) and correctly evaluates it in dollars.
$\text{Var}(R) = (0.2 \times 140) + (0.5 \times 80)$ $= 68$ 6 $R \sim N(181, 68)$	6 The candidate's Var(total cost) is incorrect because they do not square 0.2 and 0.5.
$= P\left(z < \dfrac{190 - 181}{\sqrt{68}}\right)$ 7	7 The candidate applies the standardising equation with their values, using consistent units (dollars), so the method mark is awarded.
$= P(z < 1.091)$ $= \Phi\, 1.091$ 8	8 The candidate uses the correct method to find the required probability.
$= 0.8623$ 9	9 Due to the earlier error in the variance, the candidate's final answer is incorrect. Mark for (b) = 3 out of 5

How the candidate could have improved their answer

(b) The candidate's variance calculation was incorrect: they should have squared 0.2 and 0.5 as they were multiples of the random variables.

Example Candidate Response – low	Examiner comments
7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models.	
A book of this type is chosen at random.	
(a) Find the probability that the number of minor errors is at least 200 more than the number of major errors. MIN MAJ [5]	
N(380, 140) N(210, 80)	
M = ~~380 + [200+210]~~	
M = 380 − [200 + 210]	
~~M = 790~~	
M = −30 // **1**	**1** This is a correct calculation leading to a correct answer.
Vx = 140 + 80 + 200	
Vx = 420 **2**	**2** The candidate's calculation for the variance is incorrect due to the misconception that adding 200 affects the variance.
$Z = \frac{0 - -30}{\sqrt{420}}$ **3**	**3** The candidate uses their values in the standardising equation and a method mark is awarded.
Z = 1.46 //	
P = 0.92838 // **4**	**4** The candidate calculates the wrong probability area, so the method mark is not awarded.
P = 0.93	Mark for (a) = 2 out of 5

How the candidate could have improved their answer

(a) The candidate should have omitted the value 200 from their variance calculation because this value does not affect the variance. It was not clear in their working that a probability of >0 was required. If they had included a sketch of the normal curve with mean -30, they would have seen that the probability should have related to a small area (<0.5).

Example Candidate Response – low, continued	Examiner comments
7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models. A book of this type is chosen at random. The costs of correcting a minor error and a major error are 20 cents and 50 cents respectively. **(b)** Find the probability that the total cost of correcting the errors in the book is less than \$190. [5] $Mi \sim N(380, 140)$ $Ma \sim N(210, 80)$ $P(20Mi + 50Ma < 190)$ $E\left(\frac{20}{100}Min + \frac{50}{100}Maj\right) = \frac{20}{100}E\,min + \frac{50}{100}E\,maj$ $= \frac{20}{100}(380) + \frac{50}{100}(210)$ $= \cancel{18\,100}\quad 181$ (5) $Var\left(\frac{20}{100}Min + \frac{50}{100}Maj\right) = 20\,Var\,Min + 50\,Var\,Maj$ $= 20(140) + 50(80)$ $= 6\,800$ (6) $P\left(z < \frac{190 - 18100}{\sqrt{6800}}\right) =$ $P(z <$ (7) cent $= 100\%$ $P(T_{cost} < \$190)$	(5) The candidate shows their calculations for E(total cost) leading to the correct answer in dollars. (6) The candidate calculates the variance incorrectly. (7) The candidate applies the standardising equation incorrectly using a mixture of dollars and cents, so the method mark is not awarded. Mark for (b) = 1 out of 5

How the candidate could have improved their answer

(b) The candidate's variance formula was incorrect because they needed to include 20^2 and 50^2. They were confused between units (\$ and cents) which led to inconsistencies in the standardising equation.

Common mistakes candidates made in this question

- Incorrect calculations for the mean and variance (in both parts of the question).
- Finding the large area for the probability in part **(a)** when they should have found the small area, or vice versa.
- Working with inconsistent units (\$ or cents) in part **(b)**, or incorrectly converting between different units.
- A sketch of the normal curve in **(a)** would assist candidates in checking their answer.
- It is good practice for a candidate to state the probability to be calculated as this clearly communicates their intention.

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