

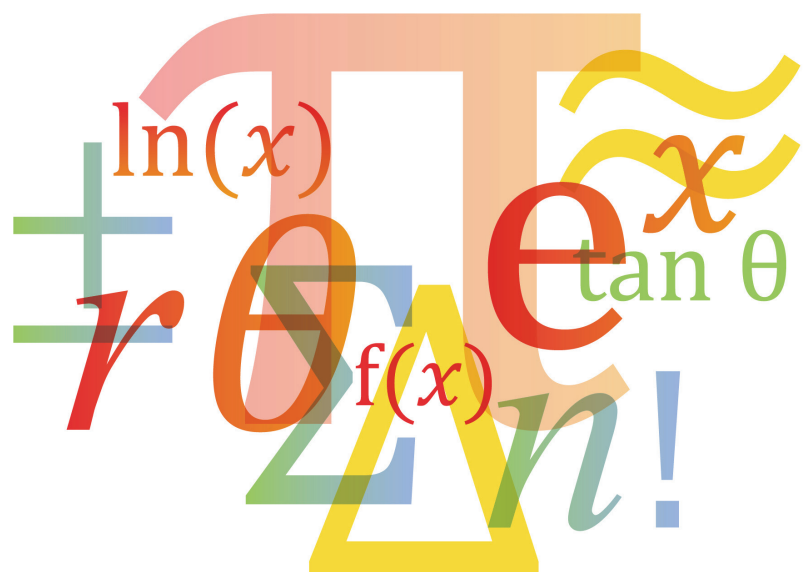


Cambridge Assessment
International Education

Example Candidate Responses – Paper 5

Cambridge International AS & A Level
Mathematics 9709

For examination from 2020



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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709 and to show how different levels of candidates' performance (high, middle and low) relate to the syllabus requirements.

In this booklet, candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

November 2020 Question Paper 52
November 2020 Paper 52 Mark Scheme

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

Example Candidate Response – low	Examiner comments
<p>1 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.</p> <p>(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]</p> <p>let x be the r.v. "no. of throws until a 4 is obtained".</p> <p>$X \sim \text{Geo}(p)$ ①</p> <p>$p = \frac{1}{6}$</p> <p>$q = 1 - \frac{1}{6} = \frac{5}{6}$</p> <p>$P(X < 6) = 1 - P(X = 6)$ ②</p> <p>$= 1 - \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)$ ③</p> <p>≈ 0.933 $= 0.933$ (3 sf)</p>	<p>① The candidate makes a clear statement that they intend to use the geometric approximation.</p> <p>② The candidate misinterprets the given criteria and is calculating the probability of not obtaining a 4 on the 6th throw.</p> <p>③ The expression is not in the correct form for any marks to be awarded. Mark for (a) = 0 out of 2</p>

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

- Before using the geometric approximation formula, the candidate could have identified the success criteria. This may have suggested a less complex approach to them for solving the question.
- The candidate needed to use the correct inequality for 'at least 3 times'.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- Misinterpreting the success criteria given: assuming that 'at least 3 times' does not include '3 times'.
- Using an inequality for less than rather than greater than.
- Arithmetical errors when evaluating expressions.
- Often candidates were not awarded marks because they misread or misinterpreted the questions.
- It is good practice for a candidate to identify the approximation for both p and q .
- It is good practice for a candidate to identify the approximation for both p and q .

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high

Examiner comments

1 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

$$X \sim \text{Geo}\left(\frac{1}{6}\right)$$

$$\begin{aligned} P(X < 6) &= \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^3 \\ &\quad + \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^4 \quad \text{1} \\ &= \left(\frac{1}{6}\right) \times \left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^4\right) \\ &= 0.538 \quad \text{2} \end{aligned}$$

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

$$\begin{aligned} P(X \geq 3) &= 1 - \left(\frac{1}{6} \times \left(\frac{5}{6}\right)^9 + \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^8 + \left(\frac{5}{6}\right)^{10} \right) \\ &= 0.225 \end{aligned}$$

$$\begin{aligned} P(X \geq 3) &= 1 - \left(\left(\frac{5}{6}\right)^{10} + \binom{10}{1} \times \frac{1}{6} \times \left(\frac{5}{6}\right)^9 \right. \\ &\quad \left. + \binom{10}{2} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^8 \right) \quad \text{3} \\ &= 1 - (0.1615 + 0.3230 + 0.2907) \\ &= 1 - 0.7752 \\ &= 0.225 \end{aligned}$$

1 The candidate makes a clear statement of the required terms of the geometric approximation and is awarded a method mark.

2 The candidate gives a correct answer to 3 significant figures, so the final mark is awarded. Mark for (a) = 2 out of 2

3 The candidate clearly states the required terms of the binomial approximation correctly so is awarded the method mark and one accuracy mark. Mark for (b) = 3 out of 3

Total mark awarded = 5 out of 5

How the candidate could have improved their answer

- The candidate could have used a more efficient approach for the geometric approximation such as

$$P(X < r) = 1 - \left(\frac{5}{6}\right)^{r-1}$$

- (b) It was not necessary to show all intermediate steps in a calculation so the candidate could have used the calculator more effectively by evaluating the unsimplified expression directly.

Example Candidate Response – middle **Examiner comments**

1 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

$P(4) < 6\text{-throws.}$

$n = 6$

$r = 6$

$p = \frac{1}{6}$

$q = \frac{5}{6}$

${}^6C_6 \times \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^0$ 1

$= 2.1433 \times 10^{-5}$

$1 - 2.1433 \times 10^{-5}$ 2

$= 0.999$ 3

- 1 This is assumed to be a term from a geometric approximation although there is no supporting statement.
- 2 The candidate uses a value from a geometric approximation term so the method mark is awarded.
- 3 The candidate's final answer is incorrect. Mark for (a) = 1 out of 2

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

$n = 10$

$r = 2 \text{ or } 1 \text{ or } 0$

$p = \frac{1}{6}$

$q = \frac{5}{6}$

${}^{10}C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^8 + {}^{10}C_1 \times \left(\frac{1}{6}\right)^1 \times \left(\frac{5}{6}\right)^9 + {}^{10}C_0 \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{10}$

$= 0.0291 + 0.0243 + 9.09 \times 10^{-3}$ 4

$= 0.06249$

$1 - 0.06249$ 5

$= 0.938$

- 4 The candidate states unsimplified terms correctly but evaluates them inaccurately.
- 5 Although the calculation is incorrect, there is clear evidence to support the value from the correct unsimplified expression, so two marks are awarded at this stage. Mark for (b) = 2 out of 3

Total mark awarded = 3 out of 5

How the candidate could have improved their answer

- The candidate calculated the probability of achieving the required outcome in less than n throws as $1 - (\text{not success})^n$. This is a common misconception when using the geometric approximation and the correct formula is $1 - (\text{not success})^{n-1}$.
- To avoid rounding errors, the candidate needed to state intermediate values to 4 significant figures.

Example Candidate Response – low

Examiner comments

1 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

let x be the d.r.v \ll no. of throws until a 4 is obtained.

$$x \sim \text{Geo}(p) \quad 1$$

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(x < 6) = 1 - P(x = 6) \quad 2$$

$$= 1 - \left(\frac{5}{6}\right)^5 \times \frac{1}{6} \quad 3$$

$$= 0.933 \quad (3 \text{ sf})$$

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

let x be the d.r.v \ll no. of times a 4 is obtained.

$$x \sim B(n, p) \quad 4$$

$$p = \frac{1}{6}, n = 10$$

$$q = \frac{5}{6}$$

$$x \sim B(10, \frac{1}{6})$$

$$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= {}^{10}C_0 \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \times \left(\frac{1}{6}\right)^1 \times \left(\frac{5}{6}\right)^9 \quad 5$$

$$+ {}^{10}C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^8 + {}^{10}C_3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^7$$

$$= 0.930 \quad (3 \text{ sf}) \quad 6$$

1 The candidate makes a clear statement that they intend to use the geometric approximation.

2 The candidate misinterprets the given criteria and is calculating the probability of not obtaining a 4 on the 6th throw.

3 The expression is not in the correct form for any marks to be awarded. Mark for (a) = 0 out of 2

4 The candidate makes a clear statement that they intend to use the binomial approximation.

5 The candidate gives terms in the binomial form and is awarded the method mark.

6 The candidate's final answer is incorrect because of the earlier error, so no further marks are awarded. Mark for (b) = 1 out of 3

Total mark awarded = 1 out of 5

How the candidate could have improved their answer

- Before using the geometric approximation formula, the candidate could have identified the terms that fulfil the criteria. This may have suggested a less complex approach to them for solving the problem.
- The candidate needed to use the correct inequality for 'at least 3 times'.

Common mistakes candidates made in this question

- Misinterpreting the success criteria given: assuming that 'at least 3 times' does not include '3 times'.
- Using an inequality for less than rather than greater than.
- Arithmetical errors when evaluating expressions.
- Rounding too early in calculations so that the final answer is inaccurate. It is good practice to use 4 significant figures for intermediate values.
- Attempting to use the same probability approximation for both parts.
- It is good practice for a candidate to identify the approximation they are using.

Question 2

Example Candidate Response – high

Examiner comments

2 A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes. [2]

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

$$\begin{aligned}
 P(RBB) &= \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \\
 &= \frac{30}{336} \\
 P(X=1) &= \frac{30}{336} \times \frac{3!}{2!} \\
 &= \frac{15}{56}
 \end{aligned}$$

(b) Draw up the probability distribution table for X . [3]

$$\begin{aligned}
 P(X=0) &= \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{6}{336} = \frac{1}{56} \\
 P(X=1) &= \frac{15}{56} \\
 P(X=2) &= \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3!}{2!} = \frac{15}{28} \\
 P(X=3) &= \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}
 \end{aligned}$$

X	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{15}{28}$	$\frac{5}{28}$

1 The candidate states clearly how they have interpreted the question and uses correct notation throughout.

2 As this is a 'show that' question, the candidate gives an unsimplified expression for the probability.

3 The candidate gives the correct unsimplified expression for 3P_1 and a method mark is awarded.

4 The candidate reaches the given answer supported by fully correct, clear workings so the final mark is awarded. Mark for (a) = 2 out of 2

5 The candidate clearly shows their calculations for each outcome, with correct probabilities.

6 The candidate's table includes all the required outcomes and all the probabilities are correct so 3 marks are awarded. Mark for (b) = 3 out of 3

Example Candidate Response – high, continued

Examiner comments

(c) Given that $E(X) = \frac{15}{8}$, find $\text{Var}(X)$. [2]

$$\text{Var}(X) = \sum x^2 p - (E(X))^2$$

$$\sum x^2 p = 0^2 \times \frac{1}{56} + 1^2 \times \frac{15}{56} + 2^2 \times \frac{15}{28} + 3^2 \times \frac{5}{28}$$

$$= 0 + \frac{15}{56} + \frac{15}{7} + \frac{45}{28}$$

$$= \frac{225}{56}$$

$$\text{Var}(X) = \frac{225}{56} - \left(\frac{15}{8}\right)^2$$

$$= \frac{225}{448}$$

7 The candidate substitutes their value for $\sum x^2 p$ into the stated variance formula with the given value of $E(X)$ and is awarded a method mark.

8 The candidate reaches a correct fraction and the final mark is awarded. Mark for (c) = 2 out of 2

Total mark awarded = 7 out of 7

How the candidate could have improved their answer

- The candidate could have justified their expression for $P(X = 1)$ in part (a), e.g., by stating there are 3C_2 ways that one red ball can be taken when three balls are selected.
- In part (b), the candidate could have evaluated the unsimplified expression directly as it is not necessary to show all intermediate stages of a numerical calculation.

Example Candidate Response – middle

Examiner comments

2 A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes. [2]

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

Handwritten work for part (a):

$$RBB = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} = \frac{5}{56}$$

$$B.R.B = \frac{5}{8} \times 3 = \frac{15}{56}$$

(1) Marked next to the first calculation.
 (2) Marked next to the second calculation.
 (3) Marked next to the final result.

On the left, there are some scribbles: 'r' above '00000' and 'b' above '000'.

(b) Draw up the probability distribution table for X . [3]

Handwritten probability distribution table:

x	0	1	2	3
$P(X=x)$	$\frac{3}{256}$	$\frac{15}{56}$	$\frac{15}{28}$	$\frac{5}{28}$

(4) Marked above the table.
 (5) Marked next to the value $\frac{5}{28}$.
 (6) Marked below the value $\frac{3}{256}$.

(c) Given that $E(X) = \frac{15}{8}$, find $\text{Var}(X)$. [2]

Handwritten work for part (c):

$$(1)^2 \times \frac{15}{56} + (2)^2 \times \frac{15}{28} + (3)^2 \times \frac{5}{28}$$

$$- \left(\frac{15}{8}\right)^2 = 0.695$$

(7) Marked next to the final result.

1 The candidate does not explain what they are calculating but we assume it is the probability of the first scenario RBB.

2 The candidate multiplies by 3 and is awarded a method mark.

3 The candidate lists three scenarios that fulfil the criteria. This supports their multiplication by 3 so the final mark is awarded for their proof. Mark for (a) = 2 out of 2

4 The probability distribution table is in the expected form with at least one probability entered so the first independent mark is awarded.

5 The candidate's probability distribution table is correctly set up and includes correct values so two marks are awarded. Their erased calculations in part (a) appear to support these values.

6 $P(X=0)$ is incorrect so the final independent mark is not awarded. It is unclear how the candidate obtains this value as they show no supporting working. Mark for (b) = 2 out of 3

7 The candidate's unsimplified expression for calculating the variance is correct but the final answer is inaccurate so only the method mark is awarded. Mark for (c) = 1 out of 2

Total mark awarded = 5 out of 7

How the candidate could have improved their answer

- Including further explanation would have improved the clarity of their proof in (a).
- In part (b), the candidate could have shown some working to justify the values stated in the table. They could have checked that the sum of probabilities was one and realised there was an error.
- In part (c), the candidate could have checked their calculation.

Example Candidate Response – low

Examiner comments

2 A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

$$\begin{aligned}
 X = 1 &= R_1 B_2 B_3 \text{ or } B_1 R_2 B_3 \text{ or } B_1 B_2 R_3 \\
 &= \left(\frac{1}{8} \times \frac{3}{7} \times \frac{2}{6} \right) \times 3 \quad \text{repeated } \times 3 \\
 &= \frac{1}{56} + \frac{1}{56} + \frac{1}{56} \\
 &= \frac{3}{56} \quad \text{2}
 \end{aligned}$$

(b) Draw up the probability distribution table for X . [3]

x	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{3}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

$$\begin{aligned}
 P(X=0) &= \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} & P(X=2) &= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \right) \times 3 \\
 &= \frac{1}{56} \\
 P(X=1) &= \left(\frac{1}{8} \times \frac{3}{7} \times \frac{2}{6} \right) \times 3 & &= \frac{15}{28} = \frac{30}{56} \\
 P(X=3) &= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \right) \\
 &= \frac{5}{28} = \frac{10}{56} \quad \text{4}
 \end{aligned}$$

1 The candidate restarts the question so only this work will be considered for credit. The values in their denominators show the effect of not replacing the chosen balls, and they identify three different scenarios so, despite the error in the numerator, the method mark is awarded.

2 The candidate's answer is incorrect so no mark is awarded. Mark for (a) = 1 out of 2

3 The candidate gives another probability table on the next page so the second version is marked.

4 The candidate calculates three further probabilities accurately and corrects their error in $P(X=2)$, so the second independent mark is awarded.

Example Candidate Response – low, continued

Examiner comments

x	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{3}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

(c) Given that $E(X) = \frac{15}{8}$, find $\text{Var}(X)$. [2]

$E(X) = \frac{15}{8}$ $\text{Var}(X) = \sum x^2 p - [E(X)]^2$

$\text{Var}(X) = \left(0^2 \times \frac{1}{56} + 1^2 \times \frac{3}{56} + 2^2 \times \frac{30}{56} + 3^2 \times \frac{10}{56} \right) - \left(\frac{15}{8} \right)^2$

$= 3 \frac{45}{56}$

5 The candidate's value of $P(X = 1)$ is not as given in part (a), so the probabilities do not total 1 and no mark is awarded.
Mark for (b) = 2 out of 3

6 The candidate's unsimplified expression for the variance uses the values in their table so the method mark is awarded.
Mark for (c) = 1 out of 2

Total mark awarded = 4 out of 7

How the candidate could have improved their answer

- In part (a), the candidate needed to conclude their proof with the expression given in the question.
- For part (b), the candidate could have used the value given in part (a) to avoid any error in the probability distribution table. They could have checked that the sum of probabilities was 1 and located their error then crossed out the unwanted table.

Common mistakes candidates made in this question

- Not identifying the different orders in which the balls could be taken.
- Simply stating an arithmetic expression that generated the given answer.
- Omitting '0 red balls' from the probability distribution table.
- Constructing a probability distribution table with probabilities that do not sum to 1.
- Using their inaccurate answer for part (a) in the table rather than the given value.
- Not stating an unsimplified expression to support a solution in part (c).
- Substituting the values from the probability distribution table inaccurately into the variance formula.
- Not subtracting $[E(X)]^2$ in the variance calculation.
- It is good practice to clearly state the general formula for calculating variance.
- It is good practice to cancel fractions to the simplest form, although uncancelled fractions are acceptable unless the question requires a particular form.

Question 3

Example Candidate Response – high

Examiner comments

3 Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

$$X \sim N(10.1, 1.3^2) \quad 1$$

$$P(X > 11.3) = P\left(Z > \frac{11.3 - 10.1}{1.3}\right) = 1 - \Phi(0.923) \quad 2$$

$$= 0.178 \quad 3$$

$$P(X > 11.3) = 0.178 \quad 4$$

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t . [3]

$$P(X > t) = 0.75$$

$$P\left(Z > \frac{t - 10.1}{1.3}\right) = 0.75$$

$$1 - \Phi\left(\frac{t - 10.1}{1.3}\right) = 0.75$$

$$\Phi\left(\frac{t - 10.1}{1.3}\right) = 0.25$$

$$\frac{t - 10.1}{1.3} = \Phi^{-1}(0.25)$$

1 The candidate's use of correct mathematical notation demonstrates their understanding of the question.

2 The candidate uses the formula appropriately to standardise the 11.3 so a method mark is awarded.

3 The candidate gives an expression for the appropriate area.

4 The candidate's final answer is correct. Mark for (a) = 3 out of 3

Example Candidate Response – high, continued

Examiner comments

$$\frac{t - 10.1}{1.3} = -\Phi^{-1}(0.75)$$

$$\frac{t - 10.1}{1.3} = -0.674 \quad 5$$

$$t - 10.1 = -(0.674)(1.3)$$

$$t = (-0.674)(1.3) + 10.1$$

$$t = 9.22 \text{ (3 sf)} \quad 6$$

$$t = 9.22 \text{ minutes}$$

- (c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

$$X \sim N(10.1; 1.3)$$

$$P(8.9 < X < 11.3) = P\left(\frac{8.9 - 10.1}{1.3} < Z < \frac{11.3 - 10.1}{1.3}\right)$$

$$\Phi(0.923) - \Phi(-0.923) =$$

$$\Phi(0.923) - (1 - \Phi(0.923)) = \quad 7$$

$$= 0.8220 - 0.178 = 0.644 \quad 8$$

$$0.644 \times 90 = 57.96 \text{ days} \approx 58 \text{ days} \quad 10$$

9

5 The candidate uses the z-value from tables provided and substitutes the value into the standardisation formula to form an equation. Two marks are awarded.

6 The candidate's answer is correct to 3 significant figures so an accuracy mark is awarded. Mark for (b) = 3 out of 3

7 The candidate omits the final bracket but this is condoned as their intention is clear.

8 The candidate uses correct notation and their values from part (a) to calculate the required probability.

9 The candidate calculates the expected number of days and is awarded the method mark.

10 Although the candidate's final answer is correct, there is evidence of approximation on the last line, so the accuracy mark is not awarded. Mark for (c) = 2 out of 3

Total mark awarded = 8 out of 9

How the candidate could have improved their answer

The candidate could have interpreted their answer without using an approximation when finding the number of days, the condition is fulfilled.

Example Candidate Response – middle

Examiner comments

3 Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

mean = 10.1
sf dev = 1.3

$$P(X > 11.3)$$

$$1 - P\left(X < \frac{11.3 - 10.1}{1.3}\right)$$

$$1 - P(0.923077) \quad \textcircled{1}$$

$$P(X < 0.923077)$$

$$0.07692 \Rightarrow \text{Ans } 0.5307$$

$$P(X > 11.3) \approx 0.5307$$

$$\approx 0.531$$

$$1 - 0.8220 = 0.178$$

$$\approx 0.18 \quad \textcircled{2}$$

1 The candidate omits the X in their inequality, but this is condoned as it can be implied by their other working.

2 Although the candidate states their final answer to only 2 significant figures, which is not acceptable, they show the correct value to 3 significant figures on the line above and so a mark is awarded. Mark for (a) = 3 out of 3

3 The candidate does not use the tables provided to state the 75% critical z -value.

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t . [3]

$$P(X > t) = 0.75$$

$$P\left(X > \frac{t - 10.1}{1.3}\right) = 0.75 \quad \textcircled{3}$$

$$\frac{t - 10.1}{1.3} = 0.6745 \quad \textcircled{4}$$

$$t = 10.976 \quad \textcircled{5}$$

$$\approx 10.98$$

4 The candidate forms an equation using the standardisation formula. Although their inequality is incorrect as it should be negative, the method mark is awarded.

5 The candidate solves the equation correctly but the earlier error means their answer is incorrect. The accuracy mark is not awarded. Mark for (b) = 1 out of 3

Example Candidate Response – middle, continued	Examiner comments
<p>(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]</p> <p>.....</p> <p>.....</p> $P(8.9 < X < 11.3)$ $P(X > 8.9) \quad \quad P(X < 11.3)$ $1 - P(X < 8.9) \quad \quad P\left(< \frac{11.3 - 10.1}{1.3}\right)$ $1 - P\left(X < \frac{8.9 - 10.1}{1.3}\right) \quad \quad P(Z < 0.6923) \quad \textcircled{6}$ <p>.....</p> $1 - (X < -0.92307)$ <p>.....</p> $0.178 \quad \quad \quad 0.7556$ <p>.....</p> $0.7556 - 0.178$ <p>.....</p> $0.5576 \quad \textcircled{7}$ <p>.....</p> $0.5576 \times 90 = 50.184 \quad \textcircled{8}$ <p>.....</p> $= 50 \text{ days} \quad \textcircled{9}$	<p>6 The candidate states a correct normal standardisation formula but makes an error.</p> <p>7 The candidate's probability is incorrect because of the error, so no mark is awarded.</p> <p>8 The candidate uses their probability appropriately to determine the expected number of days and the method mark is awarded.</p> <p>9 The candidate interprets their answer to give an integer value and is awarded a follow-through mark. Mark for (c) = 2 out of 3</p> <p>Total mark awarded = 6 out of 9</p>

How the candidate could have improved their answer

- The candidate could have followed the instructions on the paper to give answers to 3 significant figures. They could have checked their calculations for accuracy.
- A sketch of the normal distribution curve would have helped them in both parts (b) and (c) to find the correct probability, using the symmetry of the curve where appropriate.

Example Candidate Response – low

Examiner comments

3 Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

let x be time pia takes to run
 $P(X > 11; 3)$
 $z = \frac{11,3 - 10,1}{1,3}$
 $= 0,923$ 1
 $P(Z > 0,923) = \Phi 0,923$
 $= 0,8212 + 0,0008$
 $= 0,822$
 $P(X > 11,3) = 0,822$ 2

1 The candidate substitutes the given values correctly into the standardisation formula and is awarded the first method mark.

2 The candidate finds the probability from the z -value, but then calculates an incorrect area, so the second method mark is not awarded.
 Mark for (a) = 1 out of 3

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t . [3]

let x be the time Pia takes to run
 $P(X > t) = 0,75$
 ~~$P(X > t) = 0,25$~~
 $z = \frac{t - 10,1}{1,3}$
 $\Phi^{-1} 0,75 = \frac{t - 10,1}{1,3}$

Example Candidate Response – low, continued

Examiner comments

$$0,674 = \frac{t - 10,1}{1,3}$$

$$0,8762 = \frac{t - 10,1}{1,3}$$

$$10,9762 = t$$

$$11,0 = t$$

- (c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

let x be the time pia takes to run.

$$P(8,9 < X < 11,3)$$

$$z = \frac{8,9 - 10,1}{1,3}$$

$$= -0,9230$$

$$P(z < -0,9230) = 1 - \Phi(0,923)$$

$$= 1 - 0,822$$

$$= 0,178$$

$$P(8,9 < X < 11,3) = 0,822 - 0,178$$

$$= 0,644$$

- 3 The candidate gives a critical value from the tables and is awarded the independent mark. They form an equation using the standardisation formula and the method mark is awarded but their final answer is incorrect. Mark for (b) = 2 out of 3

- 4 The candidate finds the probability correctly but makes no attempt to calculate the number of days so one mark is awarded. Mark for (c) = 1 out of 3

Total mark awarded = 4 out of 9

How the candidate could have improved their answer

- A sketch of the normal distribution curve would have helped them in both parts (b) and (c) to find the correct probability, using the symmetry of the curve where appropriate.
- The candidate could have reread the question in part (c) to check that they had answered it fully.

Common mistakes candidates made in this question

- Inaccuracy in using the normal standardisation formula to calculate z or to form an equation.
- Not identifying the correct probability area and finding $P(X < 11,3)$ instead.
- Errors in using the tables to convert the z -value to a probability.
- Not using the tables provided to state the critical value in part (b).
- Misinterpreting the criteria in (b) and using a positive z -value.
- Omitting to find the expected number of days in part (c).
- Not stating the expected number of days as an integer.
- Indicating the final answer is a rounding of the calculated value.
- Not using the symmetry of the normal distribution when calculating the required probabilities.
- It is good practice to state the units for a final answer as it helps to check the answer is reasonable.

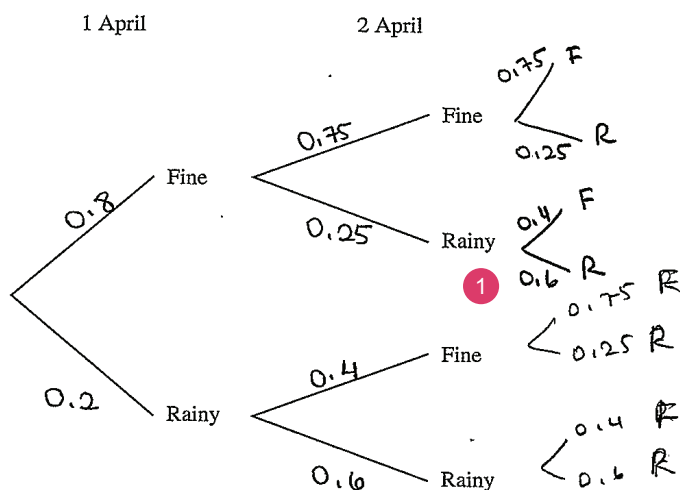
Question 4

Example Candidate Response – high

Examiner comments

- 4 In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

- (a) Complete the tree diagram to show the probabilities. [1]



- (b) Find the probability that 2 April is fine. [2].

$$\dots (0.8 \times 0.75) + (0.2 \times 0.4) \dots$$

$$= 0.68 \dots$$

1 The candidate states all the required values on their tree diagram and is awarded a mark. Mark for (a) = 1 out of 1

2 The candidate states clearly how they will find the probability and the method mark is awarded.

3 This is an exact answer so it does not need to be stated to 3 significant figures. Mark for (b) = 2 out of 2

Example Candidate Response – high, continued

Examiner comments

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

$$\begin{aligned}
 & \cancel{(0.8 \times 0.75 \times 0.75)} + \cancel{(0.8 \times 0.25 \times 0.4)} \\
 & \cancel{= 0.45} + 0.08 \\
 & = 0.53 \\
 & (0.8 \times 0.75 \times 0.25) + (0.8 \times 0.25 \times 0.6) \quad \text{4} \\
 & = 0.15 + 0.12 \\
 & = 0.27 \quad \text{5}
 \end{aligned}$$

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

$$\begin{aligned}
 & \text{3 April rainy} = \cancel{0.27} + (0.8 \times 0.75 \times 0.25) + (0.8 \times 0.25 \times 0.6) \\
 & + (0.2 \times 0.4 \times 0.25) + (0.2 \times 0.6 \times 0.6) = 0.362 \quad \text{6} \\
 & \text{Probability} = \frac{0.27}{0.362} = \frac{135}{181} \quad \text{7} \quad \text{8}
 \end{aligned}$$

4 The candidate states the correct unsimplified expression and the additional branches in their tree diagram support the statement. Two marks are awarded.

5 The candidate's answer is correct and is awarded the accuracy mark. Mark for (c) = 3 out of 3

6 The candidate calculates the probability that 3 April is rainy and is awarded the independent mark.

7 The candidate does not state the conditional probability formula but uses their value from part (c) correctly and the method mark is awarded.

8 The candidate's final answer is given in exact form and is correct, so is awarded the accuracy mark. Mark for (d) = 3 out of 3

Total mark awarded = 9 out of 9

How the candidate could have improved their answer

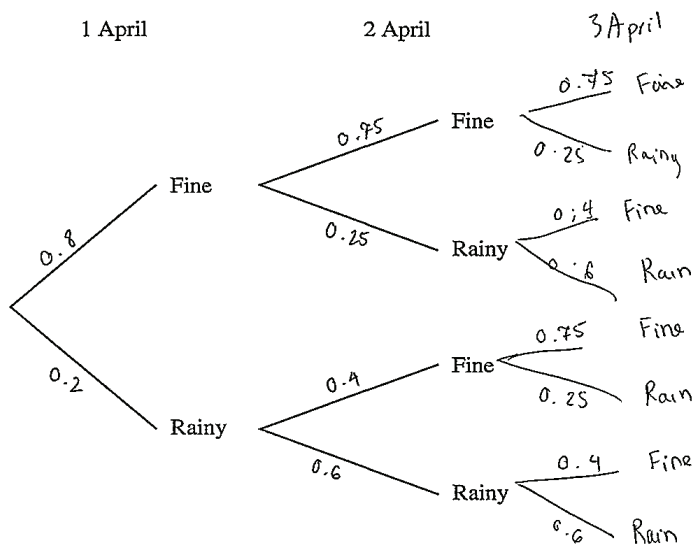
- The candidate could have labelled the additional branches in the tree diagram with '3 April' to show these were not relevant to part (a).
- In parts (b), (c) and (d), the candidate could have made an initial statement to explain their calculation. For (d), this could have included the conditional probability formula.

Example Candidate Response – middle

Examiner comments

4 In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

..... $0.8 \times 0.75 + 0.2 \times 0.4 = 17 \dots = 0.68$ 1

..... 25

1 The candidate gives a correct unsimplified expression and evaluates it correctly. There is no requirement to convert to a decimal. Mark for (a) = 1 out of 1

Example Candidate Response – middle	Examiner comments
<p>Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.</p> <p>(c) Find the value of $P(X \cap Y)$. [3]</p> <p>$P(X \cap Y) = 0.8 \times 0.75 \times 0.75 + 0.8 \times 0.25 \times 0.4 = 0.53$</p> <p>(2)</p>	<p>(2) The candidate's extended tree diagram in part (a) includes 3 April so supports this work although their calculation is incorrect. A method mark is awarded. Mark for (b) = 2 out of 2</p>
<p>(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]</p> <p>$P(X Y)$ = $\frac{P(X \cap Y)}{P(Y)}$</p> <p>$P(X Y) = \frac{0.53}{0.2 \times 0.6 + 0.2 \times 0.4 + 0.8 \times 0.75 + 0.8 \times 0.25 \times 0.6 + 0.2 \times 0.4 \times 0.75 + 0.2 \times 0.6 \times 0.6}$</p> <p>= $\frac{0.53}{2.65} = 1.32$</p> <p>(3)</p> <p>(4)</p>	
<p>Total mark awarded = 6 out of 9</p>	

How the candidate could have improved their answer

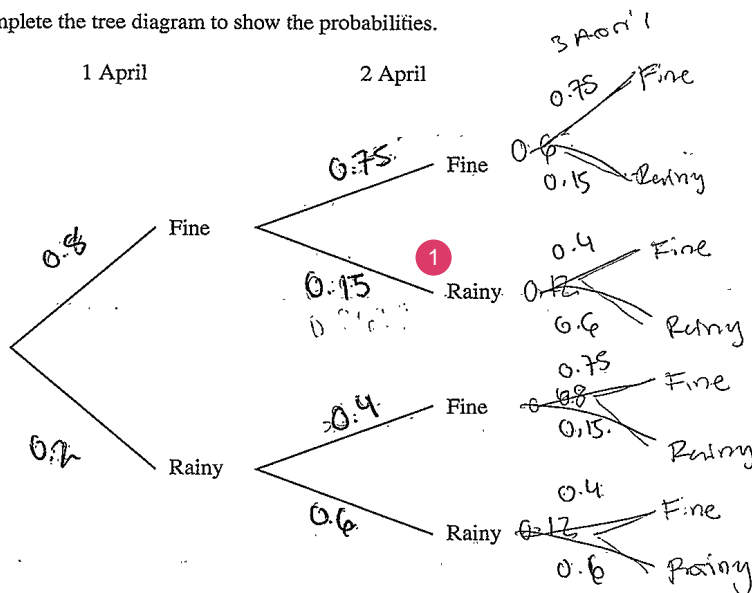
- In parts (b) and (c), the candidate could have made an initial statement to explain their calculation.
- The candidate could have left their answer to part (b) as a rational number, which is exact and saves time.
- For part (c), the candidate could have read the question again at the end to ensure that they had met all the requirements.
- In part (d), the candidate could have clarified the notation they used for conditional probability. Their solution would have been clearer with all the terms in the denominator on one line.

Example Candidate Response – low

Examiner comments

4 In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

$$\begin{aligned}
 P(2 \text{ April is fine}) &= P(1 \text{ April is fine} \cap 2 \text{ April is fine}) \\
 &+ P(1 \text{ April is Rainy} \cap 2 \text{ April is fine}) \\
 &= 0.8 \times 0.75 + 0.2 \times 0.4 \\
 &= 0.68
 \end{aligned}$$

1 The candidate makes an arithmetical error on this branch so no marks are awarded. This error is repeated throughout the question. Mark for (a) = 0 out of 1

2 The candidate clearly explains the process and substitutes correct values from their tree diagram, giving a correct final answer. All marks are awarded. Mark for (b) = 2 out of 2

Example Candidate Response – low, continued

Examiner comments

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

$$\begin{aligned}
 P(X \cap Y) &= P(X) \times P(Y) \quad \textcircled{3} \\
 &= 0.8 \times (0.8 \times 0.75 \times 0.15 + 0.8 \times 0.15 \times 0.6) \\
 &= 0.8 \times (0.8 \times 0.75 \times 0.15 + 0.8 \times 0.15 \times 0.4) \\
 &= 0.1104 = \frac{69}{625}
 \end{aligned}$$

3 The candidate assumes the events are independent, which is a misunderstanding of the question. Mark for (c) = 0 out of 3

4 The candidate makes a clear statement of the conditional probability and supports the work that follows.

5 The candidate calculates the denominator correctly and the values correspond with their tree diagram in part (a). They substitute their answer from part (c) appropriately as the numerator. The method mark is awarded, together with a follow-through independent mark, but the earlier error means the accuracy mark cannot be awarded. Mark for (d) = 2 out of 3

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

$$\begin{aligned}
 &P(\text{1 April is fine} \mid \text{3 April is rainy}) \\
 &= \frac{P(\text{1 April is fine} \cap \text{3 April is rainy})}{P(\text{3 April is rainy})} \quad \textcircled{4} \\
 &= \frac{0.8 \times 0.15 \times 0.6 + 0.8 \times 0.75 \times 0.15 + 0.2 \times 0.4 \times 0.15 + 0.2 \times 0.6 \times 0.6}{\frac{92}{205}} \quad \textcircled{5} \\
 &= \frac{92}{205}
 \end{aligned}$$

Total mark awarded = 4 out of 9

How the candidate could have improved their answer

- The candidate could have checked the probabilities in their tree summed to 1 to avoid subsequent errors.
- The candidate could have identified the branches relevant to part (c) on their tree diagram to justify their expression.
- In part (d), the candidate could have used brackets around the denominator as it does not fit on a single line.

Common mistakes candidates made in this question

- A tree diagram with probabilities on pairs of branches where the sum was not equal to 1.
- Giving probabilities greater than 1 as answers.
- In part (b), omitting the unsimplified expression to support the final answer.
- Arithmetical errors following correct unsimplified expressions.
- Assuming that X and Y were independent in part (c).
- Assuming that 1 April must be fine, and so omitting probabilities in the solution in (c).
- Misinterpreting the context and assuming that the probabilities for the weather on 3 April were the same as 1 April.
- Omitting working from part (d) that would support the values used in calculating conditional probability.
- Not using the value calculated in part (c) but recalculating and reaching a different value.

Question 5

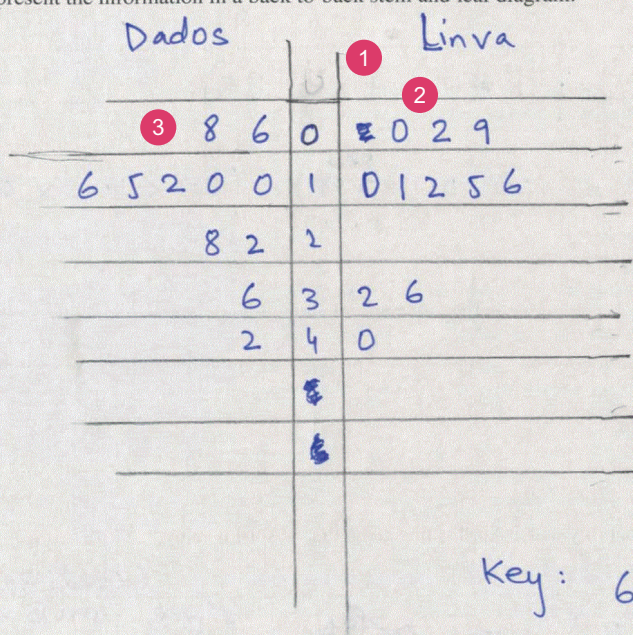
Example Candidate Response – high

Examiner comments

- 5 The following table gives the weekly snowfall, in centimetres, for 11 weeks in 2018 at two ski resorts, Dados and Linva.

Dados	6	8	12	16	10	36	42	28	10	22	16
Linva	2	11	15	16	0	32	36	40	10	12	9

- (a) Represent the information in a back-to-back stem-and-leaf diagram. [4]



Key: 6|3|2
 represents 32 cm
 for Linva
 and 36 cm for
 Dados

1 This is a correct stem so the independent mark is awarded.

2 The candidate makes an error for Linva and crosses out a value so the top row is out of alignment. The mark is not awarded.

3 The candidate correctly orders the data for Dados, uses consistent spacing between entries and correct vertical alignment. There is also a clear title so the independent mark is awarded.

4 The candidate gives a single key and explains it fully using units. Mark for (a) = 3 out of 4

Example Candidate Response – high, continued

Examiner comments

(b) Find the median and the interquartile range for the weekly snowfall in Dados. [3]

$$\text{median} = \frac{(n+1)^{\text{th}}}{2} \text{ Term} = 6^{\text{th}} \text{ term}$$

$$\text{median} = 15 \text{ cm} \quad 5$$

$$\begin{aligned} \text{IQ} &= \text{UQ} - \text{LQ} \\ &= 28 - 10 \quad 6 \end{aligned}$$

$$\text{IQ} = 18 \text{ cm} \quad 7$$

5 The candidate gives a clear explanation of how to calculate the median, with a correct value, so a mark is awarded.

6 The candidate explains clearly how to calculate the interquartile range, with correct values used, so the method mark is awarded.

(c) The median, lower quartile and upper quartile of the weekly snowfall for Linva are 12, 9 and 32 cm respectively. Use this information and your answers to part (b) to compare the central tendency and the spread of the weekly snowfall in Dados and Linva. [2]

$$\text{IQ for Linva} = 23$$

$$\text{Linva's Median} = 12$$

(Weekly)

The Average ^{snow} fall in Linva is less than & compared to Dados 8

and ~~the~~ ~~the~~ ~~also~~ the spread of it is also greater than the Dados 9
(due to its greater IQ Range)

IQ = InterQuartile

7 The candidate finds a correct value for the interquartile range and the accuracy mark is awarded. Mark for (b) = 3 out of 3

8 The candidate's comment about the central tendencies of the data is accurate and given in context so the independent mark is awarded.

9 The candidate gives an accurate comment about the spreads of the data, but do not interpret it in context, so the independent mark is not awarded. Mark for (c) = 1 out of 2

Total mark awarded = 7 out of 9

How the candidate could have improved their answer

- The candidate could have used a pencil when constructing the stem-and-leaf diagram. This would have allowed them to correct their error and maintain vertical alignment of the digits.
- The candidate could have stated values of the upper and lower quartiles before calculating the interquartile range.
- In part (c), the candidate could have interpreted the spread in context without using the word 'spread'.

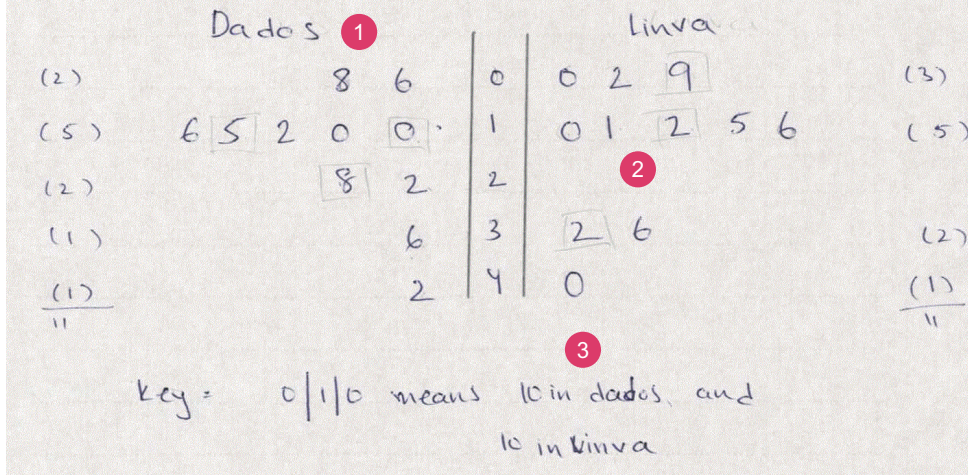
Example Candidate Response – middle

Examiner comments

5 The following table gives the weekly snowfall, in centimetres, for 11 weeks in 2018 at two ski resorts, Dados and Linva.

Dados	6	8	12	15	10	36	42	28	10	22	16
Linva	2	11	15	16	0	32	36	40	10	12	9

(a) Represent the information in a back-to-back stem-and-leaf diagram. [4]



1 The candidate orders the data for Dados correctly and places it in the diagram with good vertical alignment, appropriately labelled.

2 The candidate orders the data for Linva accurately with an appropriate heading but some values are not aligned vertically. The independent mark is not awarded.

3 The candidate gives a single key for the data, but omits units so the independent mark is not awarded. Mark for (a) = 2 out of 4

Example Candidate Response – middle

Examiner comments

(b) Find the median and the interquartile range for the weekly snowfall in Dados. [3]

$$\text{Median} = 15 \quad 4$$

$$\text{Lower quartile} = 10$$

$$\text{Upper quartile} = 28$$

$$\text{IQR} = 28 - 10 = 18 \quad 5$$

(c) The median, lower quartile and upper quartile of the weekly snowfall for Linva are 12, 9 and 32 cm respectively. Use this information and your answers to part (b) to compare the central tendency and the spread of the weekly snowfall in Dados and Linva. [2]

$$\text{median} = 12 \quad \text{LQ} = 9 \quad \text{UQ} = 32 \quad \text{IQR} = 32 - 9 = 23$$

~~Interquartile range for Linva is more spread~~

~~bigger showing it is more spread out.~~

~~Median of Dados is bigger, the values~~

~~are skewed for Dados~~

Median for both Dados and Linva are close. 6

they the values for weekly rainfall in both are

almost equally spread and also their lower and

upper quartiles are very close 1cm apart.

Their interquartile ranges are close to each other

showing they are almost equally spread out. 7

4 This is the correct median and there is some supporting evidence on the diagram.

5 The candidate identifies the quartiles and indicates them on the diagram, then shows a clear calculation for the interquartile range. All marks are awarded. Mark for (b) = 3 out of 3

6 The candidate compares the medians so this is assumed to be their comparison of the central tendencies. They do not give a generalised interpretation of the data so no marks are awarded.

7 The candidate refers to the spread of data but does not interpret it in context, so the independent mark is not awarded. Mark for (c) = 0 out of 2

Total mark awarded = 5 out of 9

How the candidate could have improved their answer

- The candidate could have used a pencil when constructing the stem-and-leaf diagram. This would have allowed them to correct errors and maintain vertical alignment of the digits.
- For part (c), the candidate could have used the context in the question to explain the differences.

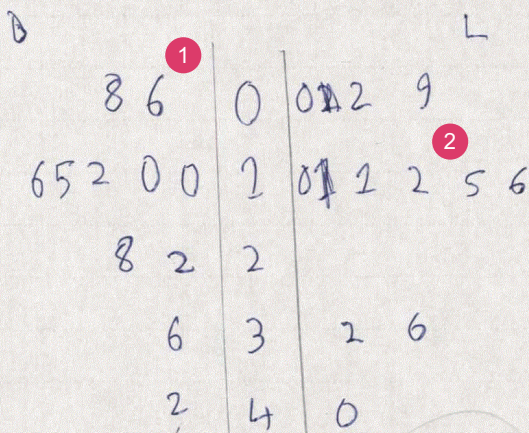
Example Candidate Response – low

Examiner comments

5 The following table gives the weekly snowfall, in centimetres, for 11 weeks in 2018 at two ski resorts, Dados and Linva.

Dados	6	8	12	15	10	36	42	28	10	22	16
Linva	2	41	15	16	0	32	86	40	10	12	9

(a) Represent the information in a back-to-back stem-and-leaf diagram. [4]



key ^D 1110 ^L means 11 for dados and 10 for Linva

1 The candidate labels the left-hand side D which is just sufficient. They order the terms correctly but the vertical alignment of 06 is outside of tolerance.

2 The candidate's vertical alignment on the right-hand side is also outside of tolerance.

3 The candidate omits the units from the key so the independent mark is not awarded. Mark for (a) = 2 out of 4

Example Candidate Response – low, continued

Examiner comments

(b) Find the median and the interquartile range for the weekly snowfall in Dados. [3]

~~6 + 2 + 10 + 2 + 12 + 15 + 16 + 22 + 28 + 36 + 42 + 20~~
~~11~~ 4 Median = 15
~~10 + 2 + 9 + 10 + 11 + 12 + 15 + 18 + 32 + 36 + 40~~
~~11~~ 5 $UQ - LQ = \text{interquartile range}$
 $22 - 10 = 12$

4 The candidate calculates the mean for Dados then crosses it out, so this is ignored. Although they give no supporting evidence, their median is correct so the independent mark is awarded.

5 The candidate states the general formula for the interquartile range and the method mark is awarded although the upper quartile is incorrect. Mark for (b) = 3 out of 3

(c) The median, lower quartile and upper quartile of the weekly snowfall for Linva are 12, 9 and 32 cm respectively. Use this information and your answers to part (b) to compare the central tendency and the spread of the weekly snowfall in Dados and Linva. [2]

~~12~~
~~11~~
 15 10 22
 12 9 32
 6
 Dados's data is less spread than Linva.
~~central tendency of dados~~
 Dados's data are closer to the center than Linva 7

6 This is a true statement about spread, but it is not given in context, so the independent mark is not awarded.

7 The candidate's second statement is not a comparison of central tendency, but on the distribution and spread of the data and they do not interpret it in context. The independent mark is not awarded. Mark for (c) = 0 out of 2

Total mark awarded = 3 out of 9

How the candidate could have improved their answer

- The candidate could have ensured that the ruled lines of the stem are vertical. Ordering the data before completing the stem-and-leaf diagram would have helped with vertical alignment.
- The candidate could have used the ordered data to identify the upper and lower quartiles.
- In (c), the candidate needed to refer to the given context in their comparisons.

Common mistakes candidates made in this question

- Omission of units in the key to the stem-and-leaf diagram.
- Stating a key that did not uniquely define the two resorts.
- Inaccurate vertical alignment of the digits.
- Not including all the given data in the stem-and-leaf diagram.
- Not ordering the data accurately in the leaves.
- Not correctly identifying the upper and lower quartiles.
- Comparing measures of central tendency, range, or interquartile range without interpreting them in context.
- Referring to rainfall rather than snowfall in comments.

Question 6

Example Candidate Response – high

Examiner comments

6 Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

$$A: 6 \quad B: 5 \quad 9$$

$$\text{Group of 6: } {}^9C_6 = \cancel{840} \quad 84$$

$$\text{Group of 3: } {}^9C_3 = 84$$

$$\therefore \text{Total ways} = 84$$

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

$$\underline{B} \underline{B} \underline{B} \underline{A} \underline{C} = 10 \text{ ways } ({}^6C_3)$$

$$\text{Total ways} = {}^9C_5 = 126$$

$$\therefore P = \frac{\text{P(Baker children)}}{\text{total ways}}$$

$$= \frac{10}{126} \quad \frac{10}{126} \quad 4$$

$$= \frac{5}{63} = \frac{5}{63}$$

$$\approx 0.111 \quad = 0.07936$$

$$\rightarrow \approx 0.0794$$

1 As the candidate gives two solutions, their second solution is marked. They clearly state that they are calculating for a group of 3, and their expression is correct, so the method mark is awarded.

2 This answer is correct and supported by the candidate's working, so the accuracy mark is awarded. Mark for (a) = 2 out of 2

3 The candidate states the correct unevaluated term and the independent mark is awarded. Note that they amend their correct value to an incorrect one.

4 The candidate calculates the total number of ways correctly and states the general expression for the conditional probability. However, their numerator, obtained in (a), is incorrect. One mark is awarded. Mark for (b) = 2 out of 3

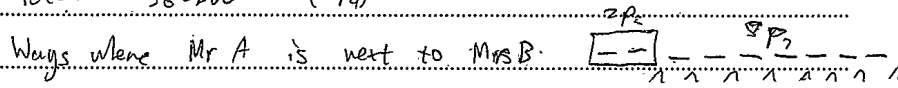
Example Candidate Response – high, continued

Examiner comments

All 9 people stand in a line.

- (c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

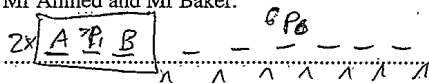
Total = 362880 (9P_9)



$= 8 \times 2P_2 \times 7P_7$ (5)
 $= 80640$

\therefore Ways where Mr A and Mr B aren't next to each other = Total - (Ways they are.)
 $= 362880 - 80640$ (6)
 $= 282240$ ways (7)

- (d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]



$= 7 \times 2 \times 7P_7 \times 6P_6$ (8)
 $= 70560$ ways (9)

5 This is an alternative form of the expression given in the mark scheme and is awarded the independent mark.

6 The candidate uses their total to calculate the number of different arrangements and is awarded the method mark.

7 This is the correct value so is awarded the accuracy mark.
 Mark for (c) = 3 out of 3

8 The candidate's diagram supports their working and they are awarded both method marks.

9 The candidate's value is correct, so the accuracy mark is awarded.
 Mark for (d) = 3 out of 3

Total mark awarded = 10 out of 11

How the candidate could have improved their answer

- In part (a), the candidate could have given only one method of determining the number of ways of dividing the people into groups.
- The candidate could have improved the clarity of their diagram in (d). For instance, they could have separated the box around the group containing Mr Ahmed and Mr Baker from the remainder of the diagram that indicated possible places to insert the group.

Example Candidate Response – middle

Examiner comments

6 Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

$9C6 \times 3C3$ 1
84 ways

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

$\frac{5C3 \times 4C2}{9C5} = \frac{10}{21} = 0.48$ 2
 $\frac{9C3}{9C5} = \frac{2}{3}$ 3

1 The candidate clearly states their approach to the problem, with a correct answer, so full marks are awarded.
Mark for (a) = 2 out of 2

2 As the candidate gives two complete solutions, only the second solution is marked.

3 In the numerator, the candidate misinterprets the context and calculates the number of ways any 3 people can be selected from 9. Their denominator is appropriate for the total number of ways the group of 5 can be selected so only the method mark is awarded.
Mark for (b) = 1 out of 3

Example Candidate Response – middle, continued

Examiner comments

All 9 people stand in a line.

- (c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

Mr A Mr A A A Mr B Mr B B B B

9!

[Mr A Mr B] x x x x x x x x

2! x 8! 4

5 9! - 2! x 8!

282240

- (d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

Mr A Mr B 7P1 7 ways

MA : MB x x x x x x 2! x 6! x 7P1 = 10080 ways

7

4 This simple diagram clarifies the candidate's thought process and supports the calculation, so the independent mark is awarded here.

5 The candidate states the appropriate unsimplified calculation and reaches the correct answer, so the remaining marks are awarded. Mark for (c) = 3 out of 3

6 The candidate works with 6! which matches the 6 crosses representing people in their diagram, but this is one less person than required so the first method mark is not awarded.

7 The candidate's 2! and 7P_1 are assumed to refer to the ways that Mr Ahmed and Mr Baker can be ordered and the number of places they can be inserted in the row. As the candidate uses these values to calculate the number of ways, the second method mark is awarded. Mark for (d) = 1 out of 3

Total mark awarded = 7 out of 11

How the candidate could have improved their answer

- In part (b), the candidate could have crossed out and rewritten the start of their solution to make it clearer to read. They could also have indicated which of their solutions was the final answer to be marked.
- For part (c), the candidate could have communicated their approach more clearly by explaining what they were calculating at each stage. Using an equals sign would have made it easier to identify their answer.
- In their diagram for part (d), using consistent notation for the other members of the group would have helped the candidate to think clearly about the question.

Example Candidate Response – low

Examiner comments

6 Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

Mr Ahmed children 2 4p sp
 ${}^9C_6 \times {}^3C_3$ ①
 84
 group of 6 = 9C_6
 = 84
 group of 3 from the remaining 3
 = 3C_3
 = 1

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

B B B ② Total number of
 ${}^3C_3 \times {}^6C_2$ ③ ${}^9C_5 = 126$ (total number of ways)
 6C_2
 $P(\text{Group all 3 Baker son}) = \frac{3}{126}$ ④
 = $\frac{1}{42}$

1 This is a clear statement of the required calculation and, with the working on the right-hand side, leads to the correct answer. Full marks are awarded. Mark for (a) = 2 out of 2

2 The candidate does not make it clear how they are using the diagram to help them.

3 The candidate crosses out their correct expression for the numerator and uses a different value instead, so the independent mark is not awarded.

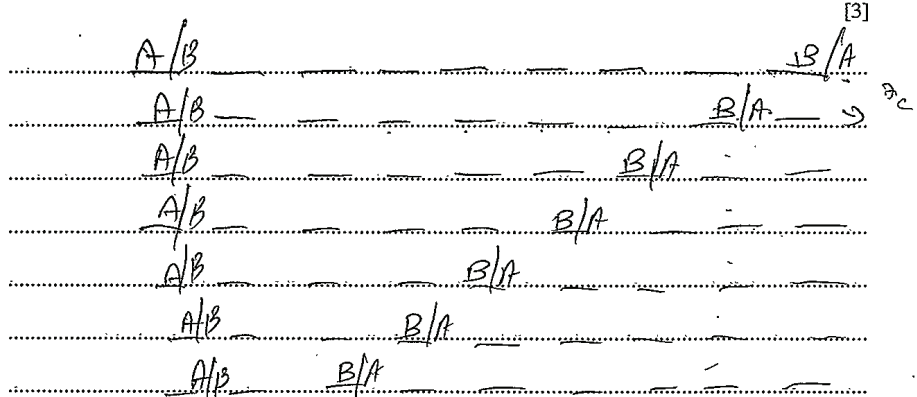
4 The candidate forms an expression for probability with the correct denominator but an incorrect numerator. Only the method mark is awarded. Mark for (b) = 1 out of 3

Example Candidate Response – low, continued

Examiner comments

All 9 people stand in a line.

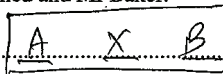
(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]



$$= 2(7! \times 7) \quad 5$$

$$= 2 \times 7! \times 7 = 560$$

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]



$$7! \times 3! \quad 6$$

$$= 30,240$$

5 The candidate gives 7! as the number of different arrangements formed by the remaining 7 people and this corresponds to '7' in their diagram. As it is multiplied by a positive integer, the independent mark is awarded.

No method mark is awarded as the diagram indicates that they have misinterpreted the question with Mr Ahmed or Mr Baker at the end of the line. Mark for (c) = 1 out of 3

6 The candidate gives 7! as the number of different arrangements of the block containing Mr Ahmed and Mr Baker with 6 remaining people. As this is multiplied by a positive integer, the first method mark is awarded. The second method mark is not awarded because the candidate's 3! indicates they have misunderstood the question. Mark for (d) = 1 out of 3

Total mark awarded = 5 out of 11

How the candidate could have improved their answer

- The candidate could have incorporated their working in part (a) into the solution.
- In (b), a statement such as $P(\text{all 3 Baker children}) = \frac{\text{number of groups with all 3 Baker children}}{\text{total number of different groups}}$ would have clarified their approach.
- A simpler diagram in part (c) could have helped them to understand the context better.

Common mistakes candidates made in this question

- Assuming that picking the group of 3 first was different from picking the group of 6 first and summing both.
- Not removing the people picked for the first group from those available to pick for the second group.
- Adding rather than multiplying the number of ways the different groups could be formed.
- Not realising that forming a group of 5 with the 3 Baker children meant candidates needed to find how many different groups of 2 could be formed from the remaining people.
- Inappropriately multiplying by 2, the number of groups with the Baker children and indicating that the order of selection is important.
- Not stating a probability having found the required values.
- If using the 'subtraction' approach in part **(c)**, not recognising that the order of Mr Ahmed and Mr Baker was important when calculating the number of ways that the group could be arranged with the 2 men standing together.
- If using the 'inserting' approach, not identifying that there are 8 places that Mr Ahmed and Mr Baker can be placed after the line of 7 people has been formed.
- If using the 'inserting' approach for part **(d)**, not identifying that there are only 7 places that the pair can be placed after the line of 7 people has been formed. Not recognising that the order is important and not multiplying by 2.
- Attempting to use the 'subtraction' approach in part **(d)**, where it is extremely difficult to identify all the scenarios that must be removed.

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