

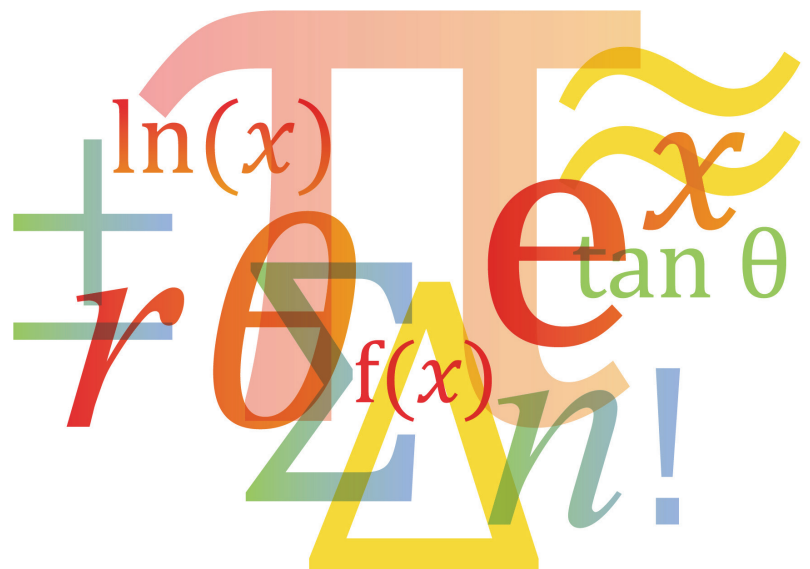


Cambridge Assessment
International Education

Example Candidate Responses – Paper 4

Cambridge International AS & A Level
Mathematics 9709

For examination from 2020



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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709 and to show how different levels of candidates' performance (high, middle and low) relate to the syllabus requirements.

In this booklet, candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

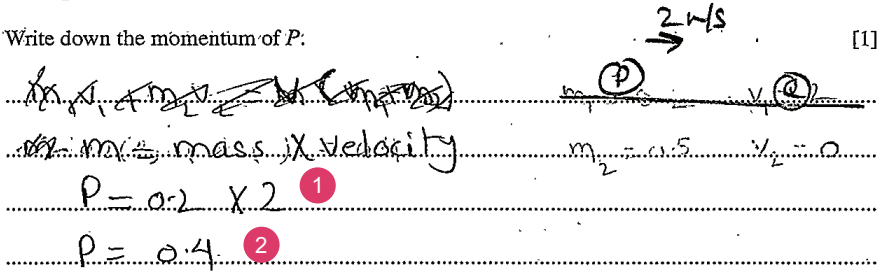
The questions and mark schemes used here are available to download from the School Support Hub. These files are:

November 2020 Question Paper 42
November 2020 Paper 42 Mark Scheme

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

Example Candidate Response – low	Examiner comments
<p>1 Two particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected towards Q with speed 2 m s^{-1}.</p> <p>(a) Write down the momentum of P. [1]</p>  <p>$p = m \times v$ $p = 0.2 \times 2$ $p = 0.4$ $p = \text{mass} \times \text{velocity}$ $p = 0.2 \times 2$ $p = 0.4$</p> <p>$m_1 = 0.2$ $v_1 = 2$ $m_2 = 0.5$ $v_2 = 0$</p>	<p>1 The candidate gives the correct definition of momentum and correctly expresses this as a product from the given information in the question.</p> <p>2 The candidate correctly evaluates the expression and is awarded the mark. Mark for (a) = 1 out of 1</p>

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

- In part (a), the candidate correctly found the required momentum.
- In part (b), the candidate assumed incorrectly that the 0.5 kg particle was moving with a speed V before impact when, in fact, it was at rest. In addition, the candidate assumed that the two particles coalesced after impact since the term $0.3(0.2 + 0.5)$ is the momentum of the combined particle. In this case, the V used should have been zero and, on the right-hand side, the two terms should be $0.3 \times 0.2 + 0.5 \times U$ where U is the required speed.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- In part (b), multiplying 0.2×0.3 to obtain 0.6 rather than 0.06 .
- Not including all the relevant terms in their equation. Candidates should remember that the total momentum before impact is equal to the total momentum after impact.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high

Examiner comments

- 1 Two particles P and Q , of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected towards Q with speed 2 m s^{-1} .

- (a) Write down the momentum of P .

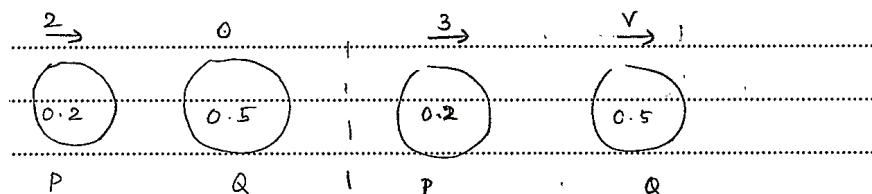
[1]

$$\begin{aligned}
 p &= m \times v \\
 &= 0.2 \times 2 \\
 &= 0.40\text{ N s}
 \end{aligned}$$

- (b) After the collision P continues to move in the same direction with speed 0.3 m s^{-1} .

Find the speed of Q after the collision.

[2]



$$\begin{aligned}
 0.40 &= 0.2 \times 3 + 0.5 \times v \\
 -0.2 &= 0.5v \\
 v &= -0.4\text{ m s}^{-1} \\
 \text{speed} &= 0.4\text{ m s}^{-1}
 \end{aligned}$$

1 The candidate uses the correct definition of momentum.

2 The candidate evaluates the required momentum correctly. Mark for (a) = 1 out of 1

3 The candidate applies the given information to the problem, equating the momentum found in part (a) to the sum of the known momentum of the 0.2 kg particle along with the momentum of the 0.5 kg particle which involves the required unknown V . However, the candidate mistakenly uses the given speed as 3 rather than 0.3 . The method used is correct and so one mark is awarded.

4 The candidate makes a numerical error in -0.2 . This leads to an incorrect answer and no further marks are awarded.

5 The numerical error seen above leads to an incorrect final answer. Mark for (b) = 1 out of 2

Total mark awarded = 2 out of 3

How the candidate could have improved their answer

- The candidate found the correct answer to part (a).
- The solution to part (b) was incorrect due to misreading the value of the speed given in the question and this led to an incorrect final answer. Careful reading of the question would have helped greatly, particularly where several different decimal values are given.

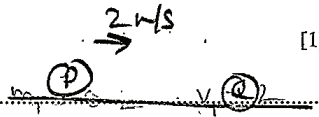
Example Candidate Response – low

Examiner comments

1 Two particles P and Q , of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected towards Q with speed 2 m s^{-1} .

(a) Write down the momentum of P .

[1]

~~$m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$~~ 

~~$m_1 v_1 = \text{mass} \times \text{velocity}$~~ $m_2 = 0.5$ $v_2 = 0$

$P = 0.2 \times 2$ ①

$P = 0.4$ ②

(b) After the collision P continues to move in the same direction with speed 0.3 m s^{-1} .

Find the speed of Q after the collision.

[2]

$m_1 v_1 + m_2 v_2 = v_3 (m_1 + m_2)$ at P

$0.2(2) + 0.5V = 0.3(0.2 + 0.5)$ ③ $m_1 = 0.2$ $v_1 = 2$

$0.4 + 0.5V = 0.21$ $m_2 = 0.5$ $v_2 = 0$

$0.5V = -0.19$

$V = -0.38$ ④

① The candidate gives the correct definition of momentum and correctly expresses this as a product from the given information in the question.

② The candidate correctly evaluates the expression and is awarded the mark. Mark for (a) = 1 out of 1

③ The given information in the question is incorrectly translated. $0.5V$ is the momentum of the 0.5 kg particle before impact but this is given as zero. On the right-hand side of this equation, the candidate assumes the particles coalesce after impact which is also incorrect. The method mark is not awarded.

④ The calculation continues to find V , but this is not what was required in the question and no further marks are available. Mark for (b) = 0 out of 2

Total mark awarded = 1 out of 3

How the candidate could have improved their answer

- In part (a), the candidate correctly found the required momentum.
- In part (b), the candidate assumed incorrectly that the 0.5 kg particle was moving with a speed V before impact when, in fact, it was at rest. In addition, the candidate assumed that the two particles coalesced after impact since the term $0.3(0.2 + 0.5)$ is the momentum of the combined particle. In this case, the V used should have been zero and, on the right-hand side, the two terms should be $0.3 \times 0.2 + 0.5 \times U$ where U is the required speed.

Common mistakes candidates made in this question

- In part (b), multiplying 0.2×0.3 to obtain 0.6 rather than 0.06 .
- Not including all the relevant terms in their equation. Candidates should remember that the total momentum before impact is equal to the total momentum after impact.

Question 2

Example Candidate Response – high

Examiner comments

2 A car of mass 1800 kg is travelling along a straight horizontal road. The power of the car's engine is constant. There is a constant resistance to motion of 650 N.

(a) Find the power of the car's engine, given that the car's acceleration is 0.5 m s^{-2} when its speed is 20 m s^{-1} . [3]

$$P = Fv \quad F = ma$$

$$F = 1800 \text{ kg} \times 0.5 \text{ m s}^{-2}$$

$$F = 900 \text{ N}$$

$$F_{\text{drive}} = 900 \text{ N} - 650 \text{ N} \quad 1$$

$$F_{\text{drive}} = 250 \text{ N}$$

$$\therefore P = 250 \text{ N} \times 20 \text{ m s}^{-1} \quad 2$$

$$P = 5000 \text{ W} \quad 3$$

(b) Find the steady speed which the car can maintain with the engine working at this power. [2]

$$a = 0 \text{ m s}^{-2}$$

$$\therefore F_{\text{drive}} - 650 \text{ N} = 0$$

$$F_{\text{drive}} = 650 \text{ N}$$

$$5000 \text{ W} = 650 \text{ N} \times v \quad 4$$

$$v = 7.69 \text{ m s}^{-1} \quad 5$$

1 The expression given for the driving force has the correct terms but incorrect sign as both terms should be positive.

2 The candidate uses the relationship $P = Fv$ correctly and even although the value of F is incorrect, a mark is awarded.

3 Due to the earlier incorrect sign, the candidate supplies an incorrect answer and is not awarded the final mark. Mark for (a) = 1 out of 3

4 As the motion has constant speed, the force used in this equation should be the friction force and the candidate correctly uses this. The power = 5000 found in part (a) is incorrect but the method is correct and is awarded a mark.

5 Even although the candidate uses power = 5000 which is not correct, their working continues to their answer of 7.69 which is correct to 3 significant figures for this power. They are awarded a follow-through mark resulting in full marks for this part of the question. Mark for (b) = 0 out of 2

Total mark awarded = 1 out of 5

How the candidate could have improved their answer

- In part (a), the candidate should have written down Newton's second law for the car as $DF - 650 = 1800 \times 0.5$ and this would have led to the correct sign in both terms. When the equation $P = Fv$ was used, this would then give the correct value of $P = 31000 \text{ W}$.
- In part (b), although the candidate was awarded full marks due to the follow-through mark, they would have obtained the correct answer if they had used $P = 31000$.

Example Candidate Response – low

Examiner comments

2 A car of mass 1800 kg is travelling along a straight horizontal road. The power of the car's engine is constant. There is a constant resistance to motion of 650 N.

(a) Find the power of the car's engine, given that the car's acceleration is 0.5 m s^{-2} when its speed is 20 m s^{-1} . [3]

$P = Fv$ (1)
 $P = 650 \times 20$ (2)
 $P = 13000 \text{ W}$ (3)

1 In the equation $P = Fv$, the candidate uses F as the frictional force and does not take into account the acceleration. They are not awarded the method mark.

2 However, the candidate uses the equation $P = Fv$ with the value of $F = 650$ and the correct value of $v = 20$, so the B1 mark is awarded.

3 Due to the earlier error, the candidate does not achieve the correct final answer. Mark for (a) = 1 out of 3

(b) Find the steady speed which the car can maintain with the engine working at this power. [2]

$\frac{13000}{v} = 1800 \times 0.5 + 650$ (4)
 $\frac{13000}{v} = 900 + 650$
 $v = \frac{13000}{1550}$
 $v = 8.387$
 $v = 8.39 \text{ m s}^{-1}$

4 The candidate includes the effect of acceleration here in their term 1800×0.5 , but in this part of the question the car moves with constant speed. The right-hand side of this expression should be 650 only, so no method mark is awarded, and no further marks are available. Mark for (b) = 0 out of 2

Total mark awarded = 1 out of 5

How the candidate could have improved their answer

- In part (a), the candidate should have written down Newton's second law and used the given acceleration. This would have led to the correct driving force.
- In part (b), the driving force used included a contribution from the acceleration. However, the acceleration used only applied to part (a) since in part (b) the wording refers to constant speed. If the candidate had used the correct driving force of 650 N in part (b), then they could still have scored full marks despite their error in part (a). This is a question where a follow-through mark can be awarded provided candidates have used the correct method.

Common mistakes candidates made in this question

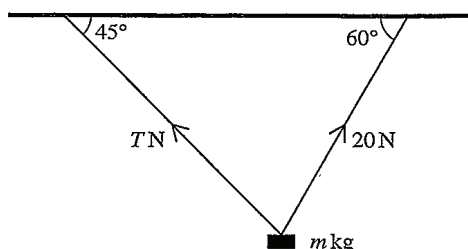
- In (a), using the driving force as $1800 \times 0.5 - 650$ rather than the correct form where both terms are positive.
- Omitting the frictional effect.
- Using forces other than friction in part (b). It was vital that the driving force was the friction force only.
- Using acceleration in part (b) even although the question involved constant speed and hence zero acceleration.

Question 3

Example Candidate Response – high

Examiner comments

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m .

[5]

$$\uparrow T \sin 45 + 20 \sin 60 = 10m \quad (1)$$

$$T \sin 45 + 10\sqrt{3} = 10m \quad (1)$$

$$\rightarrow T \cos 45 - 20 \cos 60 = 0 \quad (2)$$

$$T \cos 45 = 20 \cos 60$$

$$T = 14.1 \text{ N} \quad (3)$$

$$\textcircled{1} \Rightarrow T \sin 45 + 10\sqrt{3} = 10m$$

$$14.1 \sin 45 + 10\sqrt{3} = 10m$$

$$10m = 27.3 \quad (4)$$

$$m = 2.7 \text{ kg} \quad (5)$$

1 The candidate correctly resolves the vertical forces.

2 The candidate resolves horizontal forces correctly.

3 The value of the tension is correct to 3 significant figures.

4 The candidate makes a correct attempt to solve the equations to find the mass m .

5 The candidate gives an answer for m but only to 2 significant figures and so the final mark is not awarded.

Total mark awarded = 3 out of 5

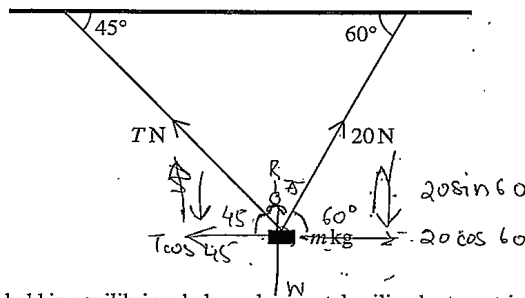
How the candidate could have improved their answer

The candidate gave their answer correct to two significant figures but the rubric states that all answers should be given to three significant figures unless otherwise specified.

Example Candidate Response – middle

Examiner comments

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m .

[5]

$$20 \sin 60 + T \sin 45 = W \quad \text{eq(1) } \textcircled{1}$$

$$20 \cos 60 = T \cos 45 \quad \text{eq(2) } \textcircled{2}$$

$$\frac{20 \cos 60}{\cos 45} = T$$

$$14.142 = T \quad \textcircled{3}$$

Substituting value of T in eq(1);

$$20(\sin 60) + (14.142) \sin 45 = W$$

$$+ 10\sqrt{3} + 9.99 = W \quad \textcircled{4}$$

$$27.32 = W$$

$$\text{Weight} = 27.3 \text{ kg} \quad \textcircled{5}$$

1 Vertical forces are resolved correctly but given in terms of weight W which must, at some stage, be written in terms of mass.

2 The candidate resolves horizontal forces correctly.

3 The candidate solves the horizontal equation to find T . It is over specified (more than 3 significant figures are shown) but as it is correct to 3 significant figures, the mark is awarded.

4 The candidate simplifies their vertical equation but still does not include mass and so is not awarded the accuracy mark.

5 The candidate does not answer the question fully since it is the mass that is required.

Total mark awarded = 3 out of 5

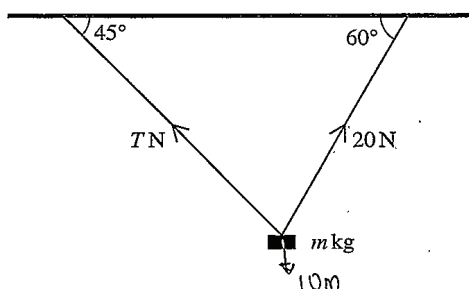
How the candidate could have improved their answer

The work of the candidate was correct, but incomplete as the request in the question was to find the mass of the block. The candidate did not use mass but gave their final answer as a weight. If the candidate had replaced their W by mg and then solved for m , all marks could have been awarded.

Example Candidate Response – low

Examiner comments

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m .

[5]

$\downarrow 10m = 20 \sin 60 + T \sin 45$ ① 1
 ~~$T \cos 45 = 20 \cos 60$ ②~~
 $\uparrow - 10m = T \cos 45 - 20 \cos 60$ ③
 ~~$20 \sin 60 + T \sin 45 = T \cos 45 - 20 \cos 60$~~
 ~~$10\sqrt{3} + \frac{\sqrt{2}}{2}T = \frac{\sqrt{2}}{2}T - 10$~~
 $10m = 20 \cos 60 - T \cos 45$ ② 2
 ①-②
 $20 \sin 60 + T \sin 45 = 20 \cos 60 - T \cos 45$
 $10\sqrt{3} + \frac{\sqrt{2}}{2}T = 10 - \frac{\sqrt{2}}{2}T$
 $\sqrt{2}T = 10 - 10\sqrt{3}$ 3
 $T = -5.18 \text{ N}$ 4
 m
 $10m = 20 \sin 60 + 5.18 \sin 45$
 $m = 2.1 \text{ kg}$ 5

1 The candidate provides a correct vertical resolution of forces involving tension and mass.

2 The candidate makes an incorrect attempt to resolve forces horizontally. Although the two forces on the right-hand side of the equation are correct, the left-hand side introduces the weight which is a vertical force.

3 The candidate solves the two equations for tension, T , and mass, m , but one of the equations has an incorrect term.

4 The solution for T is incorrect.

5 The solution for m is incorrect.

Total mark awarded = 2 out of 5

How the candidate could have improved their answer

The candidate included the weight, a vertical force, in an equation that was balancing the horizontal forces. If the weight term in this equation had been omitted, the equations would have been correct, and they could have been solved to obtain the correct answers.

Common mistakes candidates made in this question

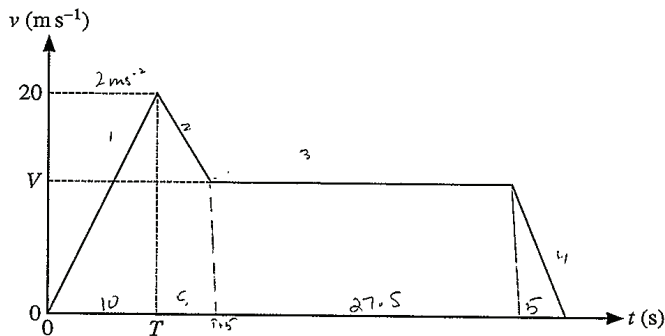
- Confusing sine and cosine when resolving the forces.
- Using the weight and not writing this as mg in order to find the required mass.
- Using mass instead of weight, leading to equations that were not dimensionally correct.

Question 4

Example Candidate Response – high

Examiner comments

4



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of 2 m s^{-2} from rest to a speed of 20 m s^{-1} over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V \text{ m s}^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T .

[1]

$$a = \frac{v - u}{t}$$

$$2 = \frac{20 - 0}{t}$$

$$T = \frac{20}{2}$$

$$= 10 \text{ s}$$

1 The candidate uses the correct constant acceleration equation with the given information and finds the correct value of $T = 10$.
Mark for (a) = 1 out of 1

Example Candidate Response – high, continued

Examiner comments

(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V . [4]

$$\text{Total distance} = \left[\frac{1}{2} \times 10 \times 20 \right] + \left[\frac{1}{2} \times 5 \times (20+V) \right] + \left[\frac{1}{2} \times V \times (27.5 + 32.5) \right]$$

$$= 100 + 2.5(20+V) + 30V$$

$$= 100 + 50 + 2.5V + 30V$$

$$= 100 + 50 + 32.5V$$

$$= 1050 + 32.5V$$

$$\text{distance travelled till speed becomes constant} = \left[\frac{1}{2} \times 10 \times 20 \right] + \left[\frac{1}{2} \times 5 \times (20+V) \right]$$

$$= 100 + 50 + 2.5V$$

$$\frac{100 + 50 + 2.5V}{3} = \frac{1050 + 32.5V}{1}$$

$$3(100 + 50 + 2.5V) = 1050 + 32.5V$$

$$450 + 7.5V = 1050 + 32.5V$$

$$450 - 1050 = (32.5 - 7.5)V$$

$$-600 = 25V$$

$$V = 24$$

2 The candidate finds the distance travelled up to the point at which the car begins to move with constant speed.

3 The candidate correctly represents the distance travelled after the car begins to move at constant speed.

4 The candidate makes a numerical error adding 100 and 50 to obtain 1050.

5 Despite the error, the candidate uses a correct method with the information given in the question and the $\frac{1}{3}$ factor positioned correctly.

6 Although the candidate's method is correct, the earlier numerical error means that their final answer for V is incorrect. Mark for (b) = 3 out of 4

Total mark awarded = 4 out of 5

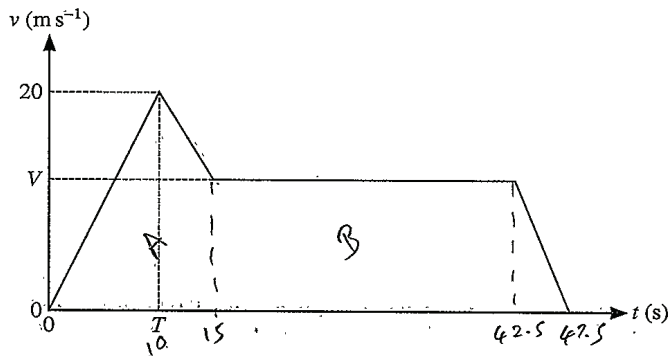
How the candidate could have improved their answer

In part (b), the candidate's only error was a numerical one.

Example Candidate Response – middle

Examiner comments

4



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of 2 m s^{-2} from rest to a speed of 20 m s^{-1} over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V \text{ m s}^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T .

[1]

.....
 acceleration = change in velocity
 time

$$a = \frac{v-u}{t}$$

$$2 = \frac{20-0}{t}$$

$$t = 10 \text{ s}$$

1 The candidate correctly uses the given acceleration in the constant acceleration formula to find the required value of T .
 Mark for (a) 1 out of 1

Example Candidate Response – middle, continued

Examiner comments

(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V . [4]

~~$\frac{1}{3}A = B$ $A = \frac{1}{3}B$~~

$A = \left(\frac{1}{2} \times b \times h\right) + \left(\frac{1}{2} \times b \times h\right) + (b \times h)$

$= \left(\frac{1}{2} \times 10 \times 20\right) + \left(\frac{1}{2} \times (20 - v) \times 5\right) + v \times 5$

$= 100 + 50 - \frac{5}{2}v + 5v$

$= 150 + \frac{5}{2}v$ 2

$B = (b \times h) + \left(\frac{1}{2} \times b \times h\right)$

$= (27.5 \times v) + \left(\frac{1}{2} \times 5 \times v\right)$

$= 27.5v + 2.5v$

$= 30v$ 3

~~$150 + 2.5v = \frac{1}{3}(30v)$~~

~~$150 = 10v - 2.5v$~~

~~$v = 20 \text{ ms}^{-1}$~~

$\frac{1}{3}A = B$

$\frac{1}{3}(150 + 2.5v) = 30v$ 4

$50 + \frac{5}{6}v = 30v$

$50 = 30v - \frac{5}{6}v$

$v = 1.71 \text{ ms}^{-1}$ 5

2 The candidate correctly expresses the distance travelled up to the point at which the car begins to move with constant speed by using the area under the graph from $t = 0$ to $t = 15$.

3 By using the area under the graph from $t = 15$ to $t = 47.5$, the candidate correctly expresses the distance travelled from the time at which the car begins to move with constant speed until it comes to rest.

4 The candidate uses the given information relating the two distances which were found earlier. The fraction of $\frac{1}{3}$ used is not correct as the $\frac{1}{3}$ stated in the question relates to the 'total distance travelled' from $t = 15$ to $t = 47.5$. In the method used here, the $\frac{1}{3}$ should in fact be $\frac{1}{2}$ when linking the two distances which they have used.

5 The candidate's answer is incorrect because of the earlier error. Mark for (b) = 2 out of 4

Total mark awarded = 3 out of 5

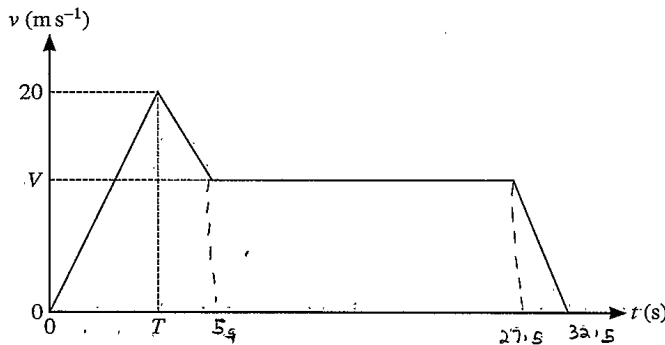
How the candidate could have improved their answer

If the candidate had used the factor $\frac{1}{2}$ rather than $\frac{1}{3}$, they could have obtained the correct answer. In questions such as this, when a fraction or ratio relates different distances, a diagram can be a great help.

Example Candidate Response – low

Examiner comments

4



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of 2 m s^{-2} from rest to a speed of 20 m s^{-1} over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V \text{ m s}^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T . [1]

$(0,0) \quad (T,20)$
 $\frac{20}{T} = 2$
 $2T = 20$
 $T = 10 \text{ s}$ ①

1 The candidate expresses the acceleration correctly using the gradient of the line from $t = 0$ to $t = T$ and finds the correct value of T as 10. Mark for (a) = 1 out of 1

(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V . [4]

$\frac{1}{2} \times 10 \times 20 + \frac{1}{2} \times 5 \times (V+20) = 100 + 50 + \frac{5}{2} V$ ②
 $= \left(150 + \frac{5}{2} V\right) \frac{1}{3}$ ③
 $V \times 22.5 + \frac{1}{2} \times 5 \times V = 25V$
 $150 + 27.5V = 50 + \frac{5}{2} V$ ④
 $100 = 80 V$
 $V = \frac{300}{80}$
 $= 3.75 \text{ m s}^{-1}$ ⑤

2 The candidate produces a correct expression representing the distance up to the point at which the car begins to move with constant speed.

3 The candidate introduces the factor $\frac{1}{3}$ incorrectly in this expression. It is not required until they find the total distance travelled.

4 The 22.5 is incorrect since these two terms do not represent the distance travelled after the car begins to move at constant speed.

5 The wrong factor of $\frac{1}{3}$ introduced earlier leads to an incorrect equation and hence their final value of V is not correct. Mark for (b) = 1 out of 5

Total mark awarded = 2 out of 5

How the candidate could have improved their answer

- Here, the candidate tried to introduce the $\frac{1}{3}$ factor too early in the calculation when they had only found part of the required distance.
- Also, when calculating the distance travelled, they did not consider one of the areas (which represented distance). A simple diagram showing how the $\frac{1}{3}$ relates to the distances would have been useful in attempting to set up the equation.

Common mistakes candidates made in this question

- Using the wrong factor to relate the distances. If total distance is related to distance up to when the car travels at constant speed, then the correct factor is $\frac{1}{3}$. If the two distances before and after it begins to travel at constant speed are used, then the factor is $\frac{1}{2}$.
- Omitting some of the areas needed to evaluate the required distance.

Question 5

Example Candidate Response – high 1

Examiner comments

5 A particle is projected vertically upwards with speed 40 m s^{-1} alongside a building of height $h \text{ m}$.

(a) Given that the particle is above the level of the top of the building for 4 s, find h . [4]

$a = -10 \quad u = 40 \quad v = 0 \quad t = ??$

$v = u + at$
 $a = -10 = 10t$

$t = 4 \text{ s}$ ①

the particle is above the building for 4 s,
then it took 2 seconds moving upwards
and 2 seconds falling back ②

$a = -10 \quad u = 40 \quad t = 2 \quad s = ??$

$$s = ut + \frac{1}{2}at^2$$

$$s = 40(2) + \frac{1}{2}(-10)(2)^2$$
 ③

$$s = 80 - 20$$

$$s = 60 \text{ m}$$
 ④

$h = 60 \text{ m}$

① The candidate correctly finds the time taken to reach the highest point.

② The candidate uses the information given in the question to find that the time taken to reach the top of the building is 2 seconds.

③ The candidate uses a constant acceleration equation to find the height of the building.

④ The candidate's evaluation is correct. Mark for (a) = 4 out of 4

Example Candidate Response – high 1, continued

Examiner comments

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed 20 m s^{-1} .

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]

Handwritten student solution for part (b):

Diagram 1: Particle 1 at $t=1$ s. Initial velocity 20 m s^{-1} , acceleration -10 . Height of building is 60 m .

Diagram 2: Particle 2 at $t=0$ s. Initial velocity 40 m s^{-1} , acceleration -10 m s^{-2} .

Equations for Particle 1 (at time $t-1$):

$$v = u + at$$

$$0 = 20 - 10(t-1)$$

$$t = 3 \text{ sec}$$

Equations for Particle 2:

$$v^2 = u^2 + 2as$$

$$0 = 400 - 20s$$

$$s = 20 \text{ m}$$

Height above ground = 80 m

Study the other particle:

At $t=3$ s, $v=0$, $a=-10$, $t=3$, $s=90$

For this particle $t=1$:

$$s = ut + \frac{1}{2}at^2$$

$$s = 15$$

$$h = 75 \text{ m}$$

Equating heights:

$$s = 20 + 10(t-1)^2$$

$$s = 40t - 5t^2$$

$$s - 60 = 20(t-1) - 5(t-1)^2$$

$$40t - 5t^2 - 60 = 20t - 20 - 5(t^2 - 2t + 1)$$

$$20t - 5t^2 - 35 = -5t^2 + 10t - 5$$

$$10t = 30$$

$$t = 3 \text{ s}$$

5 The candidate correctly expresses the height of the first particle above the ground after t seconds.

6 The candidate correctly expresses the height of the second particle above the ground after t seconds. Note their correct use of $(t - 1)$.

7 The candidate equates the two expressions for height to find the time at which the particles are at the same height.

8 The candidate makes an error in simplifying: the -5 on the right-hand side has already been included in the -35 term.

9 As a result of the error, the final answer is incorrect. Mark for (b) = 3 out of 4

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

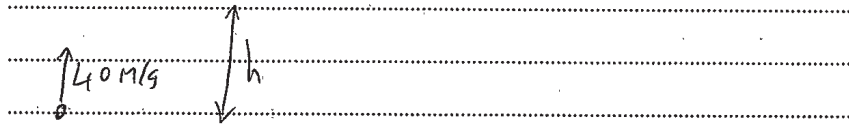
- In part (a), the candidate used one of several possible different approaches and their response was totally correct.
- In part (b), the method used by the candidate was correct, taking particular notice of the time lag of one second by using the term $(t - 1)$. Their error occurred when manipulating their equation and so the final answer was incorrect.

Example Candidate Response – high 2, continued

Examiner comments

5. A particle is projected vertically upwards with speed 40 m s^{-1} alongside a building of height $h \text{ m}$.

(a) Given that the particle is above the level of the top of the building for 4 s, find h . [4]



highest point at $0 = 40 - gt$
 $t = 4$ ①

h is when $t = 4 - \frac{4}{2} = 2$ ②

$h = 40(2) - \frac{9}{2}(2)^2$ ③

$h = 80 - 20 = 60 \text{ m}$ ④

① The time taken for the particle to reach the highest point is correctly evaluated.

② The candidate correctly uses the information given to determine the time taken for the particle to reach the top of the building.

③ The candidate uses a constant acceleration equation with the value $t = 2$ as found earlier.

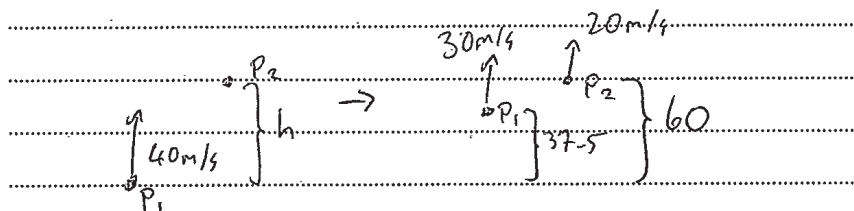
④ The height of the building is correctly evaluated.
 Mark for (a) = 4 out of 4

Example Candidate Response – high 2, continued

Examiner comments

5.(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed 20 m s^{-1} .

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]



After 1 second P_1 travels $s = 40(1) - \frac{g}{2}(1)$ 5
 $= 37.5 \text{ m}$ 6

at a speed of $v = 40 - 10t$ 7
 $v = 40 - 10(1)$
 $= 30 \text{ m/s}$ 7

~~$s = 30t$~~ $s = 30t - \frac{g}{2}t^2$ 8

~~$(s + (60 - 37.5))$~~
 $s = 20t_1 - \frac{g}{2}t_1^2 + (60 - 37.5)$ 9
 $s = 20t_1 - 5t_1^2 + 22.5$

$30t_1 - 5t_1^2 = 20t_1 - 5t_1^2 + 22.5$ 10
 $10t_1 = 22.5$
 $t_1 = 2.25 \text{ s}$

$t = 2.25 + 1$
 $= 3.25 \text{ s}$ 11

5 The candidate uses the alternative method outlined in the mark scheme, finding the distance travelled by the first particle in the first second of motion.

6 The candidate's calculation is incorrect as this should be $40 - 5 = 35$, not 37.5.

7 The candidate finds the correct speed of the first particle after one second of motion.

8 This is the distance travelled by the first particle in t_1 seconds after the second particle is projected.

9 The candidate calculates the distance travelled by the second particle in the first t_1 seconds of its motion. The distance in the bracket is the distance between the particles after one second which means that the two expressions for s can be equated. Their error of 37.5 is carried through.

10 They equate correctly the two expressions for distance but 22.5 should be 25.

11 The candidate's final answer is incorrect due to their error earlier. Mark for (b) = 2 out of 4

Total mark awarded = 6 out of 8

How the candidate could have improved their answer

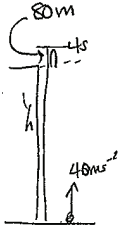
- In part (a), the candidate correctly found the required height of the building.
- In part (b), the candidate used an alternative method which was to find where the first particle was at the instant the second particle was projected. To do this, they found the distance travelled in one second and the speed of the particle after one second. The candidate correctly evaluated the speed but made a numerical error in finding the distance travelled. The method they used was correct throughout, but the numerical error meant that it was not possible to achieve the correct result.

Example Candidate Response – middle

Examiner comments

5 A particle is projected vertically upwards with speed 40 m s^{-1} alongside a building of height $h \text{ m}$.

(a) Given that the particle is above the level of the top of the building for 4 s, find h . [4]



$$\begin{aligned}
 s &= ? \\
 u &= 40 & v^2 &= u^2 + 2as \\
 v &= 0 & 0 &= 40^2 + 2(-10)s \quad \text{1} \\
 a &= -10 & 20s &= 1600 \\
 t &= 4 & s &= 80 \text{ m} \quad \text{2} \\
 \\
 d &= ? \\
 u &= 40 & d &= ut + \frac{1}{2}at^2 \\
 v &= 0 & d &= 40(2) + \frac{1}{2}(-10)(2)^2 \quad \text{3} \\
 a &= -10 & d &= 80 - 20 \\
 t &= 2 \text{ s} & d &= 60 \text{ m} \\
 \\
 h &= 80 \text{ m} - 60 \text{ m} \quad \text{4} \\
 &= 20 \text{ m}
 \end{aligned}$$

1 This is correct use of a constant acceleration equation to find s , the distance to the highest point.

2 The candidate evaluates the height correctly.

3 The candidate uses $t = 2$ but does not explain fully where this value comes from. They find d which is not shown on a diagram.

4 The candidate subtracts $80 - 60$ but gives no reasoning. This does not represent the required height. Mark for (a) = 2 out of 4

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed 20 m s^{-1} .

Denoting the time after projection of the first particle by $t \text{ s}$, find the value of t for which the two particles are at the same height above the ground. [4]

$$\begin{aligned}
 s &= s & s &= s - 20 \\
 u &= 40 & u &= 20 \\
 v &= 0 & v &= 0 \\
 a &= -10 & a &= -10 \\
 t &= t & t &= t + 1 \\
 \\
 s &= ut + \frac{1}{2}at^2 & \text{6} \\
 s_1: s &= 40t + \frac{1}{2}(-10)t^2 & s_2: s - 20 &= 20(t+1) + \frac{1}{2}(-10)(t+1)^2 \\
 s_1: &= 40 - 5t^2 \quad \text{5} & &= 20t - 20 - 5(t^2 - 2t + 1) + 20 \\
 & & &= 20t - 5t^2 + 10t - 5 \\
 & & &= 30t - 5t^2 - 5 \\
 \\
 \text{when } s_1 &= s_2 \\
 40 - 5t^2 &= 30t - 5t^2 - 5 \quad \text{7} \\
 40 + 5 &= 30t \\
 45 &= 30t \\
 t &= 1.5 \text{ s} \quad \text{8}
 \end{aligned}$$

5 The candidate obtains a correct expression for the displacement of the first particle from the ground after t seconds.

6 In the candidate's equation, the right-hand side shows the correct displacement of the second particle from its starting point after t seconds, with correct use of $(t - 1)$. However, the 20 on the left-hand side is their incorrect value of the height of the building.

7 The candidate uses a correct method to equate the two displacements, but the equation involves an error as they use 20 instead of 60 for d .

8 The final answer is incorrect because of their earlier error. Mark for (b) = 3 out of 4

Total mark awarded = 5 out of 8

How the candidate could have improved their answer

In part **(a)**, the candidate found the maximum height correctly as 80 m. They did not explain where their value of $t = 2$ came from, then they used this value to find d which they did not explain. The final incorrect answer comes from subtracting $80 - 60$. If they had explained that $t = 2$ was the time taken to reach the top of the building, and then used this to find the height of the building, they could have obtained the correct answer. They did find the correct height of the building, $d = 60$, but should not have subtracted it from 80.

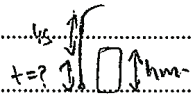
Example Candidate Response – low

Examiner comments

5 A particle is projected vertically upwards with speed 40 m s^{-1} alongside a building of height $h \text{ m}$.

(a) Given that the particle is above the level of the top of the building for 4 s, find h . [4]

$v = 40 \text{ m s}^{-1}; g = -10 \text{ m s}^{-2}; t = ?; v = 0$
 $v = u + at$
 $0 = 40 - 10t$
 $\frac{-40}{-10} = t$
 $t = 4 \text{ s}$ ①
 $S = ut + \frac{1}{2}at^2$
 $S = 40(4) + \frac{1}{2}(-10)(4)^2$
 $S = 80 \text{ m}$ ②
 ③



(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed 20 m s^{-1} .

Denoting the time after projection of the first particle by $t \text{ s}$, find the value of t for which the two particles are at the same height above the ground. [4]

$(t-1)$ $S_A = S_B$
 $S = ut + \frac{1}{2}at^2$
 $S_B = 20(t-1) - 5t^2$ ⑤
 $S_B = 20t - 20 - 5t^2$
 $S_A = S_B$ ⑥
 $40t - 5t^2 = 20t - 20 - 5t^2$
 $40t - 20t - 5t^2 + 5t^2 = -20$
 $-20t = -20$
 $t = \frac{-20}{-20}$
 $t = 1 \text{ s}$ ⑦

① The candidate correctly evaluates the time taken to reach the highest point.

② The candidate uses $t = 4$ correctly to find the greatest height reached by the particle.

③ Further work is needed to find the required height of the building. The candidate does not use the information given in the question relating to the time that the particle is above the building. Mark for (a) = 2 out of 4

④ A correct expression for the distance travelled by the first particle after t seconds.

⑤ The candidate attempts to find the distance travelled by the second particle after t seconds. This is incorrect since $(t - 1)$ should appear in both terms, not only the first one.

⑥ The candidate states incorrectly that the distances travelled by both particles are the same. These distances should differ by 60 m which is the height of the building, but they did not find this in part (a).

⑦ Because the candidate uses the incorrect relationship $S_A = S_B$, their value of t is incorrect. Mark for (b) = 1 out of 4

Total mark awarded = 3 out of 8

How the candidate could have improved their answer

In part **(a)**, the candidate correctly found the maximum height of the particle. However, they did not continue beyond this. They should have used the information given in the question that the particle spent 4 seconds above the building and interpreted this as taking 2 seconds going upwards and 2 seconds going downwards. They could have used this to find the distance the particle would fall from rest at its highest point in 2 seconds: 20 m. By subtracting this from 80, they would have found the required height of the building: 60 m.

Common mistakes candidates made in this question

- In part **(a)**, using 4 seconds (given in the question) as the time it took for the particle to reach its highest point. In fact, this referred to the total time that the particle spent above the building.
- In part **(b)**, errors in finding the distance travelled by the second particle. This was often because they did not use the fact that the particles were projected 1 second apart. They could incorporate this by using the time $(t - 1)$ for the second particle instead of using the same t value for both particles.
- Equating the two distances travelled by the particles but, in fact, they started their motion 60 m apart as found in part **(a)**.

Question 6

Example Candidate Response – high

Examiner comments

- 6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ .

(a)

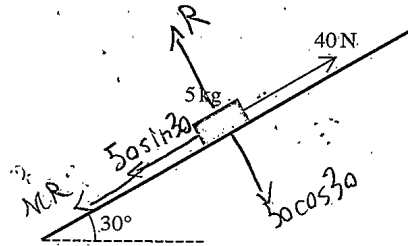


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$.

[4]

$$R = 50 \cos 30 \quad 1$$

at limiting equilibrium:

$$\sum F_x = 0$$

$$40 = 50 \sin 30 + \mu R \quad 2$$

$$\therefore 40 = 50 \sin 30 + \mu (50 \cos 30)$$

$$\mu = \frac{15}{50 \cos 30}$$

$$\mu = \frac{\sqrt{3}}{5} \quad 3$$

when the particle is sliding up the plane

$$\mu < \frac{1}{5}\sqrt{3} \quad 4$$

1 The candidate provides a correct expression for the normal reaction.

2 All the candidate's terms are correct but since motion is taking place, the equals sign should be a > sign.

3 The candidate uses $F = \mu R$ correctly but with an equals sign rather than an inequality.

4 The candidate states the given answer without providing a convincing argument.
Mark for (a) = 3 out of 4

Example Candidate Response – high, continued

Examiner comments

6 (b)

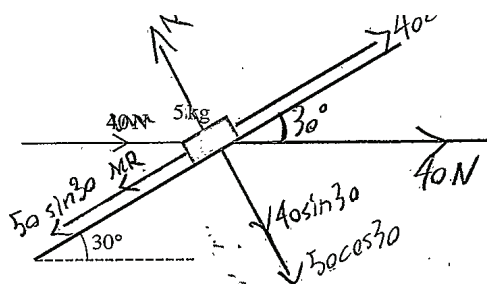


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

$$R = 20 + 50 \cos 30 \quad 5$$

$$40 \cos 30 = 50 \sin 30 + \mu (20 + 50 \cos 30) \quad 6$$

$$40 \cos 30 - 25 = \mu (20 + 50 \cos 30)$$

$$\mu = \frac{40 \cos 30 - 25}{20 + 50 \cos 30} \quad 7$$

$$\boxed{\mu = 0.152} \quad 8$$

5 The candidate provides a correct expression for the normal reaction.

6 The candidate's expression for the friction term is correct and they use $F = \mu \times R$.

7 The candidate rearranges the above equation correctly.

8 All the candidate's working is correct, and their answer is correct to 3 decimal places as required. Mark for (b) = 4 out of 4

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

In part (a), the candidate assumed that no motion was taking place which was incorrect, hence they formed an equation rather than an inequality. When resolving forces along the slope, the equation should be stating that the force acting up the slope is positive. This would then lead to the correct inequality which is given in the question.

Example Candidate Response – high, continued

Examiner comments

6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ .

(a)

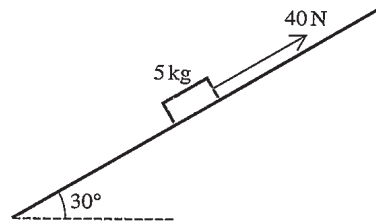


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$.

[4]

$F_c = 5g \cos 30^\circ = 25\sqrt{3}$ (contact force)

$F_{\text{along slope}} = 50 \sin 30^\circ - 40 = 15\text{N}$ or 15N up the slope
 $F_f \geq F_{\text{along slope}} = 15\text{N}$ F_f is frictional force

~~$F_f = 15 = \mu \times 25\sqrt{3}$~~ $\mu = \frac{15}{25\sqrt{3}} = \frac{\sqrt{3}}{5}$

~~$F_f = \mu \times 25\sqrt{3}$~~ $F_f = \mu \times F_c = \mu \times 25\sqrt{3}$

$F_f = \mu \times 25\sqrt{3} > F_{\text{along slope}}$, so $\mu \times 25\sqrt{3} > 15$
 $\mu > \frac{15}{25\sqrt{3}} = \frac{\sqrt{3}}{5}$

$\mu > \frac{1}{5}\sqrt{3}$

1 The candidate gives a correct expression for the normal reaction on the block.

2 The candidate resolves the forces along the block correctly for a case of no motion, but as motion is taking place the equals sign needs to be an inequality.

3 The candidate uses $F = \mu \times R$ for friction correctly but introduces an incorrect inequality.

4 There is an attempt to state the given inequality, but it should read $<$ rather than $>$.
 Mark for (a) = 3 out of 4

Example Candidate Response – high, continued

Examiner comments

6 (b)

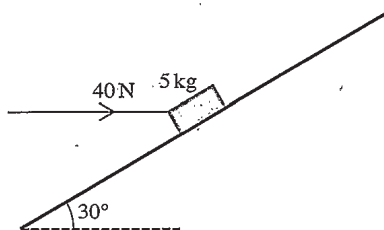


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

$F_{\text{along slope}} = 40 \cos 30^\circ - 5g \sin 30^\circ = 9.64 \text{ N}$ (5)

$F_c (\text{contact force}) = 5g \cos 30^\circ + 40 \sin 30^\circ = 63.30 \text{ N}$ (6)

$F_f = \mu \times F_c = \mu \times 63.30$

F_f is force of friction

$F_f > F_{\text{along slope}}$ for block not to move

$\mu \times 63.30 > 9.64$ (7) $\mu > \frac{9.64}{63.3} = 0.152$ (8)

so $\mu > 0.152$

(5) The candidate resolves forces parallel to the slope correctly, giving the frictional force.

(6) The candidate resolves forces perpendicular to the slope, giving the correct normal reaction.

(7) The candidate links the values of F and R using $F < \mu \times R$ and substitutes the numerical values correctly.

(8) The value of the coefficient of friction is correct to 3 decimal places as requested. Mark for (b) = 4 out of 4

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

- In part (a), the candidate used an incorrect inequality throughout the question, and this meant that they were not awarded the final mark. The candidate needed to make a statement that motion would take place when $40 > F + 5g \sin 30$, leading to $F < 15$. This would have resulted in the given inequality.
- The candidate's response to part (b) was fully correct.

Example Candidate Response – middle

Examiner comments

6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ .

(a)

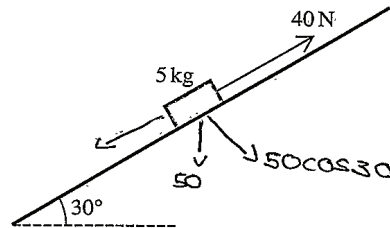


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{3}\sqrt{3}$.

[4]

..... $R = 50 \cos 30$ $F = \mu R$

..... $R = 25\sqrt{3}$ ① $\therefore F = 25\sqrt{3}\mu$

..... ~~$40 - 25\sqrt{3}\mu = 0$~~

..... ~~$40 = 25\sqrt{3}\mu$~~

..... ~~$= \frac{40}{25\sqrt{3}}$~~

..... $\mu = \frac{50 \sin 30}{25}$

..... 25μ

..... $40 - 25 - 25\sqrt{3}\mu = 0$ ②

..... $15 = 25\sqrt{3}\mu$

..... $\mu = \frac{\sqrt{3}}{5}$ ③

1 The candidate correctly evaluates the normal reaction.

2 The candidate suggests that the forces acting along the slope are in balance. In fact, there should be a net force acting up the plane which should be shown by replacing the = sign with a > sign.

3 The candidate reaches a correct extreme value of the coefficient of friction, but they do not provide a convincing argument that this is the maximum value. Mark for (a) = 3 out of 4

Example Candidate Response – middle, continued

Examiner comments

6 (b)

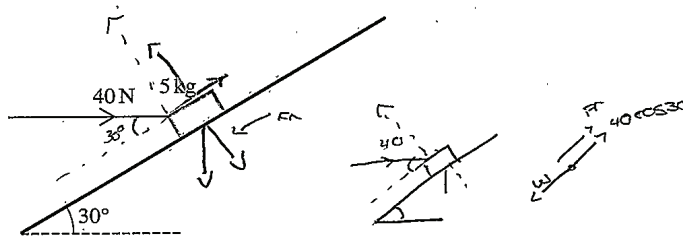


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

$40 \cos 30 = 50 \sin 30$ $R = 50 \cos 30$
 $= 25 \text{ N}$ $R = 25 \sqrt{3}$
 $F = 25 \sqrt{3} \mu$
 $F = \mu R$ 4
 ~~$40 \cos 30 + 50 \sin 30 - 25 - F = 0$~~ $40 \sin 30 + R = 50 \cos 30$
 ~~$25 \sqrt{3} \mu = -25 + 20 \sqrt{3}$~~ $R = 23.3 \text{ N}$
 $F = 23.3 \mu$
 $40 \sin 30 + R = 50 \cos 30$ $-20 + 25 \sqrt{3} \mu$
 $R = \frac{-20 + 25 \sqrt{3} \mu}{5 \sqrt{3}}$ $F = 5 \sqrt{3} \mu$
5 $R = 5 \sqrt{3} \mu$
 $40 \cos 30 - 25 - 5 \sqrt{3} \mu = 0$
 $5 \sqrt{3} \mu = -25 + 20 \sqrt{3}$ 6
 $40 \cos 30 + F - 25 = 0$
 $F + (-25 + 20 \sqrt{3}) = 0$
 $40 \cos 30 + 23.3 - 25 = 0$ 7
 $5 \sqrt{3} = 25 - 40 \cos 30$

4 The candidate attempts to find the normal reaction but the sign of the term $40 \sin 30$ is incorrect.

5 Although the candidate does not state this, the first two terms of this equation represent the friction term.

6 The candidate uses the relationship $F = \mu \times R$ but with an incorrect expression for the normal reaction R .

7 The candidate does not find a final value of the coefficient of friction as requested. Mark for (b) = 2 out of 4

Total mark awarded = 5 out of 8

How the candidate could have improved their answer

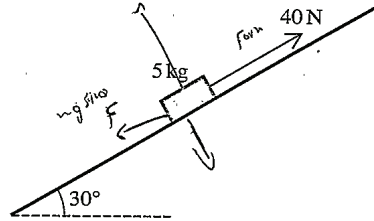
- In part (a), the candidate made no attempt to show that motion was taking place. All their equations related to equilibrium. If they had stated that the force acting on the block up the slope was positive, this could have led to the required inequality.
- In part (b), the candidate found an incorrect value of the normal reaction R because of a sign error in resolving forces perpendicular to the slope. A better diagram showing all forces might have helped them correct this. They could have attempted to find the coefficient of friction at the end.

Example Candidate Response – low

Examiner comments

6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ .

(a)



$$F + mg \sin \theta = 40 \text{ N}$$

Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$. [4]

Mass = 5 kg 1

normal component $R \Rightarrow Mg \cos \theta \Rightarrow (25\sqrt{3} / 43.3)$

Friction $\Rightarrow (Mg \sin \theta) \Rightarrow 25 \text{ N} - 40 \text{ N} \Rightarrow (25 \text{ N})$
 $\Rightarrow 15 \text{ N}$

$F = \mu R$

$(\mu \Rightarrow \frac{1}{5} \sqrt{3}) \Rightarrow 0.346$ 3

$(\mu \Rightarrow 0.577 \text{ N})$ 2

1 The candidate gives a correct value for the normal reaction.

2 The candidate does not explain that they are resolving along the slope.

3 The candidate states the given answer without proof.
 Mark for (a) = 1 out of 4

Example Candidate Response – low, continued

Examiner comments

(b)

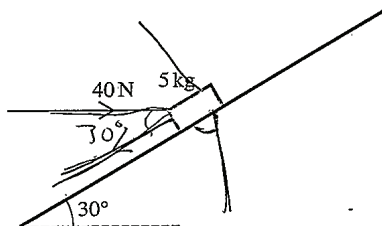


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

① $R \Rightarrow 5 \times 10 \times \cos 30 = 40 \sin 30$ 4

$\Rightarrow \underline{23.3 \text{ N}}$

② $F \Rightarrow Mg \sin 30 + 4 \cos 30$ 5

$F \Rightarrow \underline{7.54 \text{ N}}$

$F = \mu R$

$\mu \Rightarrow \underline{0.152}$ 6

(a)

4 The candidate attempts to find the normal reaction but the sign of the term $40 \sin 30$ is incorrect.

5 The candidate attempts to find friction but the force of 4 N should be 40 N.

6 The candidate states the answer without proof. Mark for (b) = 1 out of 4

Total mark awarded = 2 out of 8

How the candidate could have improved their answer

- In part (a), the candidate needed a clear explanation of resolving forces along the plane. Once friction had been found in the form $F < 15$, they could have used $F = \mu \times R$ to prove the given inequality convincingly. This means showing all working when an answer is given.
- In part (b), they made a sign error in finding the normal reaction and they did not use the correct forces when finding the friction term. If they had done this, they could have evaluated the coefficient of friction as $\frac{F}{R}$ leading to 0.152 to three decimal places.

Common mistakes candidates made in this question

- A common error was to find the required value in part (a) but not offering a convincing proof that it was a maximum. This was generally due to using an equals sign instead of the correct inequality.
- Using the same normal reaction in part (b) as in part (a) instead of a two-term expression.
- Sign errors in the evaluation of friction and normal reaction.
- Using sine and cosine incorrectly in the components when resolving forces.

Question 7

Example Candidate Response – high

Examiner comments

7 A particle P moves in a straight line, starting from a point O with velocity 1.72 m s^{-1} . The acceleration $a \text{ m s}^{-2}$ of the particle, $t \text{ s}$ after leaving O , is given by $a = 0.1t^{\frac{3}{2}}$.

(a) Find the value of t when the velocity of P is 3 m s^{-1} . [4]

$$v = \int (0.1 t^{\frac{3}{2}}) = \frac{1}{25} t^{\frac{5}{2}} + c \quad 1$$

$$c = 1.72 \quad 2$$

$$3 = \frac{1}{25} t^{\frac{5}{2}} + 1.72 \quad 3$$

$$t^{\frac{5}{2}} = 73.28$$

$$t = \cancel{81.874} \quad 5.575 \quad 4$$

(b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]

$$\int \left(\frac{1}{25} t^{\frac{3}{2}} + 1.72 \right) = \frac{2}{175} t^{\frac{7}{2}} + 1.72t + c \quad 5$$

$$c = 0 \quad 6$$

$$s = \frac{2}{175} (2)^{\frac{7}{2}} + 1.72(2) = \boxed{3.57 \text{ m}} \quad 8$$

1 The candidate integrates correctly to find v .

2 The candidate obtains the constant of integration correctly.

3 The candidate attempts to solve the equation to find the time at which the velocity is 3 metres per second. Their method is correct but a numerical error led to 73.28 rather than 32.

4 The candidate gives an incorrect final answer. Mark for (a) = 3 out of 4

5 The candidate integrates correctly to find an expression for the displacement.

6 The candidate finds the correct constant of integration.

7 The candidate uses the value $t = 2$ to find the required displacement.

8 The candidate's value of s is correct. Mark for (b) = 2 out of 3

Total mark awarded = 6 out of 7

How the candidate could have improved their answer

- In part (a) they could have checked for arithmetical errors in order to obtain the correct answer of $t = 4$.
- In part (b) their solution was fully correct.

Example Candidate Response – high, continued

Examiner comments

7 A particle P moves in a straight line, starting from a point O with velocity 1.72 m s^{-1} . The acceleration $a \text{ m s}^{-2}$ of the particle, t s after leaving O , is given by $a = 0.1t^{\frac{3}{2}}$.

(a) Find the value of t when the velocity of P is 3 m s^{-1} . [4]

$$a = 0.1t^{\frac{3}{2}}$$

$$v = \frac{0.1t^{2.5}}{2.5} + C \quad \textcircled{1}$$

$$v = 0.04t^{2.5} + C$$

$$t = 0 \quad v = 1.72 \quad 1.72 = 0.04(0)^{2.5} + C \quad \textcircled{2}$$

$$C = 1.72$$

~~$$v = 0.04(3)^{2.5} + 1.72$$~~

~~$$v = 0.62 + 1.72$$~~

~~$$v = 2.34$$~~

$$v = 3 = 0.04(t)^{2.5} + 1.72 \quad \textcircled{3}$$

$$1.28 = 0.04(t)^{2.5}$$

$$32 = (t)^{\frac{5}{2}} \quad \textcircled{4}$$

~~$$32 = t^{\frac{5}{2}} \quad t = \sqrt[2.5]{32}$$~~

$$\sqrt[2.5]{32} = t \quad t = 4 \text{ s} \quad \textcircled{5}$$

1 The candidate integrates correctly to find v .

2 The candidate uses an initial condition to find the constant of integration correctly.

3 The candidate sets up an equation to find the time at which $v = 3$.

4 The candidate attempts to solve the equation.

5 The candidate's solution is correct.
Mark for (a) = 4 out of 4

Example Candidate Response – high, continued

Examiner comments

- (b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]

$$v = 0.04t^{2.5} + 1.72$$

$$d = \frac{0.04}{3.5} t^{3.5} + 1.72t + C \quad 6$$

$$t=2 \quad d = \frac{0.04}{3.5} (2)^{3.5} + 1.72(2) \quad 7$$

$$d = 0.129 + 3.44$$

$$d = 3.6 \quad 8$$

6 The candidate integrates correctly to find the displacement.

7 The candidate uses the correct limits. When $t = 0$ the expression equates to zero.

8 The candidate evaluates both terms correctly.

9 The candidate's final answer is only given to 2 significant figures whereas the rubric requires 3 significant figures. Mark for (b) = 2 out of 3

Total mark awarded = 6 out of 7

How the candidate could have improved their answer

- The answer to part (a) was correct.
- In part (b) all the working was correct, but the candidate should have given the final answer as 3.57 (correct to 3 significant figures) rather than as 3.6 (to 2 significant figures). The rubric for this paper states the requirement to give all non-exact answers to 3 significant figures.

Example Candidate Response – middle

Examiner comments

7 A particle P moves in a straight line, starting from a point O with velocity 1.72 m s^{-1} . The acceleration $a \text{ m s}^{-2}$ of the particle, t s after leaving O , is given by $a = 0.1t^{\frac{3}{2}}$.

(a) Find the value of t when the velocity of P is 3 m s^{-1} . [4]

$$v = 0.15 t^{\frac{1}{2}} + C$$

$$c = 1.72$$

$$V = 0.15 t^{\frac{1}{2}} + 1.72$$

$$3 = 0.15 \sqrt{t} + 1.72$$

$$t = \left(\frac{3 - 1.72}{0.15} \right)^2$$

$$= 72.8 \text{ s}$$

$$V = \frac{1}{15} t^{\frac{3}{2}} + 1.72$$

$$t = \sqrt[2]{\frac{3 - 1.72}{\frac{1}{15}}}$$

$$t = 3.26 \text{ s}$$

(b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]

$$s = \frac{2}{75} t^{\frac{7}{2}} + 1.72 t$$

$$s = \frac{2}{75} (2)^{\frac{7}{2}} + 1.72 (2)$$

s from O to $P = 3.74 \text{ m}$

From Part

1 The candidate crosses out their work and replaces it so the replacement work is marked.

2 The candidate attempts to integrate to find v but makes an error in the coefficient of the first term which should be $\frac{1}{25}$ rather than $\frac{1}{15}$.

3 The candidate uses the correct method to find the value of t when $v = 3$, but the incorrect $\frac{1}{15}$ is seen here.

4 The candidate's final answer is incorrect due to the earlier error. Mark for (a) = 2 out of 4

5 The candidate uses the correct method of integration to find the displacement but their earlier error results in an incorrect coefficient $\frac{2}{75}$ which should be $\frac{2}{175}$.

6 The candidate applies the given limits correctly to the expression for displacement.

7 The candidate's error in part (a) results in an incorrect final answer. Mark for (b) = 2 out of 3

Total mark awarded = 4 out of 7

How the candidate could have improved their answer

- The candidate attempted to integrate but although the power of the first term was correctly increased by one, the coefficient of this term should have been divided by the new power of 2.5 which would have led to the correct expression for displacement. Their method used to solve for t was correct. If they had carried out the initial integration correctly, they could have obtained the correct answer of $t = 4$.
- In part (b) the error meant that, although they used the correct method, their final answer was incorrect.

Example Candidate Response – low	Examiner comments
<p>7 A particle P moves in a straight line, starting from a point O with velocity 1.72 m s^{-1}. The acceleration $a \text{ m s}^{-2}$ of the particle, $t \text{ s}$ after leaving O, is given by $a = 0.1t^{\frac{3}{2}}$.</p> <p>(a) Find the value of t when the velocity of P is 3 m s^{-1}. [4]</p> <p>.....</p> <p>..... $a = 0.1 t^{\frac{3}{2}}$</p> <p>..... $\Rightarrow v = v_0 + at = 1.72 + 0.1 t^{\frac{5}{2}}$ ①</p> <p>..... $v = 3$</p> <p>..... $\Rightarrow 3 = 1.72 + 0.1 t^{\frac{5}{2}}$ ②</p> <p>..... $\Rightarrow t^{\frac{5}{2}} = 12.8$</p> <p>..... $\Rightarrow t \approx \sqrt[5]{12.8} \approx 2.77 \text{ (s)}$ ③</p> <p>(b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]</p> <p>..... $v = 1.72 + 0.1 t^{\frac{5}{2}}$</p> <p>..... $\Rightarrow s = \int v dt = \int (1.72 + 0.1 t^{\frac{5}{2}}) dt$</p> <p>..... $= 1.72t + \frac{1}{35} t^{\frac{7}{2}} + C$ ④</p> <p>..... $t = 0, s = 0 \Rightarrow C = 0$</p> <p>..... $s = 1.72t + \frac{1}{35} t^{\frac{7}{2}}$</p> <p>..... ⑤</p> <p>..... $t = 2 \Rightarrow s = 1.72 \times 2 + \frac{1}{35} \times 2^{\frac{7}{2}} \approx 3.76$ ⑥</p>	<p>① The candidate's expression for acceleration is a function of t but the equation for constant acceleration is wrongly used.</p> <p>② Although the candidate uses the condition at $t = 0$, their original method is incorrect and cannot be awarded any marks.</p> <p>③ The candidate's answer is incorrect due to earlier errors. Mark for (a) = 0 out of 4</p> <p>④ Although the candidate found v by incorrect methods, they attempt to find displacement using the correct method of integration.</p> <p>⑤ Although the candidate's expression for displacement is incorrect, they use correct limits.</p> <p>⑥ The candidate's final answer is incorrect as they used the wrong method to find v. However, they are awarded all method marks here. Mark for (b) = 2 out of 3</p> <p>Total mark awarded = 2 out of 7</p>

How the candidate could have improved their answer

- In part (a), acceleration was given as a function of time, so the candidate should have used integration to find the velocity. If they had integrated and then applied the initial condition, they could have set $v = 3$ and solved for t .
- In part (b) they used the correct method of integration but with their incorrect expression for v found in part (a). If they had used the correct v , the method they used here would have produced the correct solution.

Common mistakes candidates made in this question

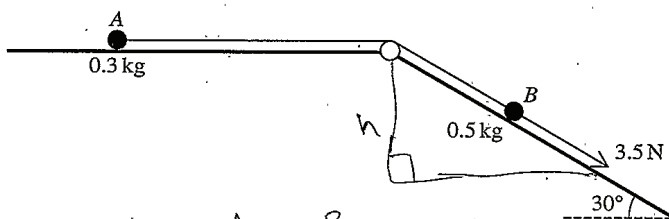
- Using the equations for constant acceleration when the acceleration was not constant. Whenever a problem involves acceleration as a function of time, then integration techniques must be applied.
- Solving the equation in part (a) which involved a fractional power.
- Not including all the terms in the expression for v in part (b), usually because the constant of integration for v had not been found correctly.

Question 8

Example Candidate Response – high

Examiner comments

8



Two particles A and B, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane (see diagram).

(a) Given that both planes are smooth, find the tension in the string (and the acceleration of B). [5]

$$\# \text{ find } a$$

$$W_{\parallel B} = mg \sin \theta = (0.5)(10)(\sin 30) = 2.5 \text{ N}$$

$$F = ma$$

$$W_{\parallel B} + 3.5 = (0.3 + 0.5)a$$

$$2.5 + 3.5 = (0.8)a$$

$$6 = 0.8a$$

$$a = 7.5 \text{ ms}^{-2} \text{ (down the slope)}$$

find T

$$T - W_{\parallel B} = ma$$

$$T - 2.5 = (0.5)(7.5)$$

$$T - 2.5 = 3.75$$

$$T = 6.25 \text{ N}$$

1 The candidate applies Newton's 2nd law to the system of two particles so does not need the tension. The equation is stated correctly.

2 The candidate solves the equation correctly and gives the correct value of a .

3 The candidate attempts to apply Newton's 2nd law to particle B. However, they do not include the driving force of 3.5 N which acts on B. Also, the signs of the terms on the left-hand side are incorrect.

4 The candidate finds a value of T but it is incorrect because of errors in their equation above. Mark for (a) = 3 out of 5

Example Candidate Response – high, continued

Examiner comments

(b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J. W_{DF}

Use an energy method to find the speed of B when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that A does not hit the pulley when it moves 0.6 m.] [4]

$$W_{\text{by force}} + E_{\text{loss B}} = W_{DF} + E_{\text{gain A+B}}$$

~~$$W = Fs$$~~

$$W_{DF} = (3.5)(0.6) = 2.1 \text{ J} \quad 5$$

$$E_{\text{loss B}} = \mu gh \quad h = 0.6 \sin(30) = 0.3 \text{ m}$$

$$E_{\text{loss B}} = (0.5)(10)(0.3) = 1.5 \text{ J} \quad 6$$

~~$$E_{\text{gain}} = \frac{1}{2}(m_B v_B^2 + m_A v_A^2) = \frac{1}{2}(m_A + m_B)v^2$$~~

$$E_{\text{gain}} = \frac{1}{2}(0.3 + 0.5)v^2 = 0.4v^2 \quad 7$$

$$2.1 + 1.5 = 1.1 + 0.4v^2 \quad 8$$

$$3.6 = 1.1 + 0.4v^2 \quad 9$$

$$2.5 = 0.4v^2$$

$$v^2 = 6.25$$

$$v = 2.5 \text{ ms}^{-1} \quad 10$$

5 The candidate finds the work done by the driving force correctly.

6 The candidate finds the loss of potential energy correctly.

7 The candidate evaluates the kinetic energy gained and correctly includes the effect of both particles.

8 The left-hand side of the candidate's work-energy equation consists of the effect of the driving force and the potential energy.

9 The right-hand side of the candidate's equation correctly shows the effect of the work done against friction and the increase in kinetic energy. All terms in the equation are correct.

10 The candidate solves the equation correctly to find the correct speed of particle B. Mark for (b) = 4 out of 4

Total mark awarded = 7 out of 9

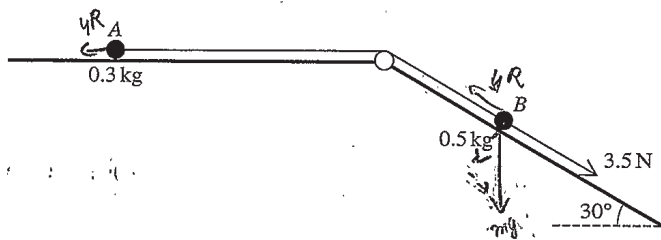
How the candidate could have improved their answer

In part (a), the candidate used the system equation correctly and this enabled them to find the acceleration. They needed to set up an equation for T from either of the two particles and they chose particle B. They stated correctly that the acceleration of 7.5 is down the plane, but two of the forces used in their equation have the wrong sign and they omitted the driving force. The correct equation would have been $3.5 + 0.5g \sin 30 - T = 0.5a$. Using the value of a they found earlier would have led to the correct value of the tension as 2.25 N.

Example Candidate Response – high, continued

Examiner comments

8



Two particles *A* and *B*, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with *A* on the horizontal plane and *B* on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and *B* can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to *B* acting down the plane (see diagram).

(a) Given that both planes are smooth, find the tension in the string and the acceleration of *B*. [5]

$mg \sin \theta + 3.5 - T = 0.5g a$ $A = 0.3$
 $0.5g \sin 30 + 3.5 - T = 0.5a$ $B = 0.5$
 $6 - T = 0.5a$ ①
 $T = ma$ $6 - 0.3a = 0.5a$ ③
 $T = 0.3a$ ② $6 = 0.8a$
 $1.25a$ ④
 $T = 0.3 \times 1.25$
 $T = 0.375$ ⑤

1 The candidate applies Newton's 2nd law correctly to particle *B*.

2 The candidate applies Newton's 2nd law correctly to particle *A*.

3 The candidate sets up an equation and then solve simultaneously using the substitution method.

4 The solution is incorrect due to a numerical error in the calculation.

5 The candidate substitutes the value of *a* in order to find *T*. The method is correct, but they use their incorrect value of *a*.

Example Candidate Response – high, continued

Examiner comments

(b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J.

Use an energy method to find the speed of B when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that A does not hit the pulley when it moves 0.6 m.] [4]

Work done by driving force = work against R = ΔE

~~$\Delta E = \frac{1}{2}(0.5)(v^2) + \frac{1}{2}(0.3)(v^2)$~~
 ~~$= 0.25v^2 + 0.15v^2$~~

increase in K.E. = $\frac{1}{2}(0.5)(v^2) + \frac{1}{2}(0.3)(v^2)$ $h = 0.6 \sin 30$
 $= 0.25v^2 + 0.15v^2$ $= 0.3$
 $= 0.4v^2$ 6

decrease in G.P.E. = $(0.5)(10)(0.3)$ 7
 $= 1.5$

work by driving - work against R = ΔE

$3.5 \times 0.6 - 1.1$ 8 $= 0.4v^2 - 1.5$ 9 10

$1 = 0.4v^2 - 1.5$

$\frac{2.5}{0.4} = v^2$

$\sqrt{6.25} = v$

$v = 2.5$ 11

~~(0.5)(3.5) = 1.1~~ $F_c = 1.1$

~~3.5 =~~

- 6 The candidate evaluates the kinetic energy of the system correctly.
- 7 This is correct for the potential energy of the system.
- 8 The candidate uses the work-energy equation. On the left-hand side is the work done by the driving force and the work done against friction.
- 9 On the right-hand side of their equation is the kinetic energy and potential energy.
- 10 All terms in the work-energy equation are correct and with the correct sign.
- 11 The candidate solves the equation and finds the correct speed of particle B. Mark for (a) = 3 out of 5

Example Candidate Response – high, continued

Examiner comments

8 If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

$$F \times 0.12 = 1.1$$

$$F = 9.167$$

$$3.5 - \frac{5.5}{6} a + 8.5 a - T = (0.5) a$$

$$\frac{19}{6} - T = 0.5 a$$

12

$$2as = v^2 - u^2$$

$$2(11.5) \frac{1}{3} (0.6) = v^2$$

$$\sqrt{46} = v$$

$$T - \frac{5.5}{6} = 0.5 \cdot 3a$$

$$T = 0.3a + \frac{5.5}{6}$$

~~0.9~~

$$0.5a + \frac{19}{6} = 0.3a + \frac{5.5}{6}$$

$$0.2a = \frac{23}{3}$$

$$a = \frac{115}{3}$$

$$\frac{6}{2} \quad 3.5 \times 0.6 - 1.1 = \frac{1}{2} \times 0.3 \times 46 + 0.25 v^2 - 1.5$$

12 Rough work is crossed out and not marked. Mark for (b) = 4 out of 4

Total mark awarded = 7 out of 9

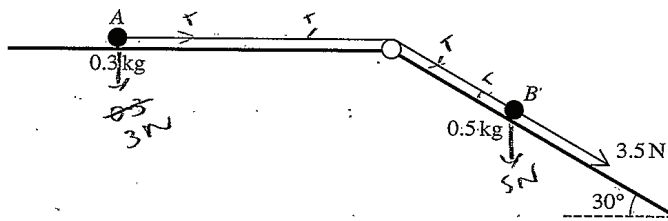
How the candidate could have improved their answer

- In part (a) the candidate has shown two relevant equations for the particles A and B correctly. They made an error in solving the equations $6 \div 0.8$ is in fact 7.5 and not 1.25 as the candidate stated. This error affected their value of T which should have been 2.25 N.
- In part (b) they used the work-energy equation correctly and obtained a correct answer.

Example Candidate Response – middle

Examiner comments

8



Two particles A and B, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane. (see diagram).

(a) Given that both planes are smooth, find the tension in the string and the acceleration of B. [5]

.....
 $F = ma$
 $3.5 - T + T + T + T + 5 \sin 30^\circ = 0.8 a$ 1
 $a = 3.75 \text{ ms}^{-2}$ 2
 $3.5 + 5 \sin 30^\circ - T = 0.5 \times 3.75$ 3
 $6 - T = 1.875$
 $T = 4.13 \text{ N}$ 4

- 1 The candidate applies the correct system equation.
- 2 The candidate's value of a is incorrect.
- 3 The candidate applies Newton's 2nd law correctly to particle B.
- 4 Since the candidate's value of a was incorrect, their value for T is also incorrect.
 Mark for (a) = 3 out of 5

Example Candidate Response – middle, continued

Examiner comments

(b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J.

Use an energy method to find the speed of *B* when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that *A* does not hit the pulley when it moves 0.6 m.] [4]

.....

 wd by driving force = wd by resistive force = gain in ke
 + gain in pe

 $1.1 = \frac{1}{2} \times 0.5 \times v^2 + \cancel{A \times 0} \times 0.5 \times 10 \sin 30 \times 0.6 +$
 $\frac{1}{2} \times m v^2 + mgh$
 $T = \frac{0.5 v^2}{2} + 1.5$
 $V = 1.26 \text{ ms}^{-1}$
 $1.1 = \frac{0.5 v^2}{2} + 1.5 + 0.674 + 1.8$
 $\cancel{6 = 0.8a}$
 $u = a$
 $a = 3.75$
 $s = 0.6$
 $V = 1.26 \text{ ms}^{-1}$
 $A \quad V = 2.12$

5 The candidate finds the kinetic energy of particle *B* but this work-energy equation must also include the effect of the kinetic energy of particle *A* which moves with the same speed.

6 The candidate finds the potential energy loss correctly.

7 The candidate's final two terms include an undefined mass *m* and so, at this stage, it is not possible to see what these terms represent.

8 The candidate's final work-energy equation does not include the kinetic energy of particle *A*. There is also no consideration of the work done by the 3.5 N driving force.

9 Due to missing terms in their work-energy equation, the candidate reaches an incorrect answer. Mark for (b) = 1 out of 4

Total mark awarded = 4 out of 9

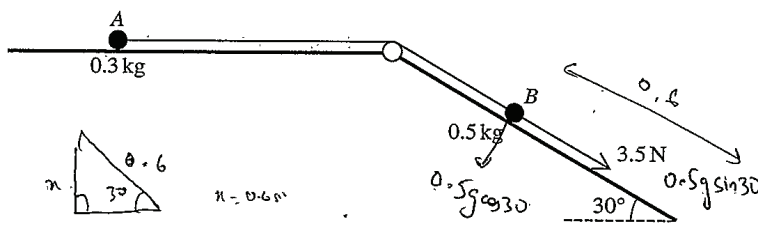
How the candidate could have improved their answer

- In part (a), the candidate has stated the system equation correctly. Their error appeared when combining the two terms on the left-hand side of the equation, resulting in an incorrect value of *a*. If they had simplified the equation to $6 = 0.8a$ and solved for *a*, they would have obtained the correct value of 7.5. They also stated the equation involving *T* correctly and, had they used *a* as 7.5, would have obtained the correct value of the tension as 2.25.
- In part (b), two terms are missing from their work-energy equation. They should have included the kinetic energy of particle *A*, $\frac{1}{2} \times 0.3 \times v^2$, and the work done against friction, 3.5×0.6 in order to obtain the correct value of *v* as 2.5.

Example Candidate Response – low

Examiner comments

8



Two particles *A* and *B*, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with *A* on the horizontal plane and *B* on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and *B* can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to *B* acting down the plane (see diagram).

(a) Given that both planes are smooth, find the tension in the string and the acceleration of *B*. [5]

For A: $T - 0.3g = 0.3a$ ①

For B: $3.5 + 0.5g \sin 30 - T = 0.5a$ ②

$T = 0.3a + 3$

$6 - (0.3a + 3) = 0.5a$ ③

$T = 0.3(3.75) + 3$

$3 = 0.8a$

$T = 4.125 \text{ N}$ ⑤

$a = 3.75 \text{ m/s}^2$ ④

① The candidate attempts to apply Newton's 2nd law horizontally to particle *A* but incorrectly includes the vertical force $0.3g$.

② The candidate applies Newton's 2nd law correctly to particle *B*.

③ The candidate attempts to solve the two equations simultaneously but the equation for *A* is incorrect.

④ Consequently, the candidate's value of *a* is also incorrect.

⑤ The candidate's solution for *T* uses the incorrect value of *a* and so their value of *T* is incorrect. Mark for (a) = 1 out of 5

Example Candidate Response – low, continued

Examiner comments

8 (b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J.

Use an energy method to find the speed of *B* when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that *A* does not hit the pulley when it moves 0.6 m.] [4]

Energy at rest = Energy at end + W_{id} by F_r

$$KE_A + PE_A + KE_B + PE_B = KE_A' + PE_A' + KE_B' + PE_B'$$

$$0 + PE_A + 0 + 0.5g(0.6 \sin 30) = \frac{1}{2}(0.3)v^2 + PE_A' + \frac{1}{2}(0.4)v^2 + 0$$

$$0.5g(0.6 \sin 30) = \frac{1}{2}(0.3)v^2 + \frac{1}{2}(0.4)v^2$$

$$\frac{3}{2} = \frac{3}{20}v^2 + \frac{1}{5}v^2$$

$$\frac{3}{2} = \frac{7}{20}v^2$$

$$v^2 = \frac{3}{7} \times \frac{20}{1}$$

$$v^2 = \frac{30}{7}$$

$$v = 2.07 \text{ m/s}$$

6 The candidate uses the correct potential energy loss.

7 The candidate makes an error in finding the kinetic energy. They use the 0.3 kg particle, but also a 0.4 kg particle which should be 0.5 kg.

8 The candidate's work-energy equation uses the incorrect kinetic energy and does not include the effect of the work done by the driving force 3.5 N. It also omits the given work done against friction of 1.1 J.

9 The candidate's value of *v* is incorrect. Mark for (b) = 1 out of 4

Total mark awarded = 2 out of 9

How the candidate could have improved their answer

- The candidate's equation for *A* in part (a) included an extra weight term (0.3 g). Their equation should have been $T = 0.3a$. The equation for *B* is correct. Solving two correct equations would have led to the correct values $a = 7.5$ and $T = 2.25$.
- In the work-energy equation for part (b), the candidate's kinetic energy of particle *B* should have been $\frac{1}{2}(0.5)v^2$ rather than $\frac{1}{2}(0.4)v^2$. The candidate omitted two terms from this equation: the work done by the 3.5 N force (3.5×0.6) and the work done against friction, given in the question as 1.1 J. With all these correct terms in the equation, they could have reached the correct solution of $v = 2.5$.

Common mistakes candidates made in this question

- Applying Newton's 2nd law to the horizontal motion of particle A but wrongly including the vertical force $0.3g$.
- Errors in the components of the $0.5g$ weight.
- Numerical errors when combining the forces and when solving the simultaneous equations.
- In part **(b)** omitting the effect of some of the forces acting on the system. The question required a specific method of using energy principles and almost all candidates followed this method. However, many did not include the kinetic energy of particle A which moves with the same speed as B . Some also forgot to include the effect of the work done by the 3.5 N driving force. Although the work done against friction was given as 1.1 J , some candidates did not include this information in their working.

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