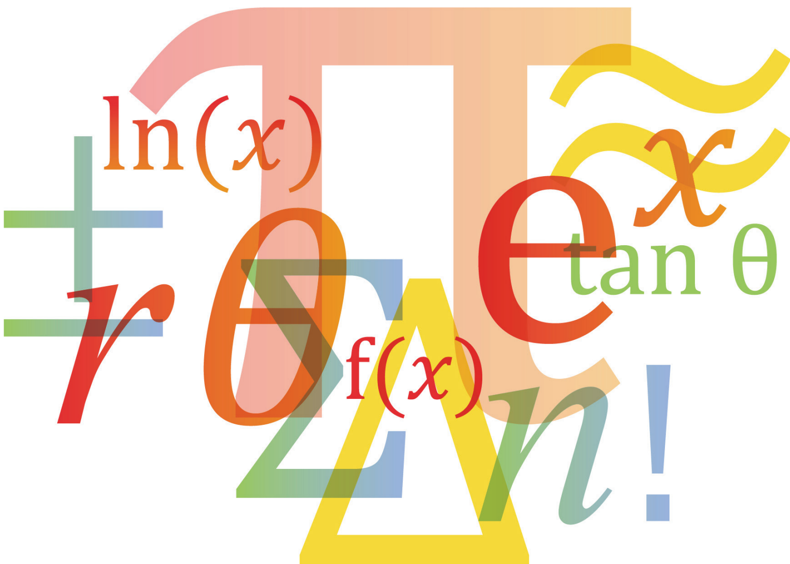
Cambridge Assessment
International Education

Example Candidate Responses – Paper 2

Cambridge International AS & A Level
Mathematics 9709

For examination from 2020



Cambridge Pathway

In order to help us develop the highest quality resources, we are undertaking a continuous programme of review; not only to measure the success of our resources but also to highlight areas for improvement and to identify new development needs.

We invite you to complete our survey by visiting the website below. Your comments on the quality and relevance of our resources are very important to us.

www.surveymonkey.co.uk/r/GL6ZNJB

Would you like to become a Cambridge International consultant and help us develop support materials?

Please follow the link below to register your interest.

www.cambridgeinternational.org/cambridge-for/teachers/teacherconsultants/

Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709 and to show how different levels of candidates' performance (high, middle and low) relate to the syllabus requirements.

In this booklet, candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

November 2020 Question Paper 22

November 2020 Paper 22 Mark Scheme

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

Example Candidate Response – low	Examiner comments
1 Solve the equation $7\cot\theta = 3\,\mathrm{cosec}\,\theta$ for $0° < \theta < 90°$. [3] $7\cot\theta = 3\left(\frac{1}{\sin\theta}\right)$ $7\cot\theta = 3\left(\frac{1}{\sin\theta}\right)$ 1 $7\cot\theta = \frac{3}{3\sin\theta}$ $7\cot\theta(3\sin\theta) - 3 = 0$ 2	1 The candidate identifies the term containing $\sin\theta$ correctly. However, the term in $\cot\theta$ is not written in terms of $\sin\theta$ and $\cos\theta$ as required for the first mark. 2 The candidate is unable to make any progress as their equation is not in terms of $\cos\theta$ only. **Total mark awarded =0 out of 3**

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

The candidate needed to be familiar with the definition of $\cot\theta$ in terms of $\sin\theta$ and $\cos\theta$. They could then have simplified the result to produce an equation in terms of $\cos\theta$ only and solved the equation.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- Candidates needed to be familiar with the definitions of $\cot\theta$ and $\mathrm{cosec}\,\theta$ in terms of $\sin\theta$ and $\cos\theta$. Further progress in solving the equation is not otherwise possible.
- Errors in simplifying a correct basic equation to obtain $\cos\theta = \frac{3}{7}$ were also common.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high	Examiner comments
1 Solve the equation $7 \cot \theta = 3 \operatorname{cosec} \theta$ for $0° < \theta < 90°$. [3] $7\frac{\cos\theta}{\sin\theta} = \frac{3}{\sin\theta}$ $7\cos\theta = 3$ (1) $\cos\theta = 3/7$ $\theta = 64.6$	(1) The candidate provides a completely correct solution which is awarded full marks. Each step is shown clearly. **Total mark awarded = 3 out of 3**

How the candidate could have improved their answer

No improvement is necessary as the candidate has shown all the necessary detail. The work was set out clearly.

Example Candidate Response – middle	Examiner comments
1 Solve the equation $7 \cot \theta = 3 \operatorname{cosec} \theta$ for $0° < \theta < 90°$. [3] $7\frac{\cos\theta}{\sin\theta} = 3\frac{1}{\sin\theta}$ (1) $\cancel{\frac{3}{7}} = \cancel{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} = \frac{3}{7}}$ $\frac{7}{3} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$ $\cancel{\sin^2\theta}$ (2) $\frac{7}{3} = \frac{1-\cos\theta}{\sin^2\theta}$ $\frac{7}{3} = \frac{\sin^2\theta}{\sin^2\theta}$ $\frac{7}{3} = \sin\theta$ $\therefore \sin^{-1}\left(\frac{7}{3}\right) = \theta$ $\theta = 25.4$	(1) The candidate uses a correct statement in terms of sine and cosine and is awarded 1 mark. (2) The candidate uses an incorrect method of solution and hence an incorrect answer so no marks are awarded. **Total mark awarded = 1 out of 3**

How the candidate could have improved their answer

The candidate made an error in their method of solution, so they were awarded only the first mark. They should have multiplied both sides of their correct equation by $\sin\theta$ to reach an equation in terms of $\cos\theta$ only.

Example Candidate Response – low	Examiner comments
1 Solve the equation $7\cot\theta = 3\operatorname{cosec}\theta$ for $0° < \theta < 90°$. [3] $7\cot\theta = 3\left(\frac{1}{\sin\theta}\right)$ $7\cot\theta = 3\left(\frac{1}{\sin\theta}\right)$ ① $7\cot\theta = \frac{3}{3\sin\theta}$ $7\cot\theta(3\sin\theta) - 3 = 0$ ②	① The candidate identifies the term containing $\sin\theta$ correctly. However, the term in $\cot\theta$ is not written in terms of $\sin\theta$ and $\cos\theta$ as required for the first mark. ② The candidate is unable to make any progress as their equation is not in terms of $\cos\theta$ only. **Total mark awarded = 0 out of 3**

How the candidate could have improved their answer

The candidate needed to be familiar with the definition of $\cot\theta$ in terms of $\sin\theta$ and $\cos\theta$. They could then have simplified the result to produce an equation in terms of $\cos\theta$ only and solved the equation.

Common mistakes candidates made in this question

- Candidates needed to be familiar with the definitions of $\cot\theta$ and $\operatorname{cosec}\theta$ in terms of $\sin\theta$ and $\cos\theta$. Further progress in solving the equation is not otherwise possible.
- Errors in simplifying a correct basic equation to obtain $\cos\theta = \frac{3}{7}$ were also common.

Question 2

Example Candidate Response – high

2 Given that $\dfrac{2^{3x+2}+8}{2^{3x}-7} = 5$, find the value of 2^{3x} and hence, using logarithms, find the value of x correct to 4 significant figures. [5]

① $\dfrac{2^{3x+2}+8}{2^{3x}-7} = 5$

$2^{3x+2}+8 = 5(2^{3x}-7)$

$2^{3x+2}+8 = 10^{3x} - 35$

$8+35 = 10^{3x} - 2^{3x+2}$

$43 = 10^{3x} - 4(2^{3x})$ 1

$43 = 5(2^{3x}) - 4(2^{3x})$

2

② Let $2^{3x} = y$

$\therefore\ 43 = 5y - 4y$ 3

$43 = y$ $\therefore\ 2^{3x} = 43$

③ $\therefore\ 2^{3x} = 43$

$\dfrac{3x\log 2}{\log 2} = \dfrac{\log 43}{\log 2}$

$\dfrac{3x}{3} = \dfrac{\log_2 43}{3}$

$x = 1{,}809$ 4

Examiner comments

1 The candidate is awarded a mark for dealing correctly with the index of $3x+2$.

2 The candidate initially makes an error in multiplying 5 and 2^{3x}. This is then re-written correctly. The slip is tolerated as the incorrect form has not been used in the final answer.

3 The candidate solves the equation correctly for 2^{3x}.

4 The candidate applies logarithms correctly to obtain a correct solution.

Total mark awarded = 5 out of 5

How the candidate could have improved their answer

The candidate could have improved their answer by not writing 5×2^{3x} as 10^{3x}. Since they did not use this incorrect term but wrote it correctly later, arriving at the correct answer, the slip was tolerated.

Example Candidate Response – middle	Examiner comments
2 Given that $\dfrac{2^{3x+2}+8}{2^{3x}-7}=5$, find the value of 2^{3x} and hence, using logarithms, find the value of x correct to 4 significant figures. [5]	
$\dfrac{2^{3x+2}+8}{\cancel{2^{3x}-7}} \times \cancel{2^{3x}-7} = 5 \times 2^{3x}-7$	
$2^{3x+2} = 5(2^{3x}-7)$ (2)	
$2^{3x+2} = 5\times 2^{3x} - 35$	
(1) $2^{3x}\times 2^{2} = 5\times 2^{3x} - 35$	1 The candidate deals with the index $3x+2$ correctly.
$2^{3x}\times 2^{2} - 5\times 2^{3x} = -35$	2 The candidate omits the term of 8 in subsequent working.
$2^{3x}(2^{2}-5) = -35$	
$2^{3x} = \dfrac{-35}{(2^{2}-5)}$ (3)	3 The candidate uses a correct method of simplification and despite the missing 8, subsequent method marks are available.
$2^{3x} = \dfrac{-35}{-1}$	
$2^{3x} = 35$	
$\log 2^{3x} = \log 35$	
$3x\log 2 = \log 35$	
$\frac{1}{3}\times 3x = \dfrac{\log 35}{\log 2}\times\frac{1}{3}$	
$x = \frac{1}{3}\times\dfrac{\log 35}{\log 2}$ (4)	4 The candidate uses logarithms correctly on their equation which is in an appropriate form.
$x = 1.70976$	
$x = 1.710$	**Total mark awarded = 3 out of 5**

How the candidate could have improved their answer

The candidate has omitted the $+8$ term in the second line of their solution. This was regarded as a slip, so any subsequent method marks were still available. The candidate needed to check their solution carefully.

Example Candidate Response – low	Examiner comments
2 Given that $\dfrac{2^{3x+2}+8}{2^{3x}-7}=5$, find the value of 2^{3x} and hence, using logarithms, find the value of x correct to 4 significant figures. [5]	
let 2^{3x} be A'	
1 $\dfrac{2A+8}{A-7} = 5$	1 The candidate does not deal correctly with the index of $3x+2$. They make an error in the coefficient of A, using 2 rather than 4.
$2A+8 = 5A-35$	
$43 = 3\cancel{A}A$	
2 3	2 Although the candidate's coefficient of A is incorrect, they use a correct method to obtain a solution for 2^{3x}.
$A = \frac{43}{3}$	
$2^{3x} = \frac{43}{3}$	
$3x\ln 2 = \ln\left(\frac{43}{3}\right)$	
$x = \frac{1}{3}\left(\ln(43/3) \times \frac{1}{\ln 2}\right)$ 3	3 The candidate uses logarithms appropriately to find a solution to their equation and is awarded one mark for the method.
$x = 1.280$	**Total mark awarded = 2 out of 5**

How the candidate could have improved their answer

The candidate needed to check their simplification of 2^{3x+2} in order to obtain the correct coefficient of 2^{3x}. This was the only error as the candidate used a correct method to solve their resulting equation.

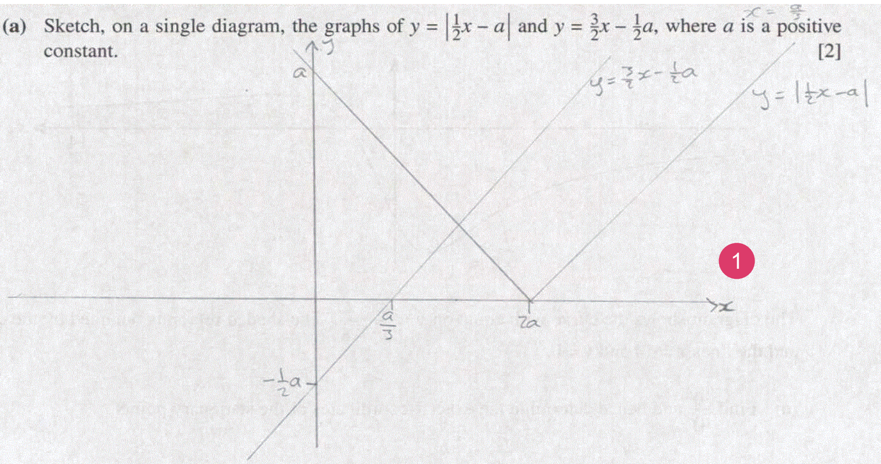
Common mistakes candidates made in this question

Many candidates chose to take logarithms of each term in the initial equation. This is clearly an incorrect approach that rarely made progress as an equation in 2^{3x} was never obtained.

Question 3

Example Candidate Response – high

3 (a) Sketch, on a single diagram, the graphs of $y = \left|\frac{1}{2}x - a\right|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]



1

(b) Find the coordinates of the point of intersection of the two graphs. [3]

$\frac{3}{2}x-\frac{1}{2}a=|\frac{1}{2}x-a|$

$\frac{3}{2}x-\frac{1}{2}a=-(\frac{1}{2}x-a)$ $\qquad$ $\frac{3}{2}x-\frac{1}{2}a=\frac{1}{2}x-a$

$\frac{3}{2}x-\frac{1}{2}a=-\frac{1}{2}x+a$ $\qquad$ $\frac{3}{2}x-\frac{1}{2}a=\frac{1}{2}x-a$

$\frac{3}{2}x+\frac{1}{2}x=a+\frac{1}{2}a$ $\qquad$ $\frac{3}{2}x-\frac{1}{2}x=-a+\frac{1}{2}a$

$4x=2(a+\frac{1}{2}a)$ $\qquad$ $x=-a+\frac{1}{2}a$

$4x=2a+a$ $\qquad$ $x=-\frac{1}{2}a$

2 $\quad$ $4x=3a$ $\qquad$ $\therefore y=$

$x=\frac{3}{4}a$ $\quad$ $y=\frac{5}{8}a$

$(\frac{3}{4}a, \frac{5}{8}a)$

(c) Deduce the solution of the inequality $\left|\frac{1}{2}x - a\right| > \frac{3}{2}x - \frac{1}{2}a$. [1]

$\cancel{x<\frac{3}{4}}$ $\quad$ $x>\frac{3}{4}a$ $\quad$ 3

Examiner comments

1 The candidate provides a correct graph. The vertex is in the correct position and the gradient of the straight line is greater than that of the modulus graph.
Mark for (a) = 2 out of 2

2 This is a correct method of solution. The candidate makes use of their graph and gives the coordinates of the point of intersection. The lack of a second y-coordinate suggests they have discounted the other x-coordinate.
Mark for (b) = 3 out of 3

3 Although the candidate identifies a correct critical value, their inequality is incorrect and no marks are awarded.
Mark for (c) = 0 out of 1

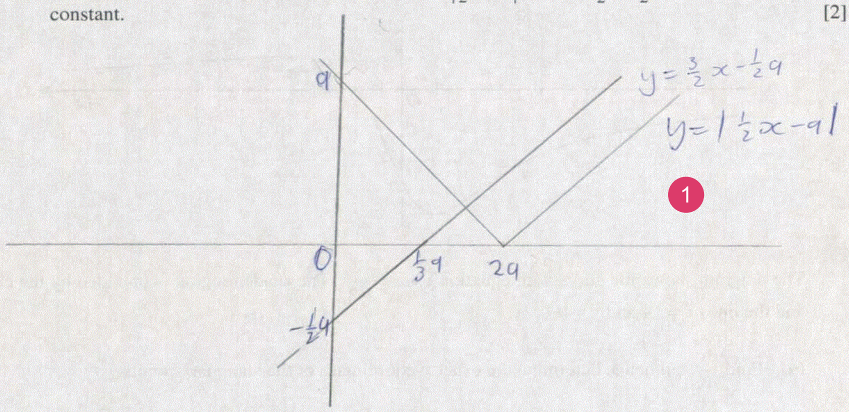
Total mark awarded = 5 out of 6

How the candidate could have improved their answer

Use of shading on the sketch in part **(a)** could have helped the candidate identify the correct region.

Example Candidate Response – middle

3 (a) Sketch, on a single diagram, the graphs of $y = \left|\frac{1}{2}x - a\right|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]



(b) Find the coordinates of the point of intersection of the two graphs. [3]

$y = |\frac{1}{2}x - a| \qquad y = \frac{3}{2}x - \frac{1}{2}a$

$|\frac{1}{2}x - a| = \frac{3}{2}x - \frac{1}{2}a$

$(\frac{1}{2}x - a)^2 = (\frac{3}{2}x - \frac{1}{2}a)^2$

$\frac{1}{4}x^2 - ax + a^2 = \frac{9}{4}x^2 - \frac{3}{2}ax + \frac{1}{4}a^2$

$\frac{8}{4}x^2 - \frac{1}{2}ax - \frac{3}{4}a^2 = 0$

$2x^2 - \frac{1}{2}ax - \frac{3}{4}a^2 = 0$ 2

$8x^2 - 2ax - 3a^2 = 0$

$x = \frac{3}{4}a \qquad x = -\frac{1}{2}a$

$y = -\frac{5}{8}a \qquad y = -\frac{5}{4}a$

(c) Deduce the solution of the inequality $\left|\frac{1}{2}x - a\right| > \frac{3}{2}x - \frac{1}{2}a$. [1]

$(\frac{1}{2}x - a)(\frac{1}{2}x - a) > (\frac{3}{2}x - \frac{1}{2}a)(\frac{3}{2}x - \frac{1}{2}a)$

$\frac{1}{4}x^2 - \frac{1}{2}ax - \frac{1}{2}ax + a^2 > \frac{9}{4}x^2 - \frac{3}{4}ax - \frac{3}{4}ax + \frac{1}{4}a^2$

$\frac{1}{4}x^2 - ax + a^2 > \frac{9}{4}x^2 - \frac{3}{2}ax + \frac{1}{4}a^2$

$-\frac{1}{2}a < x < \frac{3}{4}a$ 3

Examiner comments

1 The candidate draws a correct sketch for the modulus function and is awarded a mark. The straight-line graph, although positioned correctly, requires a steeper gradient.
Mark for (a) = 1 out of 2

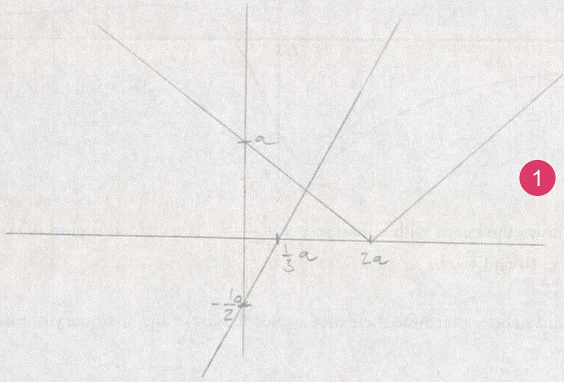
2 The candidate demonstrates a correct method of solution to obtain correct values. However, the candidate does not appear to refer to their sketch and hence does not discount one of the solutions.
Mark for (b) = 2 out of 3

3 The candidate incorrectly takes into account both x-coordinates because they did not discount one value in part **(b)**.
Mark for (c) = 0 out of 1

Total mark awarded = 3 out of 6

How the candidate could have improved their answer

- In part **(a)**, although the diagram was only a sketch, the candidate should have shown the different gradients of the lines.
- In parts **(b)** and **(c)**, the candidate should have referred to their sketch which was intended to guide them when choosing the appropriate values for the solutions. They do not appear to have taken account of the modulus part of the function when considering their answers.

Example Candidate Response – low	Examiner comments
3 (a) Sketch, on a single diagram, the graphs of $y = \left\|\frac{1}{2}x - a\right\|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]	
	1 The candidate provides a completely correct sketch which is awarded 2 marks. Mark for (a) = 2 out of 2
$y = \frac{1}{2}x - a \quad x = 0$ $y = -a$ $y = \frac{1}{2}x - a \quad y = 0$ $\frac{1}{2}x = a$ $x = 2a$ $y = \frac{3}{2}x - \frac{1}{2}a \quad x = 0$ $y = -\frac{1}{2}a$ $y = \frac{3}{2}x - \frac{1}{2}a \quad y = 0$ $\frac{3}{2}x = \frac{1}{2}a$ $x = \frac{1}{3}a$	
(b) Find the coordinates of the point of intersection of the two graphs. [3]	
$\left(\frac{1}{2}x - a\right)^2 = \frac{3}{2}x - \frac{1}{2}a$ $\frac{1}{4}x^2 - ax + a^2 = \frac{3}{2}x - \frac{1}{2}a$ $\frac{1}{4}x^2 - ax - \frac{3}{2}x + a^2 + \frac{1}{2}a = 0$ $x^2 - 4ax - 6x + 4a^2 + 2a$ $x^2 + (-4a - 6)x + 4a^2 + 2a = 0 \quad a = 1, b = 4a - 6, c = 4a^2 - 2a$ $(-4a - 6)^2 - 4(1)(4a^2 + 2a) = 0$ continued : $\cancel{8a^2 + 18a}$ $16a^2 + 48a + 36 - 16a^2 - 8a = 0 \quad 40a = -36$ $\cancel{32a^2 + 56a + 36} \quad a = -\frac{36}{40}$ $\cancel{8a^2 + 14a + 9 = 0} \quad a = \frac{9}{10}$	2 The candidate forgets to square both sides of their equation. This is an incorrect method. Mark for (b) = 0 out of 3
(c) Deduce the solution of the inequality $\left\|\frac{1}{2}x - a\right\| > \frac{3}{2}x - \frac{1}{2}a$. [1]	
$x > \frac{1}{2}a$ $x < \frac{1}{3}a$	3 No marks are awarded. The candidate is using the intercepts with the axes. Mark for (c) = 0 out of 1 **Total mark awarded = 2 out of 6**

How the candidate could have improved their answer

- Referring to the diagram in part **(a)** should have alerted the candidate to the fact that there was only one point of intersection.
- The equation obtained in part **(b)** needed to be squared on both sides.

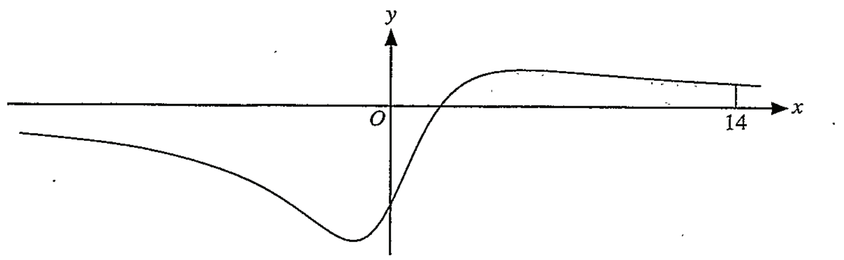
Common mistakes candidates made in this question

- The sketch in part **(a)** was meant to indicate the position of the only point of intersection between the two graphs.
- In part **(b)**, many candidates did not take this into account and gave the coordinates for two points. This then affected their answers in part **(c)**.
- When dealing with the modulus function in part **(b)**, many candidates who chose to use the 'squaring method' forgot to square both sides.

Question 4

Example Candidate Response – high

4



The diagram shows the curve with equation $\boxed{y = \dfrac{x-2}{x^2+8}}$. The shaded region is bounded by the curve and the lines $x = 14$ and $y = 0$.

(a) Find $\dfrac{dy}{dx}$ and hence determine the exact x-coordinates of the stationary points. [4]

$$\frac{dy}{dx} = \frac{1\times(x^2+8) - (x-2)\times 2x}{(x^2+8)^2} \quad -(2x^2-4x)$$

$$\frac{dy}{dx} = \frac{x^2+8-2x^2+4x}{(x^2+8)^2} \qquad (x^2+8)(x^2+8) = x^4+\underbrace{8x^2+8x^2}_{16x^2}+64$$

1

$$0 = \frac{-x^2+4x+8}{(x^4+16x^2+64)} \times (x^4+16x^2+64)$$

$$0 = -x^2+4x+8$$

$$x^2-4x-8 = 0$$

2

$x =$

Examiner comments

1 The candidate obtains a correct derivative using the quotient rule and is awarded 2 marks.

2 The candidate equates the derivative to zero and simplifies, but does not attempt to solve the resulting quadratic equation.
Mark for (a) = 2 out of 4

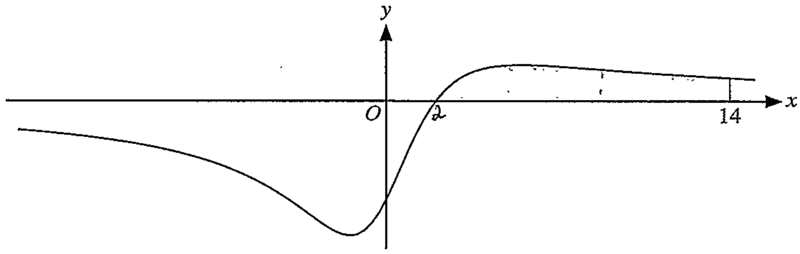
Example Candidate Response – high, continued	Examiner comments
(b) Use the trapezium rule with three intervals to find an approximation to the area of the shaded region. Give the answer correct to 2 significant figures. [3] 2 6 10 14 $0 = \frac{x-2}{x^2+8}$ $0 = x-2$ $x = 2$ $h = \frac{14-2}{3}$ $= 4$ $y_0 = \frac{2-2}{(2)^2+8} = 0$ $y_1 = \frac{6-2}{(6)^2+8} = \frac{1}{11}$ (3) $y_2 = \frac{10-2}{(10)^2+8} = \frac{2}{27}$ $y_3 = \frac{14-2}{(14)^2+8} = \frac{1}{17}$ $A_{trap} = \frac{1}{2}(4)\left[0 + \frac{1}{17} + 2\left(\frac{1}{11} + \frac{2}{27}\right)\right]$ $= 2\left[\frac{1}{17} + \frac{98}{297}\right]$ (4) $= 2\left[\cancel{to}\ \frac{1963}{5049}\right]$ $= 0.7775797188$ $= 0.78$ (5)	(3) The candidate calculates the correct y values and is awarded a mark. (4) The candidate uses a correct formula for the trapezium rule with the correct values and a mark is awarded. (5) The candidate's correct calculation with the final answer given to the required level of accuracy is awarded the final mark. Mark for (b) = 3 out of 3 **Total mark awarded = 5 out of 7**

How the candidate could have improved their answer

The candidate needed to consider the exact solutions for the quadratic equation. Although the quadratic equation did not factorise with integer coefficients, it was possible to obtain exact solutions using the quadratic formula.

Example Candidate Response – middle	Examiner comments

4



The diagram shows the curve with equation $y = \dfrac{x-2}{x^2+8}$. The shaded region is bounded by the curve and the lines $x = 14$ and $y = 0$.

(a) Find $\dfrac{dy}{dx}$ and hence determine the exact x-coordinates of the stationary points. [4]

$y = \dfrac{x-2}{x^2+8}$

$\dfrac{dy}{dx} = \dfrac{x^2+8(1) - x-2(2x)}{(x^2+8)^2}$ 1

$= \dfrac{x^2+8 - 2x^2 - 4x}{(x^2+8)^2}$

$\dfrac{-2x^2 - 4x}{x^2+8}$

$\dfrac{dy}{dx} = \dfrac{-2x(x+2)}{x^2+8}$

$0 = \dfrac{-2x(x-2)}{x^2+8}$

$0 = -2x^2 + 4x + 0$

$= -2x(x-2)$

$0 = -2x \qquad 0 = x-2$

$x = 0 \qquad x = 2$ 2

$\dfrac{-2(2)(2+2)}{2^2+8} = -\dfrac{4}{3}$

$(2,$

Examiner comments

1 The candidate attempts differentiation using the quotient rule. However, they do not obtain a correct derivative because of incorrect use of brackets and incorrect cancelling of terms.

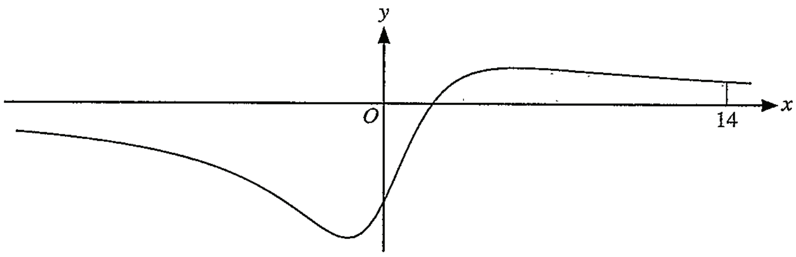
2 The candidate equates their first derivative to zero and attempts to solve to obtain two x-coordinates. Mark for (a) = 2 out of 4

Example Candidate Response – middle, continued	Examiner comments
(b) Use the trapezium rule with three intervals to find an approximation to the area of the shaded region. Give the answer correct to 2 significant figures. [3] $0 = \frac{x-2}{x^2+8}$ $\quad \frac{14-2}{3} = 4$ $\quad 2+4=6$ $0 = x-2$ $x = 2$ $\cancel{\frac{1}{2}4(}$ $y = \frac{6-2}{6^2+8} = \frac{1}{11}$ $\quad \frac{1}{2}(4)\left[\left(\frac{1}{11}+\frac{1}{17}\right)+2\left(\frac{2}{27}\right)\right]$ 4 $y = \frac{10-2}{10^2+8} = \frac{2}{27}$ $\quad = \underline{0.60}$ 5 $y = \frac{14-2}{14^2+8} = \frac{1}{17}$ 3	3 The candidate obtains the correct y-values. 4 The candidate uses the correct value of h in the formula for the trapezium rule. 5 The candidate omits $y=0$ from the formula so they substitute incorrectly into the formula. Mark for (b) = 2 out of 3 **Total mark awarded = 4 out of 7**

How the candidate could have improved their answer

- In part **(a)**, correct use of brackets around the appropriate terms would have avoided errors. The candidate needed to realise that terms cannot be cancelled unless the factor is common to each term in the numerator.
- In part **(b)**, the candidate could have checked the number of y-coordinates they used in the trapezium rule and noticed that there was a missing y-coordinate.

Example Candidate Response – low	Examiner comments
4	



The diagram shows the curve with equation $y = \dfrac{x-2}{x^2+8}$. The shaded region is bounded by the curve and the lines $x = 14$ and $y = 0$.

(a) Find $\dfrac{dy}{dx}$ and hence determine the exact x-coordinates of the stationary points. [4]

$u = x-2 \quad \dfrac{du}{dx} = 1$

$v = x^2+8 \quad \dfrac{dv}{dx} = 2x$

$\dfrac{dy}{dx} = \dfrac{x^2+8 \;-\; 2x^2-4x}{(x^2+8)^2}$ ①

$0 = \dfrac{x^2+8 \;-2x^2 \;-4x}{(x^2+8)^2}$

$-8+4x = \dfrac{x^2}{(x^2+8)^2}$

$-8+4x = \dfrac{x^2}{x^2+8}$ ②

$-16 +4x = \dfrac{x}{x^2}$

$-16x^2+4x^3 = x$

$-16x + 4x^2 = 1$

$4x^2 - 16x - 1 = 0$

$x = 4,06$ or $-0,06$

$y = \dfrac{(0,06)-7}{(0,06)^2+8}$ $\quad y = \dfrac{(4,06)-7}{(4,06)^2+8} = -0,24$

$= -0,88$ $\quad (4,06\,;\,-0,24)$

$(-0,06\,;\,-0,88)$

Examiner comments

① The candidate attempts to find the first derivative using the quotient rule. However, they simplify the second term in the numerator incorrectly.

② Although the candidate attempts to equate their first derivative to zero, they use an incorrect method of solution.
Mark for (a) = 1 out of 4

Example Candidate Response – low, continued

(b) Use the trapezium rule with three intervals to find an approximation to the area of the shaded region. Give the answer correct to 2 significant figures. [3]

$h = \frac{b-a}{n}$

$h = \frac{14-0}{2}$

$= 7$

$\int_0^{14} \frac{x-2}{x^2+8}\,dx$

n	0	1	2
x	0	7	14
y	$-\frac{1}{4}$	$\frac{5}{57}$	$\frac{1}{17}$

3

$= \frac{1}{2}(7)\left(\cancel{0}^{-\frac{1}{4}} + 2 + 2\left(\cancel{-\frac{1}{4}}\ \frac{8}{57} + \frac{1}{17}\right)\right)$

$= \cancel{1.78} \rightarrow \quad 7.2 \rightarrow$ 4

Examiner comments

3 The candidate's y values are incorrect because they use an incorrect value of h.

4 An incorrect value of h means that no further marks are available.
Mark for (b) = 0 out of 3

Total mark awarded = 1 out of 7

How the candidate could have improved their answer

- In part **(a)**, correct use of brackets may have prevented the candidate from making a sign error in the numerator of the first derivative. To solve their derivative equated to zero, they first needed to multiply both sides of their derivative by the denominator.
- In part **(b)**, an incorrect strip width h meant that there were no marks available. To help visualise the situation, the candidate could have made a small sketch showing a trapezium with three strips and the x-coordinates of the boundaries. This would have helped them to calculate the correct value for h.

Common mistakes candidates made in this question

- In part **(a)** errors with missing or incorrect brackets were common. It was essential for candidates to recognise the meaning of the word 'exact' in the context of this question as many candidates gave answers in decimal form.
- In part **(b)**, the most common error was the use of an incorrect value for the strip width h in the trapezium rule.

Question 5

Example Candidate Response – high	Examiner comments
5 The equation of a curve is $2e^{2x}y - y^3 + 4 = 0$. (a) Show that $\frac{dy}{dx} = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$. [4] $2e^{2x}y - y^3 + 4 = 0$ u v ① $u = 2e^{2x}$ $v = y$ $du = 4e^{2x}$ $dv = \frac{dy}{dx}$ $vdu + udv$ $= 4e^{2x}y + 2e^{2x}\frac{dy}{dx}$ ② $4e^{2x}y + 2e^{2x}\frac{dy}{dx} - 3\frac{dy}{dx}y^2 = 0$ $\frac{dy}{dx}\left(2e^{2x} - 3y^2\right) = -4e^{2x}y$ $\frac{dy}{dx} = \frac{-4e^{2x}y}{2e^{2x} - 3y^2}$ $\frac{dy}{dx} = \frac{\cancel{-}(4e^{2x}y)}{\cancel{-}(3y^2 - 2e^{2x})}$ $\frac{dy}{dx} = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$	1 The candidate differentiates a product using implicit differentiation and obtains a correct result. 2 The candidate differentiates the term in y^3 implicitly and obtains the correct result. 3 The candidate rearranges their correct result appropriately to obtain the given answer. They show sufficient detail to be awarded a mark. Mark for (a) = 4 out of 4

Example Candidate Response – high, continued	Examiner comments
(b) The curve passes through the point (0, 2). Find the equation of the tangent to the curve at this point, giving your answer in the form $ax + by + c = 0$. [3] ① $\frac{dy}{dx} = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$ $m = \frac{4e^{2(0)}(2)}{3(2)^2 - 2e^{2(0)}}$ $m = \frac{8}{10}$ $m = \frac{4}{5}$ 4	4 The candidate uses the given answer in part **(a)** to find the gradient of the tangent.
② $y - y_1 = m(x - x_1)$ $y - 2 = \frac{4}{5}(x - 0)$ 5	5 The candidate correctly obtains the equation of the tangent to the curve through the given point.
$y \times 5 = (\frac{4}{5}x + 2) \times 5$ $5y = 4x + 10$ $0 = 4x - 5y + 10$ 6	6 The candidate rearranges the equation of the tangent into the required form. Mark for (b) = 3 out of 3
(c) Show that the curve has no stationary points. [2] $0 = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$ $\frac{0}{y} = \frac{4e^{2x}y}{y}$ 7	7 The candidate divides each side of the derivative by y and thus overlooks the solution $y = 0$.
$\frac{0}{4} = \frac{4e^{2x}}{4}$ 8 $\ln 0 = \ln e^{2x}$ $\ln 0 = 2x$ $\Rightarrow \ln 0$ is not ~~possipl~~ possible, therefore there are no stationary poits. The gradient is never zero.	8 The candidate equates the numerator of the derivative to zero and considers the exponential term, showing that it cannot be zero. Mark for (c) = 1 out of 2 **Total mark awarded = 8 out of 9**

How the candidate could have improved their answer

In part **(c)** the candidate needed to consider the solution $y = 0$ as well as the exponential term.

Example Candidate Response – middle	Examiner comments
5 The equation of a curve is $2e^{2x}y - y^3 + 4 = 0$. **(a)** Show that $\frac{dy}{dx} = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$. [4] $\frac{dy}{dx} \Rightarrow 2 \cdot 2e^{2x}y + y'2e^{2x} - 3y^2y' = 0$ $4e^{2x}y + y'(2e^{2x} - 3y^2) = 0$ $y'(3y^2 - 2e^{2x}) = 4e^{2x}y$ $y' = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$ ↳ hence show	1 The candidate uses the product rule to differentiate the first term implicitly and obtains the correct result. 2 The candidate differentiates the term of y^3 implicitly and obtains the correct result. 3 The candidate rearranges their derivative correctly, showing sufficient detail. Mark for (a) = 4 out of 4
(b) The curve passes through the point (0, 2). Find the equation of the tangent to the curve at this point, giving your answer in the form $ax + by + c = 0$. [3] $\left.\frac{dy}{dx}\right\|_{x=0, y=2} = \frac{4e^{2(0)}(2)}{3(2)^2 - 2e^{2(0)}}$ $= \frac{8}{10}$ $= 0.8$ $y - 0 = \frac{8}{10}(x - 2)$ $10y = 8x - 16$ $\cancel{10y}$ $8x - 10y - 16 = 0$ $a = 8$ $b = -10$ $c = -16$	4 The candidate uses the given result for part **(a)** to obtain the gradient of the tangent. 5 The candidate uses x and y coordinates incorrectly and so does not obtain the equation of the tangent correctly. Mark for (b) = 1 out of 3
(c) Show that the curve has no stationary points. [2] $0 = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$ $0 = 4e^{2x}y$ $0 = e^{2x}$ hence x does not exist	6 The candidate equates the numerator of the derivative to zero and imply that the exponential term cannot be zero. 7 The candidate does not consider the solution $y = 0$. Mark for (c) = 1 out of 2 **Total mark awarded = 6 out of 9**

How the candidate could have improved their answer

- A check of the solution in part **(b)** should have alerted the candidate that the coordinates had been used incorrectly when attempting the equation of the tangent.
- In part **(c)** the candidate needed to consider the solution $y = 0$ as well as the exponential term.

Example Candidate Response – low

5 The equation of a curve is $2e^{2x}y - y^3 + 4 = 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{4e^{2x}y}{3y^2 - 2e^{2x}}$. [4]

$2e^{2x}y - y^3 + 4 = 0$

$2(2e^{2x}y) - (3y^{3-1} + 2e^{2x}) = 0$

$\dfrac{4e^{2x}y}{3y^{3-1} - 2e^{2x}}$ 1

$\therefore \dfrac{4e^{2x}y}{3y^2 - 2e^{2x}}$

(b) The curve passes through the point $(0, 2)$.

Find the equation of the tangent to the curve at this point, giving your answer in the form $ax + by + c = 0$. [3]

$m = \dfrac{4e^{2(0)}y}{3y^2 - 2e^{2(0)}}$

$= \dfrac{4e^0 y}{3y^2 - 2e^0}$

$= \dfrac{4(1)y}{3y^2 - 2(1)}$

$= \dfrac{4(2)}{3(2)^2 - 2}$ 2

$m = 4/5$ gradient of tangent

$y - 2 = 4/5(x - 0)$ 3

$5(y) = (4/5x + 2)5$

$\therefore -4x + 5y - 10 = 0$ 4 $\rightarrow$ equation of tangent.

(c) Show that the curve has no stationary points. [2]

$y = 4/5x + 2$

$2 = 4/5(0) + 2$ 5

$2 = 2$

$\therefore$ no stationary points

Examiner comments

1 The candidate makes no attempt at implicit differentiation, so no marks are available.
Mark for (a) = 0 out of 4

2 The candidate uses the given result in part **(a)** to find the gradient of the tangent.

3 The candidate finds the equation of the tangent using the correct gradient and the given point.

4 The candidate gives the equation of the tangent in an acceptable form.
Mark for (b) = 3 out of 3

5 The candidate does not make use of the answer given in part **(a)**, so no marks are awarded.
Mark for (c) = 0 out of 2

Total mark awarded = 3 out of 9

How the candidate could have improved their answer

- In part **(a)** the candidate needed to use implicit differentiation.
- In part **(c)** the candidate could have used the given result in part **(a)**.

Common mistakes candidates made in this question

- In part **(a)**, the most common mistakes included not using implicit differentiation and not recognising that the differentiation of a product was also needed.
- In part **(a)**, not showing sufficient detail in the stages to obtaining the given answer.
- In part **(b)**, the most common error was in omitting to give the equation of the tangent in the required form.
- Some candidates mistakenly found the equation of the normal.
- In part **(c)**, the most common mistake was not considering the solution $y=0$ and why this would also mean there are no stationary points.

Question 6

Example Candidate Response – high

6 (a) Find $\int\left(\frac{8}{4x+1}+\frac{8}{\cos^2(4x+1)}\right)\mathrm{d}x$. [4]

$$\int\left(\frac{8}{4x+1}+\frac{8}{\cos^2(4x+1)}\right)dx=\int\left(2\times\frac{4}{4x+1}+2\times\frac{4}{\cos^2(4x+1)}\right)dx$$

$$=\int\left(2\times\frac{4}{4x+1}+2\times4\sec^2(4x+1)\right)dx$$

$$=2\ln(4x+1)+2\tan(4x+1)+c$$

1

(b) It is given that $\int_0^{\frac{1}{2}\pi}(3+4\cos^2\tfrac{1}{2}x+k\sin 2x)\,\mathrm{d}x=10$.

Find the exact value of the constant k. [6]

$$\int_0^{\frac12\pi}\left(3+4\cos^2\tfrac12 x+k\sin 2x\right)dx=10$$

2

$$\to\int_0^{\frac12\pi}\left(3+2+2\cos x+k\sin 2x\right)dx=10$$

3

$$\to\left[5x+2\sin x-\cancel{\tfrac{1}{2}}\,\frac{1k\cos 2x}{2}\right]_0^{\pi/2}=10$$

$$\to\left(\frac{5\pi}{2}+2+\cancel{0}\,\frac{1}{2}k\right)-\left(0+0-\frac{1}{2}k\right)=10$$

$$\to\frac{5\pi}{2}+2+k=10$$

4

$$\to k=0.146$$

5

Examiner comments

1 The candidate provides a completely correct solution and is awarded full marks.
Mark for (a) = 4 out of 4

2 The candidate makes correct use of the double angle formula.

3 The candidate integrates each term correctly.

4 The candidate applies limits correctly, equates the expression to zero and attempts to find k.

5 The candidate does not give the answer in exact form as required and is not awarded the mark.
Mark for (b) = 5 out of 6

Total mark awarded = 9 out of 10

How the candidate could have improved their answer

The candidate should have checked that their final answer to part **(b)** was in the correct form.

Example Candidate Response – middle	Examiner comments
6 **(a)** Find $\int\left(\frac{8}{4x+1}+\frac{8}{\cos^2(4x+1)}\right)\mathrm{d}x$. **[4]** $\int\left(\frac{8}{4x+1}+\frac{8}{\cos^2(4x+1)}\right)dx$ 1 $\cancel{2\ln 4x+1 + 2\ln\cos^2(4x+1) + k}$ $\cos 2A = 2\cos^2\theta - 1$ 2 $2\ln 4x+1 + \frac{8}{\frac{1}{2}\cos 2A+\frac{1}{2}(4x+1)}$ $\frac{\cos 2A+1}{2}$ $2\ln 4x+1 + \frac{2(8)}{\cos 2A+(4x+1)}$ $= 2\ln 4x+1 + 4(\frac{1}{2})\ln \cos 2A+(4x+1)$ 3 4 $= 2\ln 4x+2\ln\sin 2A+(4x+1)$	1 The candidate does not write the trigonometric term in a form which can be integrated. 2 The candidate recognises that integration of the first term involves a logarithm. The lack of either modulus sign or brackets is tolerated here and the mark is awarded. 3 The candidate's integration of the trigonometric term does not lead to the correct form involving $\tan x$. 4 The candidate's final answer is incorrect. Mark for (a) = 1 out of 4

Example Candidate Response – middle, continued	Examiner comments
(b) It is given that $\int_0^{\frac{1}{2}\pi}(3 + 4\cos^2 \frac{1}{2}x + k\sin 2x)\,dx = 10$. Find the exact value of the constant k. [6]	
$\int_0^{\frac{\pi}{2}} (3 + 4\cos^2 \frac{1}{2}x + k\sin 2x)\,dx = 10$	
5 $2\cos^2\theta - 1 = \cos 2A$	5 The candidate uses the appropriate double angle formula to obtain the correct form. They have already started to integrate some other terms, but as the integration appears correctly in the line below, this is tolerated and both marks are awarded.
$3x + 4\left(\frac{\cos 2\frac{1}{2}x + 1}{2}\right) + \frac{k}{2} - \cos 2x \Big] = 0$ $2\cos^2\theta = \frac{\cos 2A + 1}{2}$	
$3x + 2(\sin x + 1) + \frac{k}{2}\cos 2x$ $\cos^2\theta = \frac{\cos 2A +}{2}$	
6 $\left[3x + 2\sin x + 2x - \frac{k}{2}\cos 2x\right]_0^{\pi/2}$	6 The candidate integrates each term correctly.
~~$5x + 2\sin$~~	
$\left[5x + 2\sin x - \frac{k}{2}\cos 2x\right]_0^{\frac{\pi}{2}} = 10$	
$= \left[\frac{5\pi}{6} + 2(1) - \frac{k}{2}(-1)\right] - \left[-\frac{k}{2}\right] = 10$	
$\frac{5\pi}{6} + 2 + \frac{k}{2} + \frac{k}{2} = 10$ 7	7 The candidate attempts to apply the limits but makes a substitution error in the first term. This is regarded as a slip as their other terms are correct and they use a correct method subsequently. They equate their expression to 10 and solve for k, obtaining an incorrect value because of their error earlier. Mark for (b) = 5 out of 6
$\frac{5\pi}{6} + 2 + \frac{2k}{2} = 10 - 2$	
$\frac{5\pi}{6} + k = 8$	
~~$k = 8$~~ $k = 8 - \frac{5\pi}{6}$	
$k = 8 - \frac{5\pi}{6}$	**Total mark awarded = 6 out of 10**

How the candidate could have improved their answer

- In part **(a)**, the candidate needed to be aware of the integrals of trigonometric functions and should have attempted to write the trigonometric term in one of these forms.
- In part **(b)**, a check of the working could have alerted the candidate to their error in substitution.

Example Candidate Response – low

6 (a) Find $\int \left(\frac{8}{4x+1} + \frac{8}{\cos^2(4x+1)}\right) dx.$ [4]

$\int \left(\frac{8}{4x+1} + \frac{8}{\cos^2(4x+1)}\right) dx$

$2\cos^2(4x+1) - 1 = \cos 2(4x+1)$

$\frac{2\cos^2(4x+1)}{2} = \frac{\cos 2(4x+1)+1}{2}$

$\cos^2(4x+1) = \frac{1}{2}\cos 2(4x+1) + \frac{1}{2}$

1

$\int \left(\frac{8}{4x+1} + \frac{8}{\frac{1}{2}\cos 2(4x+1)+\frac{1}{2}}\right)$

2 $\int \left(8(4x+1)^{-1} + 8\left(\frac{1}{2}\cos(8x+2)+\frac{1}{2}\right)^{-1}\right)$

$2\ln(4x+1) + \frac{8}{-\sin(8x+2)+8}$ 3 4

(b) It is given that $\int_0^{\frac{1}{2}\pi} (3 + 4\cos^2 \frac{1}{2}x + k\sin 2x)\, dx = 10.$

Find the exact value of the constant k. [6]

$\int_0^{\frac{1}{2}\pi} (3 + 4\cos^2 \frac{1}{2}x + k\sin 2x)\, dx = 10$ 5

$3 + 4\left(\frac{1}{2}\cos x\right) + k\sin 2x)\, dx = 10$

$3 + \cancel{4}\, 2\cos x + k\sin 2x = 10$

$\int_0^{\frac{1}{2}\pi} (3 + 2\cos x + k(\cancel{2\sin x\cos x})) = 10$ ($\sin 2x$)

6 $3x + 2\sin x \cancel{+2k\, k(k} \frac{k}{2}\cos 2x = 10$

$3x + 2\sin x - \frac{k}{2}\cos 2x = 10$

$3\left(\frac{1}{2}\pi\right) + 2\sin\left(\frac{1}{2}\pi\right) - \frac{k}{2}\cos 2\left(\frac{1}{2}\pi\right) = 10$

7 $\frac{3\pi}{2} + 2 \cancel{+} \frac{k}{2}x - 1 = 10$

$\frac{3\pi}{2} + 2 + \frac{k}{2} - 1 = 10$

$\frac{3\pi}{2} + \frac{k}{2} + 1 = 10$

$\frac{3\pi}{2} - 9 = \frac{k}{2}$

$3\pi - 18 = k$

$3\pi - 18 = k$

Examiner comments

1 The candidate has not written the trigonometric term in a form that can be integrated.

2 The candidate obtains the correct integral for the first term and is awarded a mark.

3 The candidate's trigonometric term is not in the correct form to be integrated to obtain a function of $\tan x$.

4 The candidate's final answer is incorrect.
Mark for (a) = 1 out of 4

5 The candidate does not obtain the correct form of the double angle formula - a constant term is missing. No marks awarded.

6 The candidate obtains a correct form of the integral although the coefficient of x is incorrect. This is due to the missing term from the double angle formula they used earlier so one mark is awarded.

7 The candidate applies the limits correctly to their expression, equates to zero and then solves for k. Their value is incorrect because of the earlier error.
Mark for (b) = 2 out of 6

Total mark awarded = 3 out of 10

How the candidate could have improved their answer

- In part **(a)**, the candidate needed to know the integrals of trigonometric functions and attempt to rewrite the trigonometric term in one of these forms.
- In part **(b)**, knowledge of the correct double angle formula would have helped the candidate produce a correct answer.

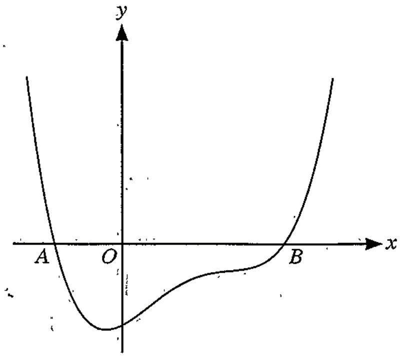
Common mistakes candidates made in this question

- In part **(a)**, the most common mistake was not writing the trigonometric term as $8\sec^2(4x+1)$. This was essential in order for the term to be integrated.
- In part **(b)**, many candidates did not recognise the need to use a double-angle formula to obtain a trigonometric form that could be integrated. The answer needed to be given in exact form, so the final mark was not available for a decimal answer.

Question 7

Example Candidate Response – high	Examiner comments

7



A curve has equation $y = f(x)$ where $f(x) = x^4 - 5x^3 + 6x^2 + 5x - 15$. As shown in the diagram, the curve crosses the x-axis at the points A and B with coordinates $(a, 0)$ and $(b, 0)$ respectively.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

$f(3) = 0$. Thus $(x-3)$ is a factor of $f(x)$. 1

(b) By first finding the quotient when $f(x)$ is divided by $(x - 3)$, show that

$$a = -\sqrt{\frac{5}{2-a}}.$$ [5]

$$
\begin{array}{r}
x^3 - 2x^2 + 5 \\
(x-3)\overline{)\,x^4 - 5x^3 + 6x^2 + 5x - 15} \\
-(x^4 - 3x^3) \\
-2x^3 + 6x^2 \\
-(-2x^3 + 6x^2) \\
0 + 5x - 15 \\
-(5x - 15) \\
0
\end{array}
$$

2

Examiner comments

1 The candidate does not show the substitution of $x = 3$ into the given equation nor evaluate each term to obtain 0. No marks are awarded.
Mark for (a) = 0 out of 2

2 The candidate divides the quartic polynomial by $x - 3$ using algebraic long division. They obtain a correct quotient from a correct process.

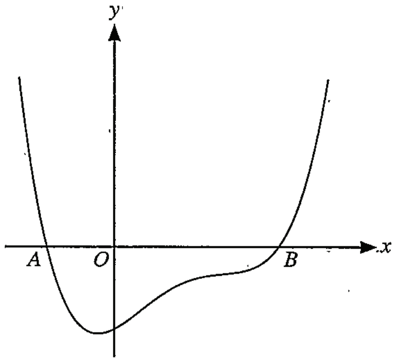
Example Candidate Response – high, continued	Examiner comments
$(x-3)(x^3-2x^2+5)=f(x)$. $\alpha^3-2\alpha^2+5=0$ $\alpha^3=+2\alpha^2-5$ $\alpha^3-2\alpha^2=-5$ $\alpha(\alpha^2-2\alpha)=-5$ $\alpha^2(\cancel{3}\alpha-2)=-5$ $\alpha^2=\dfrac{5}{2-\alpha}$ **3** $\alpha=-\sqrt{\dfrac{5}{2-\alpha}}$, for $\alpha<0$	**3** The candidate equates the quotient to zero and rearranges it correctly to obtain the required result. Mark for (b) = 5 out of 5
(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3] $f(-1)=-8$, $f(-1.5)=12.9375$ sign change in $[-1,-1.5]$. $\alpha_0=-1.25$ **4** $\alpha_1=-1.2403$ $\alpha_2=-1.2422$ $\alpha_3=-1.2418$ $\alpha_4=-1.2419$ $\alpha_5=-1.2419$ ~~$\alpha_6=$~~ $\alpha=-1.24$.	**4** The candidate chooses an appropriate starting value to use in the iterative process. They show sufficient iterations to the required level of accuracy and obtain a correct final answer. Mark for (c) = 3 out of 3 **Total mark awarded = 8 out of 10**

How the candidate could have improved their answer

In part **(a)** the candidate was expected to show that $x-3$ was a factor by substituting a value into the polynomial, evaluating the terms and reaching zero. They also needed to comment on the fact that the result was zero which meant that $x-3$ was a factor.

Example Candidate Response – middle

7



A curve has equation $y = f(x)$ where $f(x) = x^4 - 5x^3 + 6x^2 + 5x - 15$. As shown in the diagram, the curve crosses the x-axis at the points A and B with coordinates $(a, 0)$ and $(b, 0)$ respectively.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

$(3)^4 - 5(3)^3 + 6(3)^2 + 5(3) - 15$

$= 0$

1

(b) By first finding the quotient when $f(x)$ is divided by $(x - 3)$, show that

$$a = -\sqrt{\frac{5}{2-a}}.$$ [5]

$x^3 - 2x^2 + 12x + 41$ 2

$x - 3 \overline{)\, x^4 - 5x^3 + 6x^2 + 5x - 15}$

⊖ $x^4 - 3x^3 + 0 + 0 - 0$

$0 \quad -2x^3 + 6x^2 + 5x - 15$

⊖ $-2x^3 - 6x^2 + 0 - 0$

$+12x^2 + 5x - 15$

⊖ $+12x^2 + 36x - 0$

$+41x - 15$

⊖ $+41x - 123$

⊖ $+108$

Examiner comments

1 The candidate shows details of their substitution of $x = 3$ and obtains zero but makes no comment on this result. One mark is awarded.
Mark for (a) = 1 out of 2

2 The candidate attempts algebraic long division. The first two terms of the quotient are correct but the term in x and the constant term are incorrect.

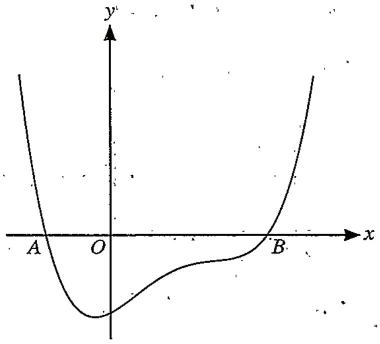
Example Candidate Response – middle, continued	Examiner comments
$\therefore$ quotient $= x^3 - 2x^2 + 12x + 41$ 3 ~~$x^3 - 2x^2 + 12x + 41$~~ ~~$a^3 - 2a^2 + 12a + 41$~~	3 The candidate does not attempt to obtain the required form using the quotient. Mark for (b) = 2 out of 5
(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3] $a = \sqrt{5/2+a}$ $a_1 = 0.6$ 4 $a_2 = 1.38675$ $a_3 = 1.21504$ $a_4 = 1.24707$ $a_5 = 1.24090$ $a_6 = 1.24208$ $a_7 = 1.24186$ $\therefore a = 1.24$	4 This has been treated as a misread as the candidate omits the negative sign from the iterative formula. The result is correct for a positive formula, so the candidate is awarded 2 marks but not the final accuracy mark. Mark for (c) = 2 out of 3 **Total mark awarded = 5 out of 10**

How the candidate could have improved their answer

- In part **(a)**, the candidate needed to comment on the fact that the result was zero which meant that $x-3$ was a factor.
- In part **(b)**, a check of the algebraic long division could have identified the error. The candidate could then have attempted rearrangement to the required form.
- In part **(c)**, a check of the solution could have alerted the candidate to the fact that they had omitted a negative sign.

Example Candidate Response – low

7



A curve has equation $y = f(x)$ where $f(x) = x^4 - 5x^3 + 6x^2 + 5x - 15$. As shown in the diagram, the curve crosses the x-axis at the points A and B with coordinates $(a, 0)$ and $(b, 0)$ respectively.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

~~$f(3) = 0$~~ $f(3) = 0$

$3^4 - 5(3)^3 + 6(3)^2 + 5(3) - 15$ **1**

$\Rightarrow 81 - 135 + 54 + 15 - 15$

$= 0$

(b) By first finding the quotient when $f(x)$ is divided by $(x - 3)$, show that

$$a = -\sqrt{\frac{5}{2-a}}.$$ [5]

$$x^3 - 2x^2 + 15$$

$x - 3 \,\big)\, x^4 - 5x^3 + 6x^2 + 5x - 15$ **2**

$-x^4 - 3x^3$

$-2x^3 + 6x^2$

$-2x^3 + 6x^2$

$15x + (-45)$

$-15x - 45$

$45 - 15$ **3**

(c) Use an iterative formula, based on the equation in part (b), to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

$a = -\sqrt{\dfrac{5}{2-a}}$ **4**

$x_1 = 1$ $x_1 = 1$

~~$x_2 = -1.2361$~~ $x_2 = 0.34476$

~~$x_3 = -2.4791$~~ 1.35090

~~$x_4 = -3.5356$~~ 3.4449

~~$x_5 = -4.$~~

Examiner comments

1 The candidate substitutes in $x = 3$ and obtains zero but makes no comment on this result. One mark is awarded.
Mark for (a) = 1 out of 2

2 The candidate carries out algebraic long division correctly to obtain the required quotient and all marks are awarded.

3 The candidate makes no attempt to equate the quotient to zero and rearrange the expression.
Mark for (b) = 3 out of 5

4 The candidate miscopies or misreads the iterative formula. Unfortunately, this produces an iterative formula which does not converge. The candidate does not use the starting value of 1 correctly in their iterative formula so no marks are available. Any crossed out work is not considered as it has been replaced.
Mark for (c) = 0 out of 3

Total mark awarded = 4 out of 10

How the candidate could have improved their answer

- In part **(a)**, the candidate needed to comment on the fact that the result was zero which meant that $x-3$ was a factor.
- In part **(b)**, they could have attempted to equate the quotient to zero and rearrange the expression.
- In part **(c)**, the candidate could have checked the iterative formula when their values were diverging. This may have alerted them to the error and hence they could have corrected it.

Common mistakes candidates made in this question

- In part **(a)**, the most common errors were not showing enough detail when substituting $x=3$ into the given equation. Very often comments on the result of zero were omitted.
- In part **(b)**, the most common error was incorrect rearrangement of the equation obtained when the quotient was equated to zero.
- In part **(c)**, errors included not giving the final answer to the correct level of accuracy and/or not giving the iteration to the required level of accuracy. Occasionally, candidates showed insufficient iterations to justify the final answer.

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: info@cambridgeinternational.org   www.cambridgeinternational.org