Cambridge Assessment International Education

Example Candidate Responses – Paper 1

# Cambridge International AS & A Level Mathematics 9709

For examination from 2020

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# Contents

# Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709 and to show how different levels of candidates' performance (high, middle and low) relate to the syllabus requirements.

In this booklet, candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

**November 2020 Question Paper 12**

**November 2020 Paper 12 Mark Scheme**

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

## How to use this booklet

<table>
<tr><th>Example Candidate Response – middle</th><th>Examiner comments</th></tr>
<tr><td>
1 The coefficient of $x^3$ in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.<br><br>
Find the value of the constant $k$. [4]<br><br>
$(1+kx)(1-2x)^5$<br>
$\binom{5}{0}(1)^5 \times (2x)^0 + \binom{5}{1}(1)^4(2x)^1 + \binom{5}{2}(1)^3(2x)^2 + \binom{5}{3}(1)^2(2x)^3$<br>
$(10x + 40x^2)(1+kx).$<br>
$10x + 10kx^2 + 40x^2 + 40kx^3.$<br>
1<br>
ans $= (0 + 10x + 40x^2 + 80x^3)(1+kx).$
</td><td>
1 The candidate identifies only one of the required terms correctly.
</td></tr>
</table>

**Answers** are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

**Examiner comments** are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

### How the candidate could have improved their answer

The candidate could have checked their expansion and algebraic simplification.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

### Common mistakes candidates made in this question

- Obtaining $+80$ rather than $-80$ in the binomial expansion.
- Only considering the $x^2$ term in their binomial expansion.
- Misunderstanding the term 'coefficient' and leaving $x^3$ in some terms.
- Using incorrect algebraic steps.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

# Question 1

## Example Candidate Response – high

**1** The coefficient of $x^3$ in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant $k$. [4]

$\cancel{(1-2x)^5}$

$\cancel{\boxed{x^0} \quad \boxed{x^1} \quad \boxed{x^2} \quad \boxed{x^3}}$

$\cancel{({}^5C_0)(1)^5(-2x)^0 \quad ({}^5C_1)(1)^4(-2x)^1}$

$\cancel{1 \times 1 \times 1 + 5 \times 1 \times -2x}$

$\cancel{1 - 10x}$

$\cancel{(1+kx)(1-10x)}$

$(1-2x)^5$

$x^0 \quad x^1 \quad \boxed{x^2} \quad \boxed{x^3}$

$({}^5C_2)(1)^3(-2x)^2 + ({}^5C_3)(1)^2(-2x)^3$

$10 \times 1 \times 4x^2 + 10 \times 1 \times -8x^3$

$40x^2 - 80x^3$

1 $(1+kx)(\ldots 40x^2 - 80x^3 \ldots)$

2 $(1 \times 80x^3) + (kx \times 40x^2) = 20x^3$

$80x^3 + 40kx^3 = 20x^3$

$\cancel{x^3}(80 + 40k) = 20x^3$

$40k = 20 - 80$

$40k = -60$

$k = \frac{-60}{40}$

$k = -\frac{3}{2}$ Ans

## Examiner comments

1 The candidate shows a good, clear method.

2 A minus sign is missing here, so only 3 of the 4 available marks are awarded.

**Total mark awarded = 3 out of 4**

## How the candidate could have improved their answer

The candidate made a good attempt at this question and just needed to check their working.

## Example Candidate Response – middle

**1** The coefficient of $x^3$ in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant $k$. [4]

$1^5 + {}^5C_1 \times 1^4 \times (-2x) + {}^5C_2 \times 1^3 \times (-2x)^2 + {}^5C_3 \times 1^2 \times (-2x)^3 +$

${}^5C_4 \times 1 \times (-2x)^4$

$10 \times 1^0 \times 4x^2 + 10 \times 1 \times -8x^3 = 40x^2 - 80x^3$ 1

$(40x^2 - 80x^3)(1 + \cancel{20}kx)$

$-80x^3 + \cancel{80k}\ 40kx^3$

2 $40kx^3 = 20x^3$

$\frac{40k}{40} = \frac{20}{40}$ $\boxed{k = \frac{1}{2}}$

## Examiner comments

1 The candidate finds the correct terms.

2 The candidate makes an error in algebraic simplification.

**Total mark awarded = 2 out of 4**

## How the candidate could have improved their answer

The candidate could have checked their algebraic simplification.

## Example Candidate Response – low

1 The coefficient of $x^3$ in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant $k$. [4]

$(1+kx)(1-2x)^5$

$\binom{5}{0}(1)^5 \times (2x)^0 + \binom{5}{1}(1)^4(2x)^1 + \binom{5}{2}(1)^3(2x)^2 \cdot + \binom{5}{3}(1)^2(2x)^3$

$(10x + 40x^2)(1+kx).$

$10x + 10kx^2 + 40x^2 + 40kx^3.$

1

ans $= (0 + 1.0x + 40x^2 + 80x^3)(1+kx).$

$10x + 10kx^2 + 40x^2 + 40kx^3 + 80x^3 + 80kx^3$

$20 = 80kx^3 + 120kx^3$ 2

$20 = 40kx^3 + 80kx^3 + 80x^3.$

$\frac{20}{120} = \frac{\cancel{120}kx^3}{\cancel{120}}$ 3

$k = 1/6.$

## Examiner comments

1 The candidate identifies only one of the required terms correctly.

2 An incorrect term results from the expansion.

3 The candidate does not simplify the expression correctly.

**Total mark awarded = 1 out of 4**

## How the candidate could have improved their answer

The candidate could have checked their expansion and algebraic simplification.

## Common mistakes candidates made in this question

- Obtaining $+80$ rather than $-80$ in the binomial expansion.
- Only considering the $x^2$ term in their binomial expansion.
- Misunderstanding the term ‘coefficient’ and leaving $x^3$ in some terms.
- Using incorrect algebraic steps.

# Question 2

## Example Candidate Response – high

**2** The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where $p$ is positive.

Find the sum to infinity of the progression. [5]

$a = 2p+6 \qquad U_2 = -2p \qquad U_3 = p+2$

$r = \dfrac{p+2}{-2p} = \dfrac{-2p}{2p+6}$

$(2p+6)(p+2) = (-2p)(-2p)$

$2p^2+4p+6p+12 = 4p^2$

$2p^2 - 10p - 12 = 0 \qquad p = 6 \qquad p = \cancel{-1}$ rejected

$(p-6)(p+1) = 0$

1

$S_\infty = \dfrac{a}{1-r} = \dfrac{2(6)+6}{1-(-\frac{2}{3})} = \dfrac{126}{5}$

$= 25.2$

$r = \dfrac{6+2}{-2\times6} = \dfrac{8}{-12} = -\dfrac{2}{3}$

## Examiner comments

1 The candidate produces a correct solution up until the final stage where they incorrectly evaluate the sum to infinity.

**Total mark awarded = 4 out of 5**

## How the candidate could have improved their answer

The candidate could have checked their final calculation.

| Example Candidate Response – middle | Examiner comments |
|---|---|
| **2** The first, second and third terms of a geometric progression are $2p+6$, $-2p$ and $p+2$ respectively, where $p$ is positive.<br><br>Find the sum to infinity of the progression. [5]<br><br>$S_\infty = \frac{a}{1-r}$ $\quad r = \frac{-2p}{2p+6}$<br><br>$S_\infty = \frac{2p+6}{1-\left(\frac{-2p}{2p+6}\right)}$ **1** $\quad 1-\left(\frac{-2p}{2p+6}\right)$<br><br>$= 1 + \frac{2p}{2p+6}$<br><br>$= 2p+6 \div \frac{4p+6}{2p+6}$<br><br>$= \frac{2p+6+2p}{2p+6} = \frac{4p+6}{2p+6}$<br><br>$\frac{p+2}{-2p} =$ | **1** This is a correct statement of the sum to infinity in terms of $p$. The candidate does not seem to realise that they can find a numerical value for $p$.<br><br>**Total mark awarded = 2 out of 5** |

## How the candidate could have improved their answer

The candidate needed to find the value of $p$ and give the sum to infinity as a numerical answer.

## Example Candidate Response – low

2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where $p$ is positive.

Find the sum to infinity of the progression. [5]

$S_\infty = \frac{a}{1-r}$ $\quad a = 2p+6 \quad r = \frac{T_2}{T_1}$

~~$S_\infty = 2$~~

$r = \frac{-2p}{2p+6}$

$r = \frac{T_3}{T_2} = \frac{p+2}{-2p}$

$\frac{-2p}{2p+6} = \frac{p+2}{-2p}$ 1

$(-2p)(-2p) = p+2(2p+6)$

$4p^2 = p + 4p + 12$

$4p^2 = 5p + 12$

$4p^2 - 5p - 12 = 0$

$(4p \quad)(p \quad)$

12 1 2 3 4
1 12 6 4

## Examiner comments

1 The candidate misses out a bracket which makes the subsequent algebra incorrect.

**Total mark awarded = 1 out of 5**

## How the candidate could have improved their answer

The candidate would have found it helpful to use brackets when working with algebraic terms.

## Common mistakes candidates made in this question

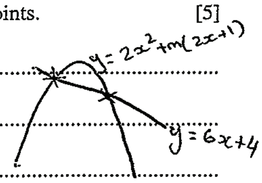
- Not realising that the sum to infinity could be given as a numerical value.
- Not using the easier method to find $p$ and ending up with an unnecessarily complicated cubic equation instead of a quadratic.
- Thinking that $a$ was equal to $p$ when working out the final answer.
- Obtaining a value of $r$ greater than 1 but not realising that this was impossible for a sum to infinity question.

# Question 3

| Example Candidate Response – high | Examiner comments |
| --- | --- |
| **3** The equation of a curve is $y = 2x^2 + m(2x + 1)$, where $m$ is a constant, and the equation of a line is $y = 6x + 4$. Show that, for all values of $m$, the line intersects the curve at two distinct points. [5] | |
| $y = 2x^2 + m(2x+1)$    $y = 6x + 4$ <br> $= 2x^2 + 2mx + m$ <br> $2x^2 + 2mx + m = 6x + 4$    $a = 2$   $b = (2m-6)$   $c = (m-4)$ <br> $2x^2 + 2mx - 6x + m - 4 = 0$ <br> $2x^2 + x(2m-6) + m - 4 = 0$ <br> $b^2 - 4ac > 0$ <br> $(2m-6)^2 - 4(2)(m-4)$ <br> $4m^2 - 24m + 36 - 8(m-4)$ <br> $4m^2 - 24m + 36 - 8m + 32$ <br> $4m^2 - 32m + 68 > 0$ <br> $4[m^2 - 8m + 17]$ <br> $4[(m-4)^2 - 16 + 17]$ <br> $4[(m-4)^2 + 1]$ <br> $4(m-4)^2 + 4$ **1** <br> $y = 6(4) + 4 = 28$ <br> $B\ (4, 28)$ <br> $y = 6(-4) + 4 = -20$ <br> $(-4, -20)$ <br> $x = 4$, $x = -4$ | **1** The candidate correctly completes the required mathematics for this question but does not clearly demonstrate that they understand the implications of their workings. <br><br> **Total mark awarded = 4 out of 5** |

## How the candidate could have improved their answer

The candidate needed to use clear reasoning and give a correct conclusion at the end of the question.

| Example Candidate Response – middle | Examiner comments |
|---|---|
| **3** The equation of a curve is $y = 2x^2 + m(2x+1)$, where $m$ is a constant, and the equation of a line is $y = 6x + 4$.<br><br>Show that, for all values of $m$, the line intersects the curve at two distinct points. [5] | |
| $y = 2x^2 + m(2x+1)$     $y = 6x + 4$   | |
| $2x^2 + m(2x+1) = 6x + 4$ | |
| $2x^2 + 2mx + m = 6x + 4$ | |
| $2x^2 + 2mx - 6x + m - 4$ | |
| a    b    c | |
| $b^2 - 4ac < 0$ | |
| $(2m-6)^2 - 4(2)(m-4) < 0$ | |
| $4m^2 \; \cancel{-48}\; {-24m} + 36 - 8m + 32$ | |
| $\dfrac{4m^2 \quad -32m \quad +68}{4 \qquad 4 \qquad 4}$ | |
| $m^2 - 8m + 17 < 0$ 1 | 1 The candidate produces correct working except for the < sign, but then doesn't finish the question.<br><br>**Total mark awarded = 3 out of 5** |
| $(m \quad)(m \quad)$ | |

## How the candidate could have improved their answer

- The candidate should have omitted the < sign.
- The candidate could have written the discriminant in completed square form and then interpreted it.

## Example Candidate Response – low

**3** The equation of a curve is $y = 2x^2 + m(2x + 1)$, where $m$ is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of $m$, the line intersects the curve at two distinct points. [5]

$2x^2 + m(2x+1) = 6x+4 \iff 2x^2 + 2mx + m = 6x+4$

$\iff 2x^2 - 6x + 2mx + m - 4 = 0$

2 points → $b^2 - 4ac > 0$ **1**

$(-6+2m)^2 - 4\times2\times(m-4) > 0 \iff 36 - 24m + 4m^2 - 8m - 32 > 0$

$\iff 4m^2 - 32m + 4 > 0$

$m = \dfrac{-(-32) \pm \sqrt{(-32)^2 - 4\times4\times4}}{2\times4} \iff m = \dfrac{32 \pm 8\sqrt{15}}{8}$ **2**

$\iff m = 4+\sqrt{15}$ OR $m = 4-\sqrt{15}$

$\iff m \simeq 7.87$ OR $m \simeq 0.127$

## Examiner comments

**1** The candidate makes a sign error. This value should be $+32$ not $-32$.

**2** The candidate does not consider 'all values of $m$' as requested in the question.

**Total mark awarded = 2 out of 5**

## How the candidate could have improved their answer

- The candidate could have found the discriminant correctly.
- The candidate needed to consider all values of $m$, not just two particular values.
- The candidate could have written the discriminant in completed square form and then interpreted it.

## Common mistakes candidates made in this question

- Assigning a particular value or values to $m$ rather than considering all possible values.
- Not equating the line and curve.
- Calculating the discriminant of the curve only.
- Algebraic errors when simplifying the equation.
- Misunderstanding what $a$, $b$ and $c$ relate to in the discriminant.
- Not realising the need to complete the square on the resulting quadratic in $m$ (or equivalent method).
- A lack of clear and correct reasoning at the end of the question.

# Question 4

## Example Candidate Response – high

**4** The sum, $S_n$, of the first $n$ terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The $k$th term in the progression is greater than 200.

Find the smallest possible value of $k$. [5]

$S_n = \frac{n}{2}(2a + (n-1)d) = n^2 + 4n$

$= na + \frac{n^2 d}{2} - \frac{nd}{2} = n^2 + 4n$

$= \frac{n^2 d}{2} - \frac{nd}{2} + na = n^2 + 4n$ 1

$\frac{d}{2} = 1$ $\quad n(a - d/2) = 4n$

$\quad a - d/2 = 4$

$d = 2$ $\quad a - 1 = 4$

$\quad a = 5$

$U_k = a + (k-1)d > 200$

$5 + (k-1)2 > 200$

$2.5 + k - 1 > 100$

$k > 100 - 1.5$

$k > 98.5$

$k = 98.5$ 2

## Examiner comments

1 The candidate compares coefficients.

2 The candidate does not realise that $k$ needs to be a whole number.

**Total mark awarded = 4 out of 5**

## How the candidate could have improved their answer

The candidate needed to realise that $k$ must be a whole number.

| Example Candidate Response – middle | Examiner comments |
| --- | --- |
| **4** The sum, $S_n$, of the first $n$ terms of an arithmetic progression is given by $$S_n = n^2 + 4n.$$ The $k$th term in the progression is greater than 200. $k$th $> 200$ <br> Find the smallest possible value of $k$. **[5]** <br> $u_1 = 1^2 + 4\times1 \Leftrightarrow a = 5$ $\quad d = 12 - 5 = 7$ (2) <br> (1) $u_2 = 2^2 + 4\times2$ <br> $= 12$ <br> $u_k = 5 + (k-1)\times 7 > 200 \quad (\Leftrightarrow) \quad 5 + 7k - 7 > 200$ (3) <br> $(\Leftrightarrow) \quad 7k < 202$ <br> $(\Leftrightarrow) \quad k < 28.9$ | (1) The candidate confuses the second term with the sum of the first two terms. <br><br> (2) The candidate finds an incorrect value of $d$. <br><br> (3) The candidate uses the correct formula for the $n$th term, with their values and 200, so the method mark is awarded. <br><br> **Total mark awarded = 3 out of 5** |

## How the candidate could have improved their answer

- The candidate confused the $n$th term with the sum of the first $n$ terms.
- The candidate did not use the correct formula for $S_2$.

| Example Candidate Response – low | Examiner comments |
|---|---|
| **4** The sum, $S_n$, of the first $n$ terms of an arithmetic progression is given by $$S_n = n^2 + 4n.$$ The $k$th term in the progression is greater than 200. Find the smallest possible value of $k$. [5] | |
| $S_n = n^2 + 4n$<br>first term : $S_1 = n^2 + 4n$<br>$S_1 = 1^2 + 4(1)$<br>$= 1 + 4$<br>$= 5$ | |
| 1 tenth term = $S_{10} = 10^2 + 4(10)$<br>$= 100 + 40$<br>$= 140$ | 1 The candidate confuses the tenth term with the sum of the first ten terms. |
| ~~t~~ thirteenth term = $S_{13} = 13^2 + 4(13)$<br>$= 169 + 52$<br>$= 221$ | |
| smallest value of $k = 13$ | **Total mark awarded = 2 out of 5** |

## How the candidate could have improved their answer

The candidate confused the $n$th term with the sum of the first $n$ terms.

## Common mistakes candidates made in this question

- Not reading the question carefully enough and confusing the $n$th term with the sum of the first $n$ terms.
- Equating the given formula for the sum of the first $n$ terms with the general formula but not realising that this was an identity, not an equation, so that coefficients could be compared.
- Not realising that $k$ needed to be a whole number.

# Question 5

| Example Candidate Response – high | Examiner comments |
|---|---|
| **5** Functions f and g are defined by<br>$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$<br>$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$ | |
| **(a)** Find the value of fg(7). [1]<br>$fg(x) : 4\left(\frac{4}{x+1}\right) - 2$<br>$fg(7) : 4\left(\frac{4}{7+1}\right) - 2$<br>$= 0$ 1 | 1 The candidate gives a correct response.<br>Mark for (a) = 1 out of 1 |
| **(b)** Find the values of $x$ for which $f^{-1}(x) = g^{-1}(x)$. [5]<br>$f^{-1}(x) : x = 4y - 2$<br>$4y = x + 2$<br>$y = \frac{x+2}{4}$<br>$g^{-1}(x) : x = \frac{4}{y+1}$<br>$x(y+1) = 4$<br>$y + 1 = \frac{4}{x}$<br>$y = \frac{4}{x} - 1$<br>$\therefore \frac{x+2}{4} = \frac{4}{x} - 1 \quad LCD = 4x$<br>$x(x+2) = 4(4) - 1(4x)$<br>$x^2 + 2x = 16 - 4x$<br>$x^2 + 6x - 16 = 0$ 2<br>$x = 2 \quad or \quad x = -8$ 3 | 2 The candidate provides a correct response up to this point.<br><br>3 The candidate does not show a method for solving the quadratic equation so even though the final answers are correct, no marks can be awarded.<br>Mark for (b) = 3 out of 5<br><br>**Total mark awarded = 4 out of 6** |

## How the candidate could have improved their answer

The candidate could have shown their method for solving the quadratic equation.

| Example Candidate Response – middle | Examiner comments |
|---|---|
| | |

**5** Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

**(a)** Find the value of fg(7). [1]

$$g(7) = \frac{4}{7+1} = \frac{1}{2}$$

$$f(g(7)) = 4\left(\tfrac{1}{2}\right) - 2 = 0$$

1

1 The candidate gives a correct response.
Mark for (a) = 1 out of 1

**(b)** Find the values of $x$ for which $f^{-1}(x) = g^{-1}(x)$. [5]

$$f^{-1}(x) = 4x - 2 = y \qquad 4y = x + 2$$

$$4y - 2 = x \qquad \boxed{y = \frac{x+2}{4}}$$

$$g^{-1}(x) = \frac{4}{x+1} = y \qquad \frac{4}{x} = y + 1$$

$$\frac{4}{y+1} = x \qquad \boxed{\frac{4}{x} - 1 = y}$$

$$\frac{x+2}{4} = \frac{4}{x} - 1$$

2

$$\frac{x+2}{4} - \frac{4}{x} + 1 = 0$$

$$1 + \frac{x+2}{4} = \frac{4}{x}$$

$$1 + x^2 + 2x = 16$$

$$x^2 + 2x - 15 = 0$$

3

$$\boxed{x = 3} \qquad \boxed{x = -5}$$

2 The candidate equates the inverse functions correctly.

3 The candidate reaches an incorrect quadratic equation and does not show their method for solving it.
Mark for (b) = 2 out of 5

**Total mark awarded = 3 out of 6**

## How the candidate could have improved their answer

- The candidate could have checked their algebraic re-arrangement to form the correct quadratic equation.
- The candidate could have shown their method for solving the quadratic equation.

## Example Candidate Response – low

**5** Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

**(a)** Find the value of fg(7). [1]

$4\left(\frac{4}{7+1}\right) - 2 = 0$ 1

**(b)** Find the values of $x$ for which $f^{-1}(x) = g^{-1}(x)$. [5]

$f'(x)$ $x = 4y - 2$

$(x-2) \div 4 = y$

$y = \frac{x-2}{4}$

$g'(x)$ $x = \frac{4}{y+1}$

$y+1 = \frac{4}{x}$

$y = \frac{4}{x} + 1$ 2

$\frac{x-2}{4} = \frac{4}{x} + 1$

$\frac{x(x-2)}{4} = \frac{4}{1} + x$

$\frac{x^2-2x}{4} - x - 4 = 0$ 3

$x^2 - 2x - 4x - 16 = 0$

$x^2 - 6x - 16 = 0$

$x = 8$ or $-2$

## Examiner comments

1 The candidate's response is correct.
Mark for (a) = 1 out of 1

2 The candidate's method is correct for both inverse functions, but the candidate makes mistakes when rearranging them.

3 The candidate reaches an incorrect quadratic equation and does not show their method for solving it.
Mark for (b) = 0 out of 5

**Total mark awarded = 1 out of 6**

## How the candidate could have improved their answer

- The candidate could have been more careful when finding the inverse functions.
- The candidate could have shown their method for solving the quadratic equation.

## Common mistakes candidates made in this question

- Not showing a method for solving the quadratic equation.
- Rearranging errors when trying to find the inverse functions.
- Algebraic errors when trying to simplify the equation formed by equating the two inverse functions.

# Question 6

## Example Candidate Response – high

**6** **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

$\Rightarrow \frac{1}{\cos x} + \frac{\tan x}{1} \cdot \frac{1}{\sin x} + \frac{1}{1}$

1 $\frac{1 + \cos x \tan x}{\cos x} \cdot \frac{1 + \sin x}{\sin x}$

$= \frac{1 \cancel{+} - \sin x}{\cos x} \cdot \frac{1 + \sin x}{\sin x}$

$\cancel{\frac{(1+\sin x)^2}{\cos x \cdot \sin x}}$

$\cancel{\Rightarrow \frac{1 + 2\sin x + \sin^2 x}{\cos x \cdot \sin x}}$

$\cancel{\frac{1 + \sin x(2 + \sin x)}{\cos x \cdot \sin x}}$

$\Rightarrow \frac{1 + \sin x - \sin x - \sin x^2}{\cos x \cdot \sin x}$

$\Rightarrow \frac{1 - \sin^2 x}{\cos x \cdot \sin x}$

$\Rightarrow \frac{\cos^2 x}{\cancel{\cos x} \cdot \sin x}$

$= \frac{\cos x}{\sin x} = \frac{1}{\tan x}$ 2

**(b)** Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

$\frac{1}{\tan x} = 2\tan^2 x$

$\Rightarrow \frac{1}{2} = \frac{\cancel{2}\tan^3 x}{\cancel{2}}$

$\sqrt[3]{\tan^3 x} = \sqrt[3]{1/2}$

$\tan x = \sqrt[3]{1/2}$

$x = \tan^{-1}\left(\sqrt[3]{1/2}\right)$

$x = 38{,}4$ 3

$180 - 38{,}4 = 141{,}6 \quad \therefore 38{,}4°$ and $141{,}6°$

## Examiner comments

1 The candidate makes a mistake with the sign here but it is corrected on the next line.

2 The rest of the proof is correct so full marks are awarded.
Mark for (a) = 4 out of 4

3 The candidate includes an invalid extra answer in the given interval, so only 1 mark is awarded.
Mark for (b) = 1 out of 2

**Total mark awarded = 5 out of 6**

## How the candidate could have improved

- The candidate included an extra, invalid answer in the interval in part **(b)**.
- The candidate could have checked the interval again at the end of the working to ensure that all answers given were valid.

## Example Candidate Response – middle

**6 (a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

First $\frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\frac{\cos x - \sin x \cos x}{\cos x(\cos x)}$

Second $\frac{1}{\sin x} + \frac{1}{1}$

$\frac{1+\sin x}{\sin x}$

$\frac{\cos x - \sin \cos x}{\cos x(\cos x)}$

$\frac{\cancel{\cos x}(1-\sin x)}{\cancel{\cos x}(\cos x)}$

$= \frac{1-\sin x}{\cos x}\left(\frac{1+\sin x}{\sin x}\right)$ 1

$= \frac{\cos^2 x}{\cancel{\cos x}(\sin x)}$ 2

$= \frac{\cos x}{\sin x}$

$= \frac{1}{\tan x}$

**(b)** Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

$\frac{1}{\tan x} = 2\tan^2 x$

$1 = 2\tan^3 x$

$\frac{1}{2} = \tan^3 x$

$\sqrt[3]{\frac{1}{2}} = \tan x$

$x = 38,4$ 3

$x = 38,4 + 180$

$= 218,4$ N/A

## Examiner comments

1 The candidate supplies a correct proof up until this point. Additional brackets should be included on this line.

2 The candidate misses out a step so the proof is incomplete.
Mark for (a) = 2 out of 4

3 The candidate supplies a correct response and is awarded full marks.
Mark for (b) = 2 out of 2

**Total mark awarded = 4 out of 6**

## How the candidate could have improved

The candidate should have included the line $= \frac{1-\sin^2 x}{\cos x \sin x}$ before using the Pythagorean identity. All necessary steps must be shown, especially in ‘Prove’ or ‘Show that’ questions.

## Example Candidate Response – low

**6** **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

L.H.S

$\frac{1}{\cos x}\left(\frac{1}{\sin x}+1\right) - \tan x\left(\frac{1}{\sin x}+1\right)$

$\frac{1}{\cos x}\times\frac{1}{\sin x} + \frac{1}{\cos x} - \frac{\cancel{\sin x}}{\cos x}\cdot\frac{1}{\cancel{\sin x}} - \frac{\sin x}{\cos x}$

$\frac{1}{\cos x}\times\frac{1}{\sin x} + \cancel{\frac{1}{\cos x}} - \cancel{\frac{1}{\cos x}} - \frac{\sin x}{\cos x}$

$\frac{1}{\cos x}\times\frac{1}{\sin x} - \frac{\sin x}{\cos x}$

$\frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x} = \frac{\cos x - \sin x(\cos x \sin x)}{\cos x(\cos x \sin x)}$ 1

$= \frac{1}{\tan x}$

**(b)** Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

$\frac{1}{\tan x} = \frac{2\tan^2 x}{1}$

$\frac{2\tan^3 x}{x} = \frac{1}{2}$ 2

$x^3 = \tan^{-1}\left(\frac{1}{2}\right)$ 3

$x = 2.9°$

$x = 177.0°$

## Examiner comments

1 The candidate carries out correct algebraic manipulation but misses out the final stages of simplification.
Mark for (a) = 2 out of 4

2 The candidate's statement is correct so the method mark is awarded.

3 The candidate's statement is incorrect so the accuracy mark is not awarded.
Mark for (b) = 1 out of 2

**Total mark awarded = 3 out of 6**

## How the candidate could have improved their answer

- The candidate could have attempted the final stages of the proof.
- The candidate appears to have misunderstood the notation $\tan^3 x$.

## Common mistakes candidates made in this question

- Missing out stages in the proof.
- Mistakes adding and multiplying the algebraic fractions.
- Making the algebra more difficult than it needed to be by not using the **lowest** common denominator when adding the fractions.
- By including extra, invalid solutions in part **(b)**, especially 141.6.
- Not realising that they needed to use the result from part **(a)** in part **(b)**.

# Question 7

## Example Candidate Response – high

7 The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

**(a)** A point moves along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the $y$-coordinate when $x = 4$. [3]

$\frac{dx}{dt} = 0.12$

$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$

$\frac{dy}{dx}\Big|_{x=4}$

$= 6(4)^{-\frac{1}{2}} - 4(4)^{-\frac{3}{2}}$

$= 2.5$

$= 2.5 \times 0.12$

$= 0.30$ units per second. 1

**(b)** Find the equation of the curve. [4]

$\int dy = \int 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\, dx$ 2

$= \cancel{12x^{\frac{1}{2}} - \frac{8}{5}x}$ $y = 12x^{\frac{1}{2}} - 8x^{-\frac{1}{2}} + C$

3 $= 12(4)^{\frac{1}{2}} - (-8)(4)^{-\frac{1}{2}} + c$

4 $= 48^{\frac{1}{2}} + 32^{-\frac{1}{2}} + c$

$c = -7.10$

$y = 12x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - 7.10$

## Examiner comments

1 The candidate gives a correct answer.
Mark for (a) = 3 out of 3

2 The candidate tries to do the working out in their head and makes a sign error. This should be $+8$ not $-8$. If they had written down $\frac{-4}{-0.5}$ they would have been awarded the mark.

3 The candidate makes errors in substituting values.

4 The candidate does not use the $y$-coordinate of 7 but equates to 0, so no mark can be awarded for the method.
Mark for (b) = 1 out of 4

**Total mark awarded = 4 out of 7**

## How the candidate could have improved

- The candidate could have shown intermediate steps in part **(b)**, for example $\dfrac{-4}{-\frac{1}{2}}$.
- The candidate should have used $y = 7$, as well as the $x$-coordinate of the given point to find $c$.

| Example Candidate Response – middle | Examiner comments |
| --- | --- |
| **7** The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$. | |
| **(a)** A point moves along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of increase of the $y$-coordinate when $x = 4$. [3] | |
| $f'(x) = 6x^{-1/2} - 4x^{-3/2}$ <br> $\frac{dx}{dt} = 0.12$ <br> $\frac{dy}{dx} = 6x^{-1/2} - 4x^{-3/2}$ <br> $\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$ <br> $0.12 \times 6(4)^{-1/2} - 4(4)^{-3/2}$ **1** <br> $= 0.3$ | **1** The final answer is correct so full marks are awarded, although some extra brackets would have helped with clarity. Mark for (a) = 3 out of 3 |
| **(b)** Find the equation of the curve. [4] | |
| gradient $= 6x^{-1/2} - 4x^{-3/2}$ <br> $= 6(4)^{-1/2} - 4(4)^{-3/2}$ <br> $= 2.5$ <br> $y = mx + c$ **2** <br> $7 = 2.5(4) + c$ <br> $7 = 10 + c$ <br> $7 - 10 = c$ <br> $-3 = c$ <br> $y = 2.5x - 3$ | **2** The candidate uses the equation of a straight line instead of integrating. Mark for (b) = 0 out of 4 |
| | **Total mark awarded = 3 out of 7** |

## How the candidate could have improved their answer

- The candidate could have produced a clearer solution by including an extra set of brackets in part **(a)**.
- The candidate needed to integrate in part **(b)** rather than use the equation of a straight line.

| Example Candidate Response – low | Examiner comments |
|---|---|
| **7** The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$. $\frac{dy}{dx}$ | |
| **(a)** A point moves along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of increase of the $y$-coordinate when $x = 4$. [3] | |
| $\frac{dy}{dx} = 6x^{-\frac{1}{2}} - 4^{-\frac{3}{2}}$ (1) | (1) The candidate misses out the $x$ but it reappears later in the solution so this does not impact on the marks awarded. |
| $\frac{dt}{dx} = 0.12.$ | |
| $\frac{dy}{dt} = \cancel{6(4)^{-\frac{1}{2}} - 4}$ | |
| $= 6(4)^{-\frac{1}{2}} - 4(4)^{-\frac{3}{2}} \times 0.12.$ (2) | (2) The candidate uses a correct method, although an extra set of brackets would have helped with clarity. |
| $4.0471497 \times 0.12$ | |
| $\frac{dy}{dt} = 0.486$ (3) | (3) The final answer is incorrect. Mark for (a) = 2 out of 3 |
| **(b)** Find the equation of the curve. [4] | |
| $7 = 6(4)^{-\frac{1}{2}} - 4(4)^{-\frac{3}{2}} + C$ | |
| $7 - 4.047 = C$ (4) | (4) The candidate has not integrated so no marks are awarded. Mark for (b) = 0 out of 4 |
| $C = 2.953$ | |
| $-\frac{3}{2}6x^{+\frac{1}{2}} - \frac{5}{2}4x^{+\frac{5}{2}}$ | |
| $-\frac{9}{8} - \frac{5}{16} = \frac{-23}{16}$ | |
| $6 + 3.20$ | |
| $y = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} + 2.953$ | **Total mark awarded = 2 out of 7** |

## How the candidate could have improved their answer

- The candidate could have produced a clearer solution by including an extra set of brackets in part **(a)**.
- The candidate needed to take more care entering the values into their calculator or make sure to check later.
- The candidate needed to integrate in part **(b)**.

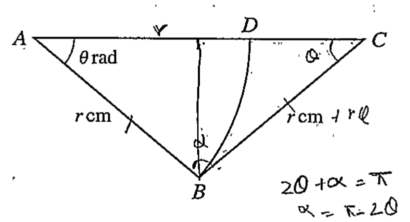
## Common mistakes candidates made in this question

- Dividing 2.5 by 0.12 or 0.12 by 2.5 rather than multiplying them.
- Treating the curve in part **(b)** as a line.
- Either not integrating or integrating incorrectly.
- Simplifying $\frac{6}{\frac{1}{2}}$ to equal 3 and $\frac{-4}{-\frac{1}{2}}$ to equal 2 or $-2$.

# Question 8

## Example Candidate Response – high

**8**




$\gamma + \alpha =$

$2\theta + \alpha = \pi$
$\alpha = \pi - 2\theta$

In the diagram, $ABC$ is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point $D$ lies on $AC$ and $ABD$ is a sector of a circle with centre $A$.

**(a)** Express the area of the shaded region in terms of $r$ and $\theta$. [3]

Area of shaded = Area of △ − Area of Sector ABD

Area of sector $= \frac{1}{2} r^2 \theta$

Area of △ $= \frac{1}{2} \times r \times r \times \sin(\pi - 2\theta)$

$= \frac{1}{2} r^2 \sin(\pi - 2\theta)$

Shaded Area $= \frac{1}{2} r^2 \sin(\pi - 2\theta) - \frac{1}{2} r^2 \theta$

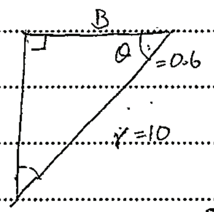
$= \frac{1}{2} r^2 \theta \left(\sin(\pi - 2\theta) - \theta\right)$ 1

**(b)** In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

Arc DB $= r\theta$

$= 10 \times 0.6$

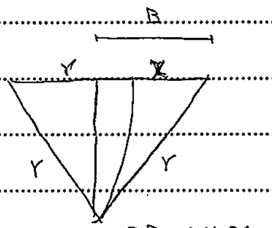
$= 6$ cm




$\cos 0.6 = \frac{B}{10}$

$\cos 0.6 \times 10 = B$

$8.3 = B$ 2




$2B = r + x$

$2B = r + x$

$16.6 = 10 + x$

$6.6 = x$

Perimeter = Arc AB + r + (6.6)

$= 6 + 10 + 6.6$

$= 22.6$ cm

## Examiner comments

1 The candidate gives a correct response.
Mark for (a) = 3 out of 3

2 The candidate uses a correct method, but premature approximation leads to an inaccurate final answer.
Mark for (b) = 3 out of 4

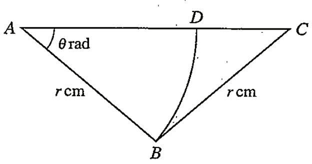
**Total mark awarded = 6 out of 7**

## How the candidate could have improved their answer

The candidate needed to work to at least four significant figures in part **(b)** so that the final answer would be correct to three significant figures when rounded.

| Example Candidate Response – middle | Examiner comments |
|---|---|

**8**




In the diagram, $ABC$ is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point $D$ lies on $AC$ and $ABD$ is a sector of a circle with centre $A$.

**(a)** Express the area of the shaded region in terms of $r$ and $\theta$. [3]

Area of triangle ABC $= \dfrac{r^2}{2}$ (1)

Area of sector ABD $= \dfrac{1}{2}r^2\theta$ (2)

Area of shaded region $= \dfrac{r^2}{2} - \dfrac{1}{2}r^2\theta$ (3)

**(b)** In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

$A\hat{B}C = \pi - 0.6 - 0.6$
$= 1.941$ radians

$\dfrac{AC}{\sin 1.941} = \dfrac{10}{\sin 0.6}$

$AC = \cancel{32.35}\ 16.52$ cm

Arc length $\cancel{D}B = r\theta$
$= 10\,(0.6)$
$= 6$ cm

~~DC length = 32.35 − 10~~
~~= 22.35~~

DC length $= 16.52 - 10$
$= 6.52$

Perimeter DCB $= 6 + 10 + 6.52$
$= 22.52$ cm (4)

**Examiner comments**

1 The candidate gives an incorrect formula for the area of triangle $ABC$.

2 The candidate gives a correct formula for the area of the sector $ABD$.

3 The candidate is awarded the final follow through mark as they subtract the correct sector area from what they think is the area of $ABC$.
Mark for (a) = 1 out of 3

4 The candidate gives a correct response.
Mark for (b) = 4 out of 4

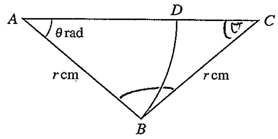
**Total mark awarded = 5 out of 7**

## How the candidate could have improved their answer

The candidate did not use a correct formula to calculate the area of triangle $ABC$.

## Example Candidate Response – low

**8**




In the diagram, $ABC$ is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point $D$ lies on $AC$ and $ABD$ is a sector of a circle with centre $A$.

**(a)** Express the area of the shaded region in terms of $r$ and $\theta$. [3]

~~Area of BCD =~~

~~Area of ABC =~~

Area of sector ABD $= \frac{1}{2}r^2\theta$ **1**

$AC^2 = r^2 + r^2 - 2(r \times r)\cos\theta$

$= r^2 + r^2 - 2r^2\cos\theta$

$= 2r^2 - 2r^2\cos\theta$

$= \sqrt{2r^2(1-\cos\theta)}$

$AC = \sqrt{2r(1-\cos\theta)}$

Area of $ABC = \frac{1}{2} \times \sqrt{2r(1-\cos\theta)} \times r$ **2**

Area of $BCD = \frac{1}{2}r\sqrt{2r(1-\cos\theta)} - \frac{1}{2}r^2\theta$ **3**

~~$= \frac{1}{2}r(r\theta$~~

$= \frac{1}{2}r\left(\sqrt{2r(1-\cos\theta)} - r\theta\right)$

**(b)** In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

Length of Arc $ABD = 10(0.6)$

$= 6$ cm **4**

~~$AC = \sqrt{2(10)(1-\cos 0.6)}$~~

~~$= \sqrt{20(1-\cos 0.6)}$~~

~~$= 0.78112$~~

$AC^2 = 10^2 + 10^2 - 2(10)(10)\cos 0.6$ **5**

$= 200 - 200\cos 0.6$

$= \sqrt{34.9328}$

$= 5.9104$

$DC = 5.9104 - 10$

$= {}^{-}4.089$

Length of $BCD = 6\text{cm} + 10\text{cm} + 4.089$

$= 20.1$ cm

## Examiner comments

**1** This is correct for the area of the sector.

**2** The candidate gives an incorrect area of triangle $ABC$.

**3** The candidate subtracts the area of the sector from their triangle area so is awarded a follow through mark.
Mark for (a) = 1 out of 3

**4** The candidate gives a correct arc length.

**5** The candidate uses the wrong angle here so no method marks are awarded.
Mark for (b) = 1 out of 4

**Total mark awarded = 2 out of 7**

## How the candidate could have improved their answer

- The candidate could have calculated the correct area of the triangle in part **(a)**.
- The candidate needed to use the correct angle in the cosine rule in part **(b)**.

## Common mistakes candidates made in this question

- Using $180$ degrees instead of $\pi$ radians in formulae that are only valid when radians are used.
- Only working out half of the area of triangle $ABC$.
- Giving the general form for the area of a triangle, $\frac{1}{2}r^2 \sin\theta$, as the answer for the area of $ABC$.
- Using $0.6$ instead of $(\pi - 2 - 0.6)$ in the cosine rule to find the length of $AC$.

# Question 9

| Example Candidate Response – high | Examiner comments |
|---|---|
| **9** A circle has centre at the point $B$ (5, 1). The point $A$ (–1, –2) lies on the circle.<br><br>**(a)** Find the equation of the circle. [3]<br><br>centre (5, 1) radius = r<br>$AB = r = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$<br>$= \sqrt{(\cancel{8}5-(-1))^2+(1-(-2))^2}$<br>$= \sqrt{(6)^2+(3)^2}$<br>$= \sqrt{36+9}$<br>$r = \sqrt{45}$<br>$r^2 = 45$<br>$(x-5)^2+(y-1)^2 = 45$. **1** | **1** The candidate produces a concise and efficient solution.<br>Mark for (a) = 3 out of 3 |
| Point $C$ is such that $AC$ is a diameter of the circle. Point $D$ has coordinates (5, 16).<br><br>**(b)** Show that $DC$ is a tangent to the circle. [4]<br><br>$C = \frac{x_1+(-1)}{2} = 5$ $\frac{y_1-2}{2} = 1.$<br>$y_1 = 4$<br>$x-1 = 10$<br>$x = 11$<br>(11, 4)<br>~~G (BDC) midpoint of DC = (8, 10)~~<br>$\frac{y_2-y_1}{x_2-x_1}$ = m of DC = $\frac{4-16}{11-5} = \frac{-12}{6} = -2$<br>~~m of B (midpoint of DC) = 8 − 5 ...~~<br>$\frac{y_2-y_1}{(x_2-x_1)}$ = m of BC = $\frac{4-1}{11-5} = \frac{3}{6} = \frac{1}{2}.$ **2**<br>$m_1 \times m_2 = -1$ $\frac{1}{2} \times -2 = -1$ so DC is tangent to circle | **2** All of the working out is correct but the candidate does not mention the fact that, since the product of the gradients equals –1, this implies the two lines are perpendicular and therefore $DC$ is a tangent to the circle. Not mentioning perpendicular lines means that the final accuracy mark is not awarded.<br>Mark for (b) = 3 out of 4 |

| Example Candidate Response – high, continued | Examiner comments |
|---|---|
| The other tangent from $D$ to the circle touches the circle at $E$. | |

**(c)** Find the coordinates of $E$. [2]

m of DE = 2

$\frac{11 + x}{2} = 5$

$y - y_1 = 2(x - x_1)$

$y - 16 = 2(x - 5)$

$x = -1$

$y - 16 = 2x - 10$

$\frac{4 + x}{2} = 1$

$y = 2x + 6$

$(x-5)^2 + (y-1)^2 = 45$

$(x-5)^2 + (2x+6-1)^2 = 45$

$x^2 + 25 - 10x + 4x^2 + 25 + 20x = 45$

$\cancel{5x^2 - 38x + 29 = 0}$

5 | 45
29

$5x^2 + 10x + 5 = 0$

$5x^2 + 5x + 5x + 5 = 0$

$5x(x+1) + 5(x+1) = 0$

$x = -1 \qquad x = -1$

$y - 1 = \pm 3 \qquad y = -2$

$y = +4$

$(-1, 4)$ 3

**Examiner comments**

3 The candidate gives a correct response.
Mark for (c) = 2 out of 2

**Total mark awarded = 8 out of 9**

## How the candidate could have improved their answer

In part **(b)**, the candidate needed to mention the fact that, since the product of the gradients of two lines was $-1$, this implied that the lines were perpendicular. All necessary steps and explanations must be given, especially in 'show that' or 'prove' questions.

## Example Candidate Response – middle

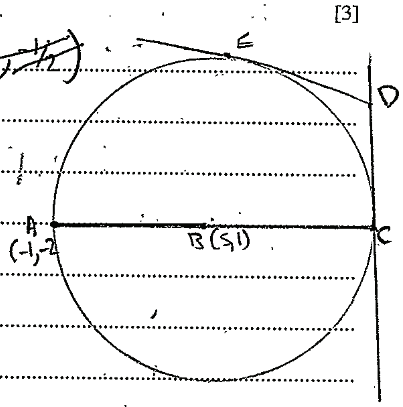
**9** A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

**(a)** Find the equation of the circle. [3]

~~$M\left(\frac{5-1}{2}, \frac{1-2}{2}\right)$~~ ~~$M(2, -\frac{1}{2})$~~

$5-(-1) = 6 = r$ (1)

~~$(x+5)^2 + (y-1$~~




$(x-5)^2 + (y-1)^2 = 6^2$

$(x-5)^2 + (y-1)^2 = 36$ (2)

Point $C$ is such that $AC$ is a diameter of the circle. Point $D$ has coordinates $(5, 16)$.

**(b)** Show that $DC$ is a tangent to the circle. [4]

$C(x, y)$ $A(-1,-2)$

~~AC x-coordinates~~

$\frac{x+(-1)}{2} = 5$

$10 = x-1$

$x = 11$

$\frac{y-2}{2} = 1$

$y-2 = 2$

$y = 4$

$C(11, 4)$

$m = \frac{4-(-2)}{11-(-1)} = \frac{1}{2}$

$m_{\perp} = -2$

$m = 1/2$

$m_{\perp} = -\frac{1}{1/2} = -2$

$D(5, 16)$ $C(11, 4)$

$m_{DC} = \frac{16-4}{5-11} = \frac{12}{-6} = -2$ (3)

## Examiner comments

1 The candidate uses an incorrect method for finding the radius.

2 The candidate uses the given centre and what they think the radius is, so is awarded the method mark.
Mark for (a) = 1 out of 3

3 All of the working out is correct but the candidate does not explain the implications of their answers.
Mark for (b) = 3 out of 4

| Example Candidate Response – middle, continued | Examiner comments |
| --- | --- |
| The other tangent from $D$ to the circle touches the circle at $E$. $D(5, 16)$<br><br>**(c)** Find the coordinates of $E$. [2]<br><br>$DC \Rightarrow y = -2x + c \quad , 16 = -2(5) + c)$<br>$c = 26$<br>$y = -2x+26$<br>$DE$<br><br>4<br><br>$\cancel{(x-5)^2 + (-2x+26)^2 = 36}$<br>$\cancel{x^2 - 10x + 25 + (4x^2 - 104x + 676) = 36}$<br>$\cancel{5x^2 - 114x + 665 = 0}$<br><br>$E($<br><br>$D$ | 4 The candidate makes no progress in this part of the question.<br>Mark for (c) $=$ 0 out of 2<br><br>**Total mark awarded = 4 out of 9** |

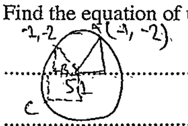
## How the candidate could have improved their answer

- The candidate needed to use a valid method to find the radius in part **(a)**.
- Having found gradients that multiply to $-1$ in part **(b)**, the candidate needed to explain what this implied, i.e., that the lines were perpendicular and hence $DC$ was a tangent to the circle.
- The candidate could have attempted a correct method in part **(c)** with the help of their diagram.

## Example Candidate Response – low

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]



Ans $AB = \sqrt{(5+1)^2+(1+2)^2}$

$= \sqrt{36+9}$

$r = \sqrt{45}$ 1

$x^2+y^2=r^2$, $x^2+y^2=45$ Ans. 2

Point $C$ is such that $AC$ is a diameter of the circle. Point $D$ has coordinates $(5, 16)$.

(b) Show that $DC$ is a tangent to the circle. [4]

Ans - $D(5,16)$ $C(11,4)$ $D(5,16)$ $m = \frac{16-4}{5-11}$, $= -2$ 3

eq. of circle = $y = \sqrt{45-x^2}$, $y = (45-x^2)^{1/2}$

$\frac{dy}{dx} = \frac{-2x}{2\sqrt{(45-x^2)}}$ $\frac{dy}{dx} = \frac{1}{2(45-x^2)^{1/2}} \cdot -2x$

The other tangent from $D$ to the circle touches the circle at $E$.

(c) Find the coordinates of $E$. [2]

4

## Examiner comments

1 The radius is correct so the candidate is awarded 1 mark.

2 The candidate does not use the given centre so cannot be awarded any further marks.
Mark for (a) = 1 out of 3

3 The candidate has found point $C$ correctly and the gradient of $CD$, but makes no further progress so they are awarded 2 marks.
Mark for (b) = 2 out of 4

4 The candidate does not attempt part **(c)**.
Mark for (c) = 0 out of 2

**Total mark awarded = 3 out of 9**

## How the candidate could have improved their answer

- The candidate needed to use the given centre in their equation in part **(a)**.
- The candidate could have found the gradient of $AB$ (or $BC$ or $AC$) in part **(b)**.
- The candidate could have attempted part **(c)**, perhaps with the help of a diagram.

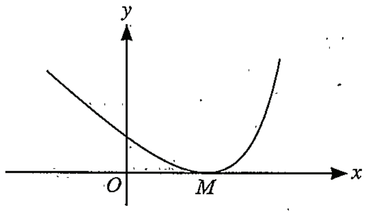
## Common mistakes candidates made in this question

- Using the equation of a straight line in part **(a)** rather than the equation of a circle.
- Thinking that the radius was 45 not $\sqrt{45}$.
- Writing the equation as $(x-5)^2+(y-1)^2=\sqrt{45}$, not 45 on the right-hand side.
- Not explaining the implications of their working in part **(b)**: if the product of the gradients of two lines equals $-1$, this implies that they are perpendicular.

# Question 10

## Example Candidate Response – high

**10**




The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point $M$, which lies on the $x$-axis.

**(a)** Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

$y = 2(3-2x)^{-2} - x$.

$dy/dx = 8(3-2x)^{-3} - 1$     Let $k = \cancel{2x}\ 3-2x$

$dk/dx = -2$

$d^2y/dx^2 = 48(3-2x)^{-4}$

$\int y = \int 2(3-2x)^{-2} - x$

$\int y = (3-2x)^{-1} - x^2 + c$. 1

## Examiner comments

1 The candidate forgets to divide by 2 when integrating $x$.

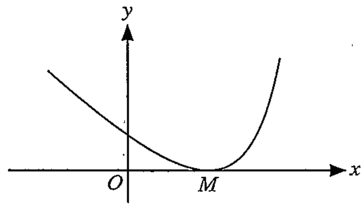
Mark for (a) = 5 out of 6

| Example Candidate Response – high, continued | Examiner comments |
|---|---|
| **(b)** Find, by calculation, the $x$-coordinate of $M$. [2]<br>$dy/dx = 0.$<br>$8(3-2x)^{-3} - 1 = 0.$ 2<br>$8(3-2x)^{-3} = 1$<br>$\cancel{(3-2x)^{-3} = \frac{1}{8}}.$<br>$\frac{8}{(3-2x)^3} = 1$<br>$\sqrt[3]{8} = \sqrt[3]{(3-2x)^3}$<br>$2 = 3 - 2x.$<br>$\frac{-1}{-2} = \frac{-2x}{-2}.$<br>$x = \frac{1}{2}.$ 3 | 2 Setting $\frac{dy}{dx}$ equal to 0 is a much easier method than setting $y = 0$.<br><br>3 The candidate gives a correct response.<br>Mark for (b) = 2 out of 2 |
| **(c)** Find the area of the shaded region bounded by the curve and the coordinate axes. [2]<br>$\text{Area} = 2(3-2x)^{-1} - x^2.$<br>$\left[(3-2x)^{-1} - x^2\right]_0^{1/2}$<br>4<br>$\left[(3-2(\tfrac{1}{2}))^{-1} - (\tfrac{1}{2})^2\right] - \left[(3-2(0))^{-1} - (0)^2\right]$<br>$= (\tfrac{1}{2} - \tfrac{1}{4}) - (\tfrac{1}{3} - 0)$<br>$= \tfrac{1}{4} - \tfrac{1}{12}$<br>$= \tfrac{1}{6}$ units$^2$. 5 | 4 The candidate clearly shows both limits substituted which is best practice. It is important when using 0 as a limit in integration questions that candidates don't assume the value of that part of the expression will equal 0.<br><br>5 Because the integral from part **(a)** is incorrect, the final answer is incorrect and the accuracy mark cannot be awarded. The method mark is awarded.<br>Mark for (c) = 1 out of 2<br><br>**Total mark awarded = 8 out of 10** |

## How the candidate could have improved their answer

In part **(a)**, the candidate could have integrated $x$ with the correct coefficient.

| Example Candidate Response – middle | Examiner comments |
|---|---|
| **10** | |




The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point $M$, which lies on the $x$-axis.

**(a)** Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\displaystyle\int y\,dx$. [6]

$y = \dfrac{2}{(3-2x)^2} \Rightarrow 2(3-2x)^{-2} - x$

$\dfrac{dy}{dx} = -4(3-2x)^{-3}(-2) - 1$

$\dfrac{dy}{dx} = 8(3-2x)^{-3} - 1$

$\dfrac{dy}{dx} = \dfrac{8}{(3-2x)^3} - 1$

$\cancel{\dfrac{d^2y}{dx^2} = -16(3-2x)^{-4}(-2)}$

$\cancel{\dfrac{d^2y}{dx^2} = 32(3-2x)^{-4}}$

$\cancel{\dfrac{32}{(3-2x)^4}}$

$\dfrac{d^2y}{dx^2} = 8(3-2x)^{-3}$

$\dfrac{d^2y}{dx^2} = -24(3-2x)^{-4}(-2)$

$\dfrac{d^2y}{dx^2} = 48(3-2x)^{-4}$

$\dfrac{d^2y}{dx} = \dfrac{48}{(3-2x)^4}$

$y\,dx = \int 2(3-2x)^{-2}\,dx$

$y\,dx = \dfrac{2(3-2x)^{-1}}{2} + c$

$y\,dx = (3-2x)^{-1} + c$

$y\,dx \int 2(3-2x)^{-2} - x + c$

$y\,dx = \dfrac{2(3-2x)^{-1}}{-2} - \dfrac{x^2}{2} + c$

$y\,dx = (3-2x)^{-1} - \dfrac{x^2}{2} + c$

$2(3-2x)^{-2} - x$

$-4(3-2x)^{-3}(-2) - 1$

$= 8(3-2x)^{-3} - 1$

$d^2y = -24(3-2x)^{-3}(-2)$

$x^1\ 4$

$1x$

**1** Full marks are awarded for this part of the question.
Mark for (a) = 6 out of 6

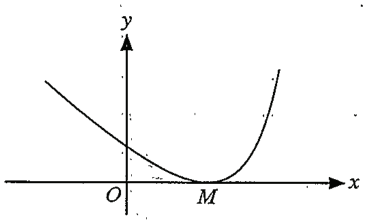
| Example Candidate Response – middle, continued | Examiner comments |
|---|---|
| **(b)** Find, by calculation, the $x$-coordinate of $M$. [2] | |
| $y = 2(3-2x)^{-2} - x$ | |
| $0 = \frac{2}{(3-2x)^2} - x$ | |
| $x = \frac{2}{4x^2 - 12x + 9} - \frac{x}{1}$    $3(3-2x) - 2x(3-2x)$ | |
| $9 - 6x - 6x + 4x^2$ | |
| $\frac{d^2y}{dx^2} =$    $4x^2 - 12x + 9$ | |
| $\frac{dy}{dx} = 8(3-2x)^{-3} - 1$ | |
| $0 =$ | |
| $x = \frac{3}{2}$    2    $\frac{48}{(3-2x)^4} > \frac{0}{1}$ | 2 The candidate makes insufficient progress in either setting $y = 0$ or $\frac{dy}{dx} = 0$ and so is awarded no marks. Mark for (b) = 0 out of 2 |
| $3 - 2x > 0$ | |
| $\frac{3}{2} > \frac{2x}{2}$ | |
| **(c)** Find the area of the shaded region bounded by the curve and the coordinate axes. [2] | |
| Area $= y \int_0^{\frac{3}{2}}$ | |
| $= \frac{1}{2} b.h$    3 | 3 The candidate uses an invalid method and so no marks are awarded. Mark for (c) = 0 out of 2 |
| $= \frac{1}{2}(\frac{3}{2})(1)$    $y = 2(3 - 2(\frac{3}{2}))^{-2} - \frac{3}{2}$ | |
| $= \frac{3}{4}$ units$^2$    $y = -\frac{3}{2}$ | **Total mark awarded = 6 out of 10** |

## How the candidate could have improved their answer

- The candidate could have solved $\frac{dy}{dx} = 0$ in part **(b)**.
- The candidate could have used their integral from part **(a)** to find the area in part **(c)**.

## Example Candidate Response – low

**10**




The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point $M$, which lies on the $x$-axis.

**(a)** Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

$$\frac{dy}{dx} = 2(3-2x)^{-2} - x = -4(3-2x)^{-3}(-2) - x$$

$$= \boxed{8(3-2x)^{-3} - x} \quad ①$$

$$\frac{d^2y}{dx^2} = -24(3-2x)^{-4}(-2)\cancel{x}$$

$$= \boxed{48(3-2x)^{-4}}\ x$$

$$\int 2(3-2x)^{-2} - x\,dx = \frac{2(3-2x)^{-1}}{-1 \times -2}$$

$$= \boxed{\frac{2(3-2x)^{-1}}{2} - x + c} \quad ②$$

## Examiner comments

① The candidate does not differentiate the $x$ term and repeats the same error with the second derivative.

② The candidate does not integrate the $x$ term.
Mark for (a) = 3 out of 6

## Example Candidate Response – low, continued

**(b)** Find, by calculation, the $x$-coordinate of $M$. [2]

$M(x, 0)$

$0 = \dfrac{2}{(3-2x)^2} - x = \dfrac{2}{9-12x+4x^2} - x$

$\cancel{0 = 2(3-2x)^{-2} - x}$

$\cancel{9-12x+4x^2} = 2 - x$ 3

$\boxed{x = 2}$

**(c)** Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

$\int_0^2 \frac{1}{2}\left[2(3-2x)^{-1}\right] - x$

$\dfrac{2(3-2x)^{-1}}{2} - x + C -$ 4

## Examiner comments

3 The candidate displays no working when solving the equation. Without working, it is not possible to determine whether the candidate has solved the equation with a calculator and so no marks are awarded.
Mark for (b) = 0 out of 2

4 The candidate has not substituted any limits so no marks are awarded.
Mark for (c) = 0 out of 2

**Total mark awarded = 3 out of 10**

## How the candidate could have improved

- The candidate could have remembered to differentiate and integrate the '$-x$' term.
- The candidate needed to show their working for solving the equation in part **(b)**.
- The candidate could have substituted limits in part **(c)**.

## Common mistakes candidates made in this question

- Forgetting to differentiate and integrate the '$-x$' term.
- Differentiating and integrating the '$-x$' term incorrectly.
- Forgetting to multiply by '$-2$' twice when differentiating $\dfrac{2}{(3-2x)^2}$.
- Forgetting to divide by '$-2$' when integrating $\dfrac{2}{(3-2x)^2}$.
- Setting $y = 0$ in part **(b)** rather than the easier $\dfrac{dy}{dx} = 0$.
- Not showing their method of solving the equations in part **(b)**.
- Assuming that when $0$ is substituted as a limit in part **(c)**, the value of the expression is $0$.
- Not showing both limits substituted in part **(c)**.

# Question 11

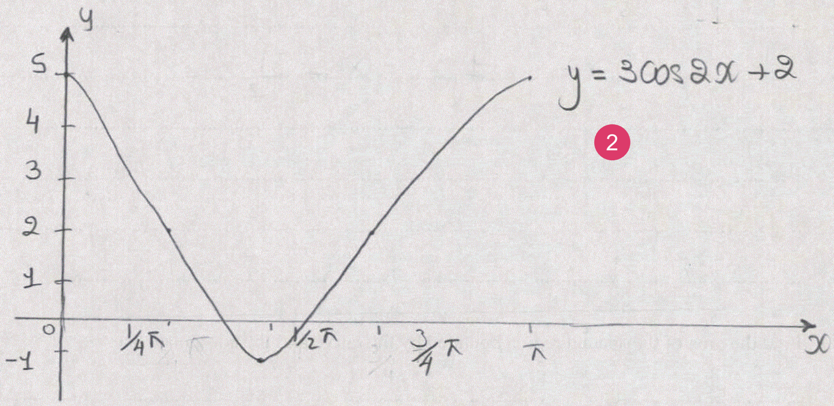
## Example Candidate Response – high

**11** A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

**(a)** State the greatest and least values of $y$. [2]

greatest value of y is 5 (1)
smallest value of y is -1

**(b)** Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]




**(c)** By considering the straight line $y = kx$, where $k$ is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

**(i)** $k = -3$ [1]

0 solutions (3)

**(ii)** $k = 1$ [1]

2 solutions

**(iii)** $k = 3$ [1]

1 solution

## Examiner comments

(1) The candidate gives a correct response.
Mark for (a) = 2 out of 2

(2) While the sketch isn't perfect, it is sufficient to be awarded both marks. An ideal sketch would be more curved, be obviously symmetrical about $x = \frac{\pi}{2}$ and level off more when $x = 0$.
Mark for (b) = 2 out of 2

(3) The candidate gives correct responses for part **(c)**.
Mark for (c)(i) = 1 out of 1
Mark for (c)(ii) = 1 out of 1
Mark for (c)(iii) = 1 out of 1

| Example Candidate Response – high, continued | Examiner comments |
|---|---|
| Functions f, g and h are defined for $x \in \mathbb{R}$ by<br>$$f(x) = 3\cos 2x + 2,$$<br>$$g(x) = f(2x) + 4,$$<br>$$h(x) = 2f(x + \tfrac{1}{2}\pi).$$<br>**(d)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]<br>Stretch on y axis with scale factor $\frac{1}{2}$ (4) followed by translation with vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ (5) | (4) The candidate uses correct terminology but refers to the wrong axis.<br><br>(5) A correct response. The candidate uses best practice: a column vector to describe the magnitude of the translation.<br>Mark for (d) = 1 out of 2 |
| **(e)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]<br>translation with vector $\begin{pmatrix} -\frac{1}{2}\pi \\ 0 \end{pmatrix}$ followed by stretch on x-axis (6) with scale factor 2 | (6) The candidate's translation is correct, with correct terminology for the stretch but, again, the axis is incorrect so only 1 mark is awarded.<br>Mark for (e) = 1 out of 2<br><br>**Total mark awarded = 9 out of 11** |

## How the candidate could have improved their answer

- The candidate could have improved their sketch by making it more curved, more obviously symmetrical about $x = \dfrac{\pi}{2}$, and by showing that it levelled off at $x = 0$.
- The candidate could have described the stretches with the correct axes in parts **(d)** and **(e)**.

## Example Candidate Response – middle

**11** A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

**(a)** State the greatest and least values of $y$. [2]

$-1 \leqslant \cos 2x \leqslant 1$

$y = 3\cos 2x + 2$
$\quad -2 \quad\quad -2$

$\dfrac{y-2}{3} = \dfrac{3\cos 2x}{3}$

$-1 \leqslant \dfrac{y-2}{3} \leqslant 1$

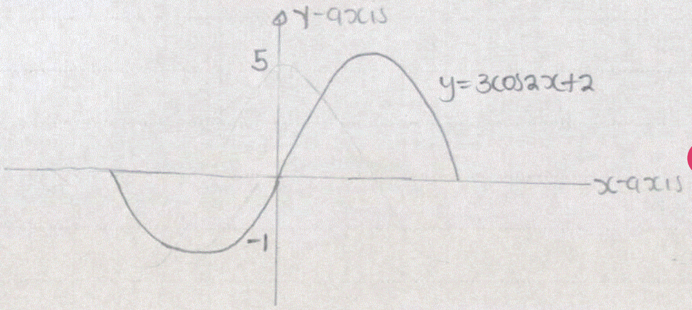
$-3 \leqslant y-2 \leqslant 3$

$-1 \leqslant y \leqslant 5$

$y \leqslant 5$ (greatest)

$y \geqslant -1$ (least)

**(b)** Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]




**(c)** By considering the straight line $y = kx$, where $k$ is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

**(i)** $k = -3$ [1]

none

**(ii)** $k = 1$ [1]

two solutions

**(iii)** $k = 3$ [1]

two solutions

## Examiner comments

1 The candidate provides a correct response.
Mark for (a) = 2 out of 2

2 The candidate's graph is incorrect so no marks are awarded here.
Mark for (b) = 0 out of 2

3 The candidate shows no method but this is not required for this question. Parts **(i)** and **(ii)** are correct but **(iii)** is incorrect so 2 marks are awarded.
Mark for (c)(i) = 1 out of 1
Mark for (c)(ii) = 1 out of 1
Mark for (c)(iii) = 0 out of 1

## Example Candidate Response – middle, continued

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}(x) = 3\cos 2x + 2,$$
$$\mathrm{g}(x) = \mathrm{f}(2x) + 4,$$
$$\mathrm{h}(x) = 2\mathrm{f}(x + \tfrac{1}{2}\pi).$$

**(d)** Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

$g(x) = f(2x)+4$

$= 3\cos 2(2x)+2+4$

$= 3\cos(4x)+6$

i) Horizontal stretch, stretch factor $\frac{1}{2}$

4

ii) vertical translation, vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

**(e)** Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [2]

i) vertical stretch, stretch factor 2

ii) Horizontal translation, vector $\begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}$

5

## Examiner comments

4 The candidate provides a correct response with correct use of terminology.
Mark for (d) = 2 out of 2

5 The candidate gives a correct response except that $\pi$ is missing, so only 1 mark is awarded.
Mark for (e) = 1 out of 2

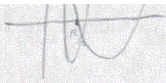
**Total mark awarded = 7 out of 11**

## How the candidate could have improved their answer

- The candidate could have drawn a correct cos graph in part **(b)**.
- The candidate's answer to part **(e)** would have been correct if they had included the $\pi$.

## Example Candidate Response – low

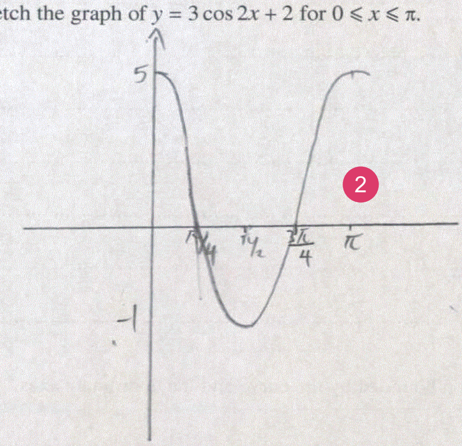
**11** A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.



**(a)** State the greatest and least values of $y$. [2]

$-1 \leqslant y \leqslant 5$ 1

**(b)** Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]




**(c)** By considering the straight line $y = kx$, where $k$ is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

**(i)** $k = -3$ [1]

$y = -3x$

no. of solutions = 3

**(ii)** $k = 1$ [1]

$y = x$

no. of solutions =

**(iii)** $k = 3$ [1]

$y = 3x$

no. of solutions =

## Examiner comments

1 The candidate gives a correct response.
Mark for (a) = 2 out of 2

2 A well-drawn sketch which is awarded full marks.
Mark for (b) = 2 out of 2

3 The candidate gives no answers for part **(c)**.
Mark for (c)(i) = 0 out of 1
Mark for (c)(ii) = 0 out of 1
Mark for (c)(iii) = 0 out of 1

| Example Candidate Response – low, continued | Examiner comments |
|---|---|
| Functions f, g and h are defined for $x \in \mathbb{R}$ by<br>$\mathrm{f}(x) = 3\cos 2x + 2,$<br>$\mathrm{g}(x) = \mathrm{f}(2x) + 4,$<br>$\mathrm{h}(x) = 2\mathrm{f}(x + \frac{1}{2}\pi).$ | |
| **(d)** Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]<br>Stretch by $1/2$ 4 | 4 The candidate does not give the direction of the stretch so no marks are awarded, similarly for the stretch in part **(e)**.<br>Mark for (d) = 0 out of 2 |
| **(e)** Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [2]<br>Stretch by factor 2<br>Translation left by $\pi/2$ along the $x$-axis 5 | 5 A column vector is better practice for describing a translation, but this answer can be awarded a mark because it did mention 'along the $x$-axis'.<br>Mark for (d) = 1 out of 2<br><br>**Total mark awarded = 5 out of 11** |

## How the candidate could have improved their answer

- The candidate could have attempted part **(c)**.
- The candidate needed to describe the directions of the stretches in parts **(d)** and **(e)**.
- The candidate could have used best practice, describing the translations with column vectors.

## Common mistakes candidates made in this question

- Substituting $x = 0$ and $x = \pi$ into the equation in part **(a)** and obtaining a greatest and least value of $5$.
- In part **(b)**, drawing a curve that resembled a parabola (i.e. it didn't level off at either end).
- Plotting points and joining them up instead of sketching a smooth cosine curve.
- Poor descriptions of the transformations in parts **(d)** and **(e)** using words such as move, up, down, left, and right. Correct descriptions use the words 'stretch' and 'translation' and best practice uses column vectors to describe translations.

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