Cambridge Assessment International Education

Example Candidate Responses – Paper 2

Cambridge IGCSE™
Additional Mathematics 0606

Cambridge O Level
Additional Mathematics 4037

For examination from 2020

Cambridge Pathway

Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge IGCSE / O Level Additional Mathematics 0606 / 4037, and to show how different levels of candidates' performance (high, middle and low) relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

0606 November 2020 Question Paper 22

0606 November 2020 Mark Scheme 22

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

This booklet goes through the paper one question at a time, showing you the high-, middle- and low-level response for each question. The candidate answers are set in a table. In the left-hand column are the candidate answers, and in the right-hand column are the Examiner comments.

Example Candidate Response – high	Examiner comments
1 Solve the inequality $(x-8)(x-10) > 35$. [4] $x^2 - 10x - 8x + 80 > 35$ $x^2 - 18x + 80 > 35$ $x^2 - 18x + 80 - 35 > 0$ $x^2 - 18x + 45 > 0$ 1 $x \geqslant 15$ or	1 The candidate is awarded one mark for expanding and simplifying to the correct three terms. A mark is awarded for solving the quadratic and another for achieving the

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

The candidate should have used their two solutions to identify whether the required values for the given expression were below and above, or between those solutions. They needed to use strict inequality symbols in their answer to be consistent with the given expression.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- Candidates often made algebraic and arithmetic errors.
- Many candidates did not use a method to identify whether the required solution set was below and above or between the two values obtained from the quadratic.
- Candidates often used non strict inequality symbols in their answer.
- A number of candidates used the word 'and' instead of 'or' or a comma in an otherwise correct range of values for x.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

Question 1

Example Candidate Response – high	Examiner comments
1. Solve the inequality $(x-8)(x-10) > 35$. [4] $x^2 - 10x - 8x + 80 > 35$ $x^2 - 18x + 80 > 35$ $x^2 - 18x + 80 - 35 > 0$ $x^2 - 18x + 45 > 0$ (1) $x \geqslant 15$ or $x \geqslant 3$	(1) The candidate is awarded one mark for expanding and simplifying to the correct three terms. A mark is awarded for solving the quadratic and another for achieving the correct values. The final mark is not awarded as the inequality is not correct. **Total mark awarded = 3 out of 4**

How the candidate could have improved their answer

The candidate should have used their two solutions to identify whether the required values for the given expression were below and above, or between those solutions. They needed to use strict inequality symbols in their answer to be consistent with the given expression.

Example Candidate Response – middle	Examiner comments
1 Solve the inequality $(x-8)(x-10) > 35$. [4] $x(x-10)-8(x-10) > 35$ $x^2-10x-8x+80 > 35$ $x^2-18x+45 > 0$ $x^2-15x-3x+45 > 0$ $x(x-15)-3(x-15) > 0$ $(x-3)(x-15) > 0$ $x=-3$ or $x=15$ 1	1 The candidate is awarded a mark for expanding and simplifying to the correct three terms. A second is awarded for solving the quadratic. No further marks are awarded as one x value is wrong. There is no attempt to express the answer as an inequality. **Total mark awarded = 2 out of 4**

How the candidate could have improved their answer

The candidate needed to write down the two correct solutions from their correct factors. Then they should have found the range of values for x using those two solutions.

Example Candidate Response – low	Examiner comments
1 Solve the inequality $(x-8)(x-10) > 35$. [4] $\cancel{-(x-8)(x-10) > 35}$ $\cancel{-x^2+10x+8x-18 > 35}$ $\cancel{-x^2+18x-53 > 0}$ $\cancel{x^2-10x-8x+18 > 35}$ $\cancel{x^2-18x-17 > 0}$ $\cancel{x^2-x-17x-17 > 0}$ $-(x-8) > 35$ $-x+8 > 35$ $-x > 27$ $x < -27$ $x-10 > 35$ $x > 45$ $x-8 > 35$ $x > 43$ $-(x-10) > 35$ $-x+10 > 35$ $-x > 25$ $x < 25$ $\therefore\ x > 45$ and $x < -27$ 1	1 The candidate selects an incorrect method, associating each factor with 35 rather than the whole expression. No marks are awarded. **Total mark awarded = 0 out of 4**

How the candidate could have improved their answer

The candidate needed to expand the brackets and produce a three term quadratic and solve it to find the two x values, before identifying whether the required solution set was below and above those values or between them.

Common mistakes candidates made in this question

- Candidates often made algebraic and arithmetic errors.
- Many candidates did not use a method to identify whether the required solution set was below and above or between the two values obtained from the quadratic.
- Candidates often used non strict inequality symbols in their answer.
- A number of candidates used the word ‘and’ instead of ‘or’ or a comma in an otherwise correct range of values for x.

Question 2

Example Candidate Response – high

2 Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$. [4]

$$\cancel{\frac{2^{2x+1}}{2^{x-1}} = 2^{5x/3} \times 2}$$

$$\cancel{(2x+1) - (x-1) = \frac{5x}{3} + 1}$$

$$\cancel{x = \frac{5x}{3} + 1}$$

$$\cancel{x - \frac{5x}{3} = 1}$$

$$\cancel{x - 5x = 3}$$

$$\cancel{-4x = 3}$$

$$\cancel{x = \frac{3}{-4}}$$

$$\frac{2^{2x+2}}{2^{x-1}} = 2^{5x/3} \times 2^1$$

$$(2x+2) - (x-1) = \frac{5x}{3} + 1$$

$$x + 3 = \frac{5x+1}{3}$$ 1

$$3x + 9 = 5x + 1$$

$$8 = 2x$$

$$x = 4$$

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·5

Examiner comments

1 The candidate is awarded three marks as the powers of 2 are dealt with correctly and the laws of indices are correctly applied, resulting in a linear equation. The fourth mark is not awarded as there is an error in solving the equation.

Total mark awarded =
3 out of 4

How the candidate could have improved their answer

- The candidate needed to clear the fraction in their linear equation, remembering that the $+1$ on the right-hand side also needed to be multiplied by 3.

Example Candidate Response – middle	Examiner comments
2 Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$. [4]	
$\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$	
$4^{x+1} \div 2^{x-1} = (2^5)^{\frac{x}{3}} \times (2^3)^{\frac{1}{3}}$	
$(2^2)^{x+1} \div (2^1)^{x-1} = (2^5)^{\frac{x}{3}} \times (2^3)^{\frac{1}{3}}$ 1	1 The candidate is awarded two marks for dealing with powers of 2 and using laws of indices correctly. No further marks are awarded as the righ-hand side of the linear equation is not formed correctly, each term being multiplied by 9 instead of by 3 as on the left-hand side.
$\Rightarrow 2(x+1)^{\times 3} - x^{\times 3} - 1^{\times 3} = 5^{\times 3} \times \frac{x}{3}^{\times 3} + 3^{\times 3} \times \frac{1}{3}^{\times 3}$	
$\Rightarrow 6(x+1) - 3x - 3 = 15x + 9$	
$6x + 6 - 3x - 3 = 15x + 9$	
$6x - 3x + 6 - 3 = 15x + 9$	
$3x - 3 = 15x + 9$ $+3 \qquad\quad +3$	
$3x = 15x + 12$ $-15x \quad -15x$	
$\frac{-12x}{-12} = \frac{12}{-12}$	
$x = -1$	**Total mark awarded = 2 out of 4**

How the candidate could have improved their answer

The candidate needed to remember that when an expression of the form $p \times q$ is multiplied by a number, they should only multiply p by that number or q by that number and not both. They also needed to consider that when applying the law of powers $\frac{x^p}{x^q} = x^{(p-q)}$ to an expression where q had more than one term, it would be easy to make an error with signs. All the terms of q needed to have their sign changed.

Example Candidate Response – low	Examiner comments
2 Find the value of x such that $\dfrac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$. [4] $\dfrac{2^{2(x+1)}}{2^{x-1}} = 2^{5(\frac{x}{3})} \times 2^{3(\frac{1}{3})}$ $2(x+1) - (x-1) = 5(\frac{x}{3}) \times 3(\frac{1}{3})$ **1** $2x+2-x+1 = \frac{5x}{3} \times 1$ $3(x+3) = \frac{5x}{\cancel{3}} \times \cancel{3}$ $3x+9 = 5x$ $9 = 5x-3x$ $\frac{9}{2} = \frac{2}{2}x$ $x = \frac{9}{2}$	**1** A mark is awarded for the correct conversion to powers of 2. The candidate then applies the laws of indices in an incorrect way with the right-hand side being expressed as $\frac{5x}{3} \times 1$ not $\frac{5x}{3} + 1$. No further marks are awarded. **Total mark awarded = 1 out of 4**

How the candidate could have improved their answer

The candidate could have applied the laws of indices correctly to the right-hand side and added the powers.

Common mistakes candidates made in this question

Many candidates made algebraic errors, including incorrect expansion of $2(x + 1)$ and sign errors in the expansion of $-(x - 1)$.

Question 3

Example Candidate Response – high

3 (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$. [5]

$m_L = \dfrac{3-1}{4-12} = -1/4$.

gradiant of ⊥ perpendicular bisector = 4.

Mid point = $\left(\dfrac{12+4}{2}, \dfrac{1+3}{2} \right)$

$= (8, 2)$

equation of the line =

$y - 2 = 4(x - 8)$

$y - 2 = 4x - 32$

$\boxed{y = 4x - 30}$ 1

(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB. [3]

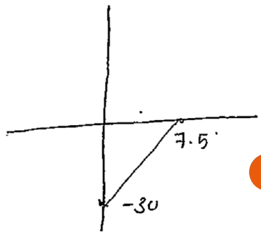
$y = 0$.

$0 = 4x - 30$

$x = 7.5$.

$x = 0$.

$y = -30$.



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$AB = \sqrt{(30)^2 - (7.5)^2}$

$= 29.04737351$

$AB \; \boxed{= 29.0}$

Examiner comments

1 The candidate is awarded all five marks as the question is answered correctly.

Mark for (a) = 5 out of 5

2 The candidate is awarded the first two marks for the correct intercepts. The final mark is not awarded as Pythagoras' is not applied correctly and hence the final answer is incorrect.

Mark for (b) = 2 out of 3

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

The candidate needed to use Pythagoras' correctly, adding the two squared values.

Example Candidate Response – middle

3 (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$. [5]

Mid point $= \left[\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right]$

$= \left[\dfrac{12+4}{2}, \dfrac{3+1}{2}\right]$

$= (8, 2)$

m of line × m of bisector = -1

m of line $= \dfrac{3-12}{4-1}$

$= \dfrac{-9}{3}$

m of line $= -3$

$-3 \times m_b = -1$

$m_b = \dfrac{-1}{-3}$

$m_b = 1/3$

~~$y - 2 = \frac{1}{3}(x$~~

$y = mx + c$

$y = \frac{1}{3}x + c$

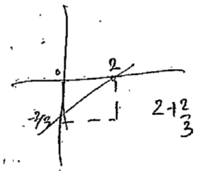
when $x = 8$, $y = 2$

$\therefore 2 = \frac{1}{3} \times 8 + c$

$2 = \frac{8}{3} + c$

$c = -\frac{2}{3}$

$\therefore y = \frac{1}{3}x - \frac{2}{3}$



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(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB. [3]

Point A $(x, 0)$

$0 = \frac{1}{3}x - \frac{2}{3}$

$\frac{2}{3} = \frac{1}{3}x$

$x = \frac{2}{3} \times 3$

$x = 2$

$A = (2, 0)$

Point B $(0, y)$

$\therefore y = \frac{1}{3}(0) - \frac{2}{3}$

$y = -\frac{2}{3}$

$B = (0, -\frac{2}{3})$

$AB = \sqrt{[2-0]^2 + [-\frac{2}{3} - 0]^2}$

$AB = \sqrt{2^2 + [\frac{2}{3}]^2}$

$AB = \sqrt{4 + \frac{4}{9}}$

$AB = 2.11$

2

Examiner comments

1 The candidate is awarded a mark for the correct mid-point. The gradient of the line joining the given points is incorrect. However, a mark is awarded for the correct method used to find the gradient of a line perpendicular to it. A further mark is awarded for using this gradient and the mid-point to find the equation of the line.

Mark for (a) = 3 out of 5

2 The candidate is awarded a mark for each of the correct intercepts with the axes for the line found in (a). The final answer is incorrect.

Mark for (b) = 2 out of 3

Total mark awarded = 5 out of 8

How the candidate could have improved their answer

The candidate needed to find the gradient of the line between the two given points correctly by using the difference in the y values divided by the difference in the x values.

Example Candidate Response – low	Examiner comments
3 (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$. [5] $m = \frac{y_2 - y_1}{x_2 - x_1}$ 1 $\frac{3-1}{4-12}$ $\frac{2}{-8} = \frac{1}{-4}$ $m = -\frac{1}{4}$ $y = -\frac{1}{4}x + c$ when $(4, 3)$ $3 = -\frac{1}{4}(4) + c$ $3 = -1 + c$ $+1 \quad +1$ $c = 4$ $\therefore \; y = -\frac{1}{4}x + 4$	1 The candidate finds the correct gradient of the line joining the two given points and is awarded one mark. No more marks are awarded as the mid-point and perpendicular gradient are not found. Mark for (a) = 1 out of 5
(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB. [3] $y = -\frac{1}{4}x + 4$ $0 = -\frac{1}{4}x + 4$ $-4 \qquad -4$ $\frac{-4}{-1/4} = \frac{-\frac{1}{4}x}{-\frac{1}{4}}$ $-\frac{4}{1} \times 4$ 2 $16 = x$ when $x = 16$ $y = -\frac{1}{4}(16) + 4$ $-4 + 4$ $y = 0$ $= (16, 0)$	2 The candidate finds the intercept of the line found in (a) with the x-axis and is awarded one mark. Mark for (b) = 1 out of 3 **Total mark awarded = 2 out of 8**

How the candidate could have improved their answer

- **(a)** The candidate needed to find the mid-point. They also needed to use the gradient they found, to work out the gradient of a line perpendicular to it. Then by using that perpendicular gradient with the mid-point they needed to find the equation of the line.
- **(b)** The candidate should have found the intercept of their line with the y-axis and then found the distance between the two intercepts by using Pythagoras'.

Common mistakes candidates made in this question

- **(a)** Many candidates used an incorrect method to find the gradient between the two given points often by mixing up the x and y coordinates within the calculation.
- **(a)** Often candidates did not find the mid-point.
- **(a)** Many candidates did not find the gradient of a perpendicular line or used an incorrect method to find it.
- **(a)** Candidates sometimes found the equation of a perpendicular line through one of the given points instead of the mid-point.
- **(a)** Candidates did not always give the equation of the line in the required $y = mx + c$ form.

Question 4

Example Candidate Response – high	Examiner comments
4 Solve the simultaneous equations. $\log_3(x+y) = 2$ $2\log_3(x+1) = \log_3(y+2)$ [6] $\log_3(x+y) = 2$ $x+y = 9$ $y = 9-x$ — ① $\log_3(x+1)^2 = \log_3(y+2)$ $(x+1)^2 = y+2$ — ② subst. ① into ② $x^2+2x+1 = 11-x$ $x^2+3x-10 = 0$ $(x+5)(x-2) = 0$ $x = -5, 2$ $x = -5$, $y = 14$ $x = 2$, $y = 7$ 1	1 The candidate uses the laws of logarithms in a correct way on both of the given equations and then eliminates one of the variables resulting in a correct quadratic equation. This is solved correctly giving two pairs of values. The final mark is lost as $x = -5$ is not rejected as being an inappropriate solution. **Total mark awarded = 5 out of 6**

How the candidate could have improved their answer

The candidate could have checked their two pairs of answers in the given equations and rejected the solution where $x = -5$ as it would have led to the log of a negative number in the second equation, which is undefined.

Example Candidate Response – middle

4 Solve the simultaneous equations.

$$\log_3(x+y) = 2$$
$$2\log_3(x+1) = \log_3(y+2)$$

[6]

$\log_3(x+y) = 2$

$3^2 = x+y$

$x+y = 9$

$2\log_3(x+1) = \log_3(y+2)$

$\log_3(x+1)^2 = \log_3(y+2)$

$\log_3(x+1)^2 - \log_3(y+2) = 0$

$\log_3\dfrac{(x+1)^2}{y+2} = 0$

$3^0 = \dfrac{(x+1)^2}{y+2}$

$1 = \dfrac{(x+1)^2}{y+2}$

$y+2 = (x+1)^2$

$y+2 = x^2+2x+1$

$y = x^2+2x-1$

$x+y = 9$

$y = 9-x$

$9-x = x^2+2x-1$

$0 = x^2+3x-8$

1

Examiner comments

1 The candidate is awarded two marks for using the laws of logarithms in a correct way to obtain two correct equations, and a third mark for obtaining a quadratic equation in x by eliminating y. The three term quadratic is incorrect and there is no attempt to solve it so no further marks are awarded here.

Total mark awarded = 3 out of 6

How the candidate could have improved their answer

The candidate could have attempted to solve their quadratic equation. They needed to check their working to find their arithmetic error.

Example Candidate Response – low	Examiner comments
4 Solve the simultaneous equations. $\log_3(x+y)=2$ $2\log_3(x+1)=\log_3(y+2)$ [6] $x+y=3^2$ $x+y=9$ —① $\log_3(x+1)^2=\log_3(y+2)$ $2\log_3(x+1)=\log_3(y+2)$ $2\log_3(x+1)-\log_3(y+2)=0$ $2\log_3\left(\frac{x+1}{y+2}\right)=0$ $\log_3\left(\frac{x+1}{y+2}\right)=0$ $\frac{x+1}{y+2}=1$ $x+1=y+2$ $x-y=1$ —② $x+y=9$ $x-y=1 \rightarrow x=y+1$ $y+1+y=9$ $2y+1=9$ $2y=8$ $y=4$ $x=5$ $\boxed{x=5,\ y=4}$	1 The candidate obtains $x+y=9$ and is awarded one mark. The candidate does not go on to obtain the second required relationship or the quadratic function. **Total mark awarded = 1 out of 6**

How the candidate could have improved their answer

After having correctly moved the coefficient of 2 from in front of the word log in the second equation to form $\log_3(x+1)^2$ the candidate could have removed the log from each side of the equation leaving an equation now not involving log. This equation they could have solved simultaneously with the first equation they had already formed correctly.

Common mistakes candidates made in this question

- Candidates often removed log from the first equation in an incorrect way, obtaining 8 from 2^3 rather than the correct 9 from 3^2.
- Many candidates made algebraic and arithmetic errors in solving the equations simultaneously.
- Candidates often did not check their answers in the given equations and realise that one pair had to be rejected as it would have required the calculation of the log of a negative number which is undefined.
- Candidates sometimes incorrectly combined two log terms where the coefficients in front of the logs were different values.

Question 5

Example Candidate Response – high

5 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

(a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$. [4]

$y = x^3 - 6x^2 + 3x + 10$

$\frac{dy}{dx} = 3x^2 - 12x + 3$

$\frac{dy}{dx} = 3(1)^2 - 12(1) + 3$

$\frac{dy}{dx} = 3 - 12 + 3$

$\frac{dy}{dx} = -6$

$y = (1)^3 - 6(1)^2 + 3(1) + 10$

$y = 1 - 6 + 3 + 10$

$y = 8$

$y = -6x + c$

$8 = -6(1) + c$

$8 = -6 + c$

$14 = c$

$y = -6x + 14$ 1

(b) Find the coordinates of the point where this tangent meets the curve again. [5]

$y = x^3 - 6x^2 + 3x + 10$

$y = -6x + 14$

$-6x + 14 = x^3 - 6x^2 + 3x + 10$

$0 = x^3 - 6x^2 + 3x + 10 + 6x - 14$

$0 = x^3 - 6x^2 + 9x - 4$

1	1	−6	9	−4
	↓	1	−5	+4
	1	−5	4	0

$x = 1$

$0 = (x-1)(x^2 - 5x + 4)$

$0 = (x-1)(x^2 - 4x - x + 4)$

$0 = (x-1)(x(x-4) - 1(x-4))$

$0 = (x-1)(x-1)(x-4)$

$x = 1, 1, 4$

$y = -6(1) + 14$

$y = 8$

$y = -6(4) + 14$

$y = -10$

$(1, 8)$ $(4, -10)$ 2

Examiner comments

1 The candidate gives a completely correct answer.

Mark for (a) = 4 out of 4

2 The candidate equates the equations correctly and the resulting cubic is solved giving one distinct root plus a repeated root as expected. The final mark is lost as the coordinates of both points are quoted whereas the correct answer is $(4, -10)$ only.

Mark for (b) = 4 out of 5

Total mark awarded = 8 out of 9

How the candidate could have improved their answer

The candidate needed to read the question carefully and note that it asked for the 'point' not 'points' so the known point where $x = 1$ was not part of the answer.

Example Candidate Response – middle

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$. [4]

$\frac{dy}{dx} = 3x^2 - 12x + 3$

$x = 1$

$\frac{dy}{dx} = 3 - 12 + 3$

$\frac{dy}{dx} = -6$

$y = x^3 - 6x^2 + 3x + 10$

$x = 1$

$y = 1 - 6 + 3 + 10$

$y = 8$

$(1, 8)$

$y = mx + c$

$y = -6x + c$

$8 = -6(1) + c$

$8 + 6 = c$

$c = 14$

1 $y = -6x + 14$

(b) Find the coordinates of the point where this tangent meets the curve again. [5]

$y = -6x + 4 \qquad y = x^3 - 6x^2 + 3x + 10$ 2

$-6x + 4 = x^3 - 6x^2 + 3x + 10$

$x^3 - 6x^2 + 9x - 6 = 0$

$x = 4.19558233\,45$

$(4.20, \cancel{0})$

$(4.2)^3 - 6(4.2)^2 + 9(4.2) - 6$

$\cancel{x}\ y = 0.048$

$(4.20, 0.048)$

Examiner comments

1 The candidate is awarded all four available marks for (a).

Mark for (a) = 4 out of 4

2 The candidate is awarded one mark for equating the answer from (a) to the given cubic equation and simplifying. A mis-copy of the linear equation leads to an incorrect cubic equation and no valid method is shown to try and solve it.

Mark for (b) = 1 out of 5

Total mark awarded = 5 out of 9

How the candidate could have improved their answer

(b) The candidate should have transcribed the linear equation found in the first part of the question correctly when using it in the second part. They should have attempted to solve the cubic equation without using a calculator, as required in the question.

Example Candidate Response – low	Examiner comments
5 **DO NOT USE A CALCULATOR IN THIS QUESTION.** **(a)** Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$. [4] $y = x^3 - 6x^2 + 3x + 10$ $\frac{dy}{dx} = 3x^2 - 12x + 3$ $\frac{dy}{dx} = 6x - 12$ **1** $y = mx + c$ $y = 6x - 12$	**1** The candidate finds the gradient function and is awarded one mark. Mark for (a) = 1 out of 4
(b) Find the coordinates of the point where this tangent meets the curve again. [5] $6x - 12 = x^3 - 6x^2 + 3x + 10$ ($+12 - 6x$) $x^3 - 6x^2 + 3x + 10 + 12 - 6x$ **2** $y = x^3 - 6x^2 - 3x + 22$ $\cancel{y = 1^3 + 6(1)^2 - 3(1) + 22}$ $\cancel{y = 1 + 6 - 3 + 22}$ $\cancel{y = 26}$ $\cancel{(1, 26)}$ $y = 1^3 + 6(1)^2 - 3(1) + 22$ $y = 26$ $(1, 26)$ $\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ $x =$	**2** The candidate is awarded one mark for equating their equation of the tangent from (a) to the cubic and simplifying to four terms. No attempt is made to solve the cubic equation. Mark for (b) = 1 out of 5 **Total mark awarded = 2 out of 9**

How the candidate could have improved their answer

- **(a)** The candidate should have found the y-coordinate of the point on the curve at the given value for x, then found the gradient of the curve at the given point. Using the given point and their calculated gradient they could have worked out the equation of the tangent.
- **(b)** The candidate could have attempted to solve the cubic equation either by substituting in values until one that satisfied the equation was found. Or, using the known point $x = 1$, and hence a known factor of $x - 1$, they could have found the corresponding quadratic factor and factorised that to obtain the answer.

Common mistakes candidates made in this question

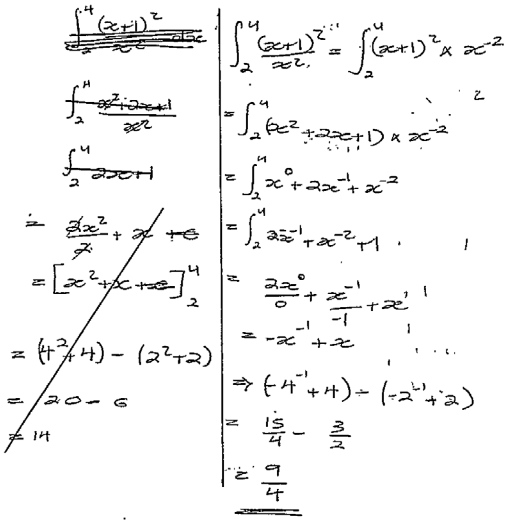
- Some candidates made errors in differentiating the given cubic equation.
- Candidates sometimes made arithmetic errors in substituting $x = 1$ into either the original cubic or the differentiated equation.
- Occasionally candidates wrote $x + 1$ as a factor rather than $x - 1$.
- A number of candidates gave $(1,8)$ as an extra answer when only $(4,–10)$ was required.

Question 6

Example Candidate Response – high	Examiner comments
6 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$. [6] $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$ $\int_2^4 \frac{x^2+2x+1}{x^2}\,dx$ $\int_2^4 1+\frac{2}{x}+\frac{1}{x^2}\,dx$ $\left[x+2\ln x - \frac{1}{x}\right]_2^4$ $\left[4+2\ln 4-\frac{1}{4}\right]-\left[2+2\ln 2-\frac{1}{2}\right]$ $6.52 - 2.89 = 3.63$ 1	1 The candidate expands the function and divides by x^2 correctly. The integral is also correct and the limits are applied in a correct way but the final result is given to 3 significant figures rather than as an exact value and the last mark is not awarded. **Total mark awarded = 5 out of 6**

How the candidate could have improved their answer

The candidate could have given the exact answer rather than an approximate one involving decimals.

Example Candidate Response – middle	Examiner comments
6 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$. [6]  1	1 The candidate is awarded two marks for expanding the numerator correctly and for dividing each term in it by x^2. A third mark is then awarded as the candidate integrates two terms correctly. No further marks are available as there is no ln term in their result. **Total mark awarded = 3 out of 6**

How the candidate could have improved their answer

The candidate needed to integrate the $\frac{1}{x}$ term correctly as $\ln x$.

Example Candidate Response – low	Examiner comments
6 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,dx$. [6]	

$$\frac{\int_2^4 (x+1)^2}{\int_2^4 x^2} \qquad x+1 \qquad x^2+2x+1$$

$$\int_2^4 \frac{x^2+2x+1}{x^2}$$

1

1 The candidate expands the numerator correctly and is awarded one mark.

$$\left[\frac{\frac{2x^3}{3}+\frac{2x^2}{2}+x}{\frac{x^3}{3}}+c\right]_2^4$$

2

2 The candidate incorrectly attempts to integrate the numerator and the denominator separately. No further marks are awarded.

$$\left[\frac{\frac{4^3}{3}+\frac{2(4)^2}{2}+4}{\frac{(4)^3}{3}}\right]-\left[\frac{\frac{2^3}{3}+\frac{2(2)^2}{2}+2}{\frac{2^3}{3}}\right]$$

$$\cancel{[1.9375]-[3.25]=-1.3125}$$

$$\cancel{39.68666667}=$$

$$1.750000002$$

Total mark awarded = 1 out of 6

How the candidate could have improved their answer

The candidate could have divided each of the three terms in the correctly expanded numerator by the single term denominator leading to three terms each of the form xn which could then be integrated individually.

Common mistakes candidates made in this question

- Candidates did not always divide each term in the expanded numerator by the denominator.
- Some candidates integrated the numerator and denominator separately.
- Many candidates did not integrate $\frac{1}{x}$ to $\ln x$.
- Candidates often made sign errors when integrating the $\frac{1}{x^2}$ term.
- Some candidates used a calculator to give a non-exact answer.

Question 7

Example Candidate Response – high	Examiner comments
7 A geometric progression has a first term of 3 and a second term of 2.4. For this progression, find	
(a) the sum of the first 8 terms, [3]	
~~$S_n = \frac{a(1-r^n)}{1-r}$, $S_8 = \frac{3(1-(0.8)^8)}{1-0.8} = \frac{3(0.16777216)}{0.2} = 2.5165824 = 2.52$~~	
$r = \frac{2.4}{3} = 0.8$	
$S_n = \frac{a(1-r^n)}{1-r}$	
$S_8 = \frac{3(1-(0.8)^8)}{1-0.8} = \frac{3(0.83222784)}{0.2} = \frac{2.49668352}{0.2} = 12.4834176 = 12.5$	**1** The candidate answers (a) and (b) correctly.
(b) the sum to infinity, [1]	Mark for (a) = 3 out of 3
$S_\infty = \frac{a}{1-r} = \frac{3}{1-0.8} = \frac{3}{0.2} = 15$	Mark for (b) = 1 out of 1
(c) the least number of terms for which the sum is greater than 95% of the sum to infinity. [4]	
$S_n > 0.95(S_\infty)$	**2** The candidate answers part (c) well until the division by lg 0.8. As this is negative, the inequality needs to be reversed to give a final integer answer of 14 rather than 13, therefore the final mark is not awarded.
$\frac{a(1-r^n)}{1-r} > 0.95(15)$	Mark for (c) = 3 out of 4
~~$3(1-r^n)$~~ $\frac{3(1-0.8^n)}{1-0.8} > 14.25$	
$\frac{3-3\cdot 0.8^n}{0.2} > 14.25$	
$3-(3\cdot 0.8^n) > 2.85$	
$3 > (3\cdot 0.8^n) + 2.85$	
$(3\cdot 0.8^n) < 0.15$	
$\lg 0.8^n < \lg 0.05$	**Total mark awarded = 7 out of 8**
$n \lg 0.8 < \lg 0.05$	
$n < \frac{\lg 0.05}{\lg 0.8}$	
$n < 13.42513488$	
$n < 13$	
∴ number of terms should be at least 13	

How the candidate could have improved their answer

Despite the error in not reversing the sign as the inequality was being divided by a negative number, the candidate should have realised that the value of *n* they had found was when the sum was equal to 95% of the sum to infinity, and thus, for it to be greater than 95%, they needed to round the value 'up' to the next whole number.

Example Candidate Response – middle

7 A geometric progression has a first term of 3 and a second term of 2.4. For this progression, find

(a) the sum of the first 8 terms, [3]

$\cancel{S_8 = \frac{2.4(1-r^8)}{1-r}}$

$\cancel{3r^{1-1} = 3}$

$\frac{3r}{3} = \frac{2.4}{3}$

$\therefore r = \frac{4}{5}$

$S_8 = \frac{3\left[1-\left(\frac{4}{5}\right)^8\right]}{1-\frac{4}{5}}$

$= \cancel{\frac{0.5033}{\frac{1}{5}}}$ $\frac{2.4966...}{\frac{1}{5}}$

$= 12.483...$

$= 13$

(b) the sum to infinity, [1]

$S_\infty = \frac{3}{1-\frac{4}{5}} = \frac{3}{\frac{1}{5}} = 15$

(c) the least number of terms for which the sum is greater than 95% of the sum to infinity. [4]

$\frac{95}{100} \times 15 = 14.25$

$\frac{3(1-\frac{4}{5}^n)}{1-\frac{4}{5}} > 14.25$

$\frac{3-2.4^n}{0.2} > 14.25$

$\frac{3}{0.2} - \frac{2.4^n}{0.2} > 14.25$

$15 - \frac{2.4^n}{0.2} = 14.25$

$\frac{-2.4^n}{0.2} = \frac{-0.75}{1}$

$-2.4^n = -0.15$

Examiner comments

1 The candidate is awarded the four available marks for (a) and (b).

Mark for (a) = 3 out of 3

Mark for (b) = 1 out of 1

2 The candidate gives the correct initial equation in (c) but the subsequent ‘simplification’ of it is processed incorrectly. There is no attempt to find the value of n and so no further marks are available.

Mark for (c) = 1 out of 4

Total mark awarded = 5 out of 8

How the candidate could have improved their answer

(c) The candidate could have divided each side of their correct expression by the 3 and multiplied by the 0.2 in order to isolate the $(1-\left(\frac{4}{5}\right)^n)$, then rearranged to make $\left(\frac{4}{5}\right)^n$ the subject. They could have continued to solve for n by cancelling the negative signs and then taking logs to get $n \log 2.4 = \log 0.15$ leading to $n = \frac{\log 0.15}{\log 2.4}$ which, though incorrect, would have been awarded a mark.

Example Candidate Response – low

7 A geometric progression has a first term of 3 and a second term of 2.4. For this progression, find

(a) the sum of the first 8 terms, [3]

$S_n = \frac{a(1-r)^n}{1-r}$ 1

$S_8 = \frac{3(1-\frac{4}{5})^8}{1-\frac{4}{5}}$ 2

$= \frac{(\frac{3}{5})^8}{\frac{1}{5}} = \frac{3}{390625} \div \frac{1}{5}$

$\frac{3}{390625} \times \frac{5}{1} = \frac{15}{390625}$

(b) the sum to infinity, [1]

$\cancel{S_8}\ S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-\frac{4}{5}}$ $\quad 3 \div \frac{1}{5}$

$3 \times \frac{5}{1} = 15$ 3

(c) the least number of terms for which the sum is greater than 95% of the sum to infinity. [4]

$\frac{95}{100} \times 15$

$\frac{57}{4} < \frac{3(1-\frac{4}{5})^n}{1-\frac{4}{5}}$ 4

$\frac{57}{4} < \left(\frac{3}{5}\right)^n \div \frac{1}{5}$

$\frac{57}{4} < \left(\frac{3}{5}\right)^n \times \frac{5}{1}$

$\frac{57}{4} < \frac{15^n}{5^n}$

$\frac{57}{4} < 3^n$

$14.25 < 3^n$

$\log 14.25 < \log 3^n$

$\frac{\log 14.25}{\log 3} \nless \frac{n \cancel{\log 3}}{\cancel{\log 3}}$

$n > 2.418$

$n > 2$

$\therefore 2$ terms

Examiner comments

1 The candidate uses an incorrect formula for S_n and will not be able to correctly calculate the sum of the first 8 terms.

2 The candidate identifies the correct values of both a and r for the progression and is awarded one mark.

Mark for (a) = 1 out of 3

3 The candidate gives a correct answer which is awarded one mark.

Mark for (b) = 1 out of 1

4 The candidate uses an incorrect formula for S_n, so no marks can be awarded.

Mark for (c) = 0 out of 4

Total mark awarded = 2 out of 8

How the candidate could have improved their answer

The candidate should have used the correct formula, printed in the question paper, for the sum of a geometric progression throughout the question.

Common mistakes candidates made in this question

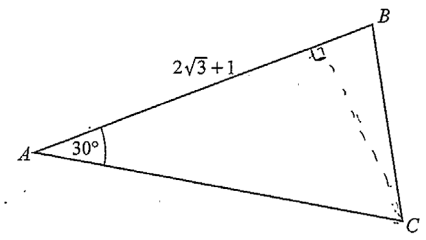
- Some candidates used a common ratio of 1.25 instead of the correct 0.8, having evaluated $\frac{3}{2.4}$ instead of $\frac{2.4}{3}$.
- Several candidates used an incorrect formula for the sum of a geometric progression. Sometimes the sum of an arithmetic progression was used.
- **(a)** Many candidates did not give the sum of the first 8 terms to sufficient accuracy.
- **(c)** Candidates often incorrectly combined $p \times q^n$ as $(pq)^n$.
- **(c)** Many candidates did not reverse the inequality sign when dividing by a negative value.
- **(c)** Often candidates did not round their answer 'up' to the next integer value or left an inequality sign in their answer.

Question 8

Example Candidate Response – high

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question lengths are in centimetres.



You may use the following trigonometric ratios.

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

(a) Given that the area of the triangle ABC is $5.5\ \text{cm}^2$, find the exact length of AC. Write your answer in the form $a+b\sqrt{3}$, where a and b are integers. [4]

$\text{Area} = \frac{1}{2}ab\sin c$

$= \frac{1}{2} \times (2\sqrt{3}+1) \times AC \times \sin 30$

$\frac{1}{4} \times (2\sqrt{3}+1) \times AC = 5.5$

$AC = 5.5 \div \frac{2\sqrt{3}+1}{4}$ 1

$= \frac{22}{2\sqrt{3}+1} \times \frac{(2\sqrt{3}-1)}{(2\sqrt{3}-1)}$

$= \frac{44\sqrt{3}-22}{12-1}$

$= \frac{44\sqrt{3}-22}{11} \rightarrow 4\sqrt{3}-2$

(b) Show that $BC^2 = c+d\sqrt{3}$, where c and d are integers to be found. [4] 2

$BC^2 = AB^2 + AC^2 - (2 \times AB \times AC \times \cos 30)$

$= (2\sqrt{3}+1)^2 + (4\sqrt{3}-2)^2 - (2 \times (2\sqrt{3}+1) \times (4\sqrt{3}-2) \times \cos 30)$

$= 12 + 4\sqrt{3} + 1 + 48 - 16\sqrt{3} + 4 - [2(24 - 2) \times \frac{\sqrt{3}}{2}]$

$= 65 + 20\sqrt{3} - [\frac{44\sqrt{3}}{2}]$

$= 65 - 2\sqrt{3}$

Examiner comments

1 The candidate is awarded full marks as sufficient detail is provided to show that a calculator has not been used.

Mark for (a) = 4 out of 4

2 The candidate evaluates all three terms correctly. The final mark is not awarded as there is an error in combining the surd terms.

Mark for (b) = 3 out of 4

Total mark awarded = 7 out of 8

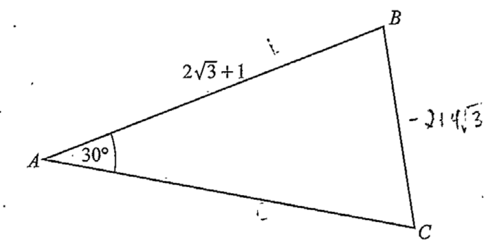
How the candidate could have improved their answer

(c) The candidate should have checked to find their arithmetic error in combining the surd terms.

Example Candidate Response – middle

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question lengths are in centimetres.



You may use the following trigonometric ratios.

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

(a) Given that the area of the triangle ABC is $5.5\,\text{cm}^2$, find the exact length of AC. Write your answer in the form $a + b\sqrt{3}$, where a and b are integers. [4]

$\frac{1}{2}ab\sin\theta = 5.5$

$\frac{1}{2}\times(2\sqrt{3}+1)(b)\times\sin 30 = 5.5$

$\frac{1}{2}\times\frac{1}{2}\times(2\sqrt{3}+1)(b) = 5.5$

$b = \frac{22}{2\sqrt{3}+1}$

$b = \frac{22(2\sqrt{3}-1)}{2\sqrt{3}(2\sqrt{3}-1)+1(2\sqrt{3}-1)}$

$\frac{44\sqrt{3}-22}{11}$

$4\sqrt{3}-2$

$-2+4\sqrt{3}$

$AC = -2+4\sqrt{3}$

1

(b) Show that $BC^2 = c + d\sqrt{3}$, where c and d are integers to be found. [4]

$a^2 = b^2 + c^2 - 2bc\cos A$

2

$a^2 = (2\sqrt{3}+1)^2 + (-2+4\sqrt{3})^2 - 2(2\sqrt{3}+1)(-2+4\sqrt{3})\cos 30$

$a^2 = 11 - 14\sqrt{3}$

$BC^2 = 11 - 14\sqrt{3}$

Examiner comments

1 The candidate is awarded all four marks as the answer is correct with all working shown.

Mark for (a) = 4 out of 4

2 The candidate is awarded a mark for the correct use of the cosine rule. No further working is seen and the answer is incorrect, so no further marks can be awarded.

Mark for (b) = 1 out of 4

Total mark awarded = 5 out of 8

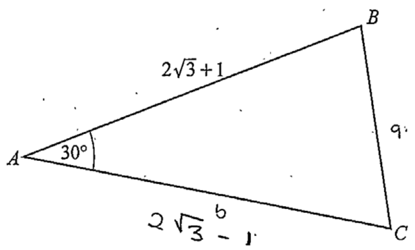
How the candidate could have improved their answer

(b) If the candidate had shown more working, it would have been possible to see if any of the brackets had been expanded correctly.

Example Candidate Response – low

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question lengths are in centimetres.



You may use the following trigonometric ratios.

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

(a) Given that the area of the triangle ABC is 5.5 cm^2, find the exact length of AC. Write your answer in the form $a+b\sqrt{3}$, where a and b are integers. [4]

$\Delta = \frac{1}{2} bc \sin A$ 1

$5.5 = \frac{1}{2} b\, 2\sqrt{3}+1 \sin 30°$

$5.5 = \frac{1}{2} b\, 2\sqrt{3}+1 \sin 30°$

$\frac{11}{} = b\, 2\sqrt{3}+1 \left(\frac{1}{2}\right)$

$11 = b\, 2\sqrt{3}+1 \times 1$

$\frac{11}{2\sqrt{3}+1} = b$

$\frac{11}{2\sqrt{3}+1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$

$\frac{11}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$ 2

$= \frac{22\sqrt{3} - 11}{4(3) - 2\sqrt{3} + 2\sqrt{3} - 1}$

$= \frac{22\sqrt{2} - 11}{12 - 1}$

$= \frac{22\sqrt{3} - 11}{11}$

$2\sqrt{3} - 1$

(b) Show that $BC^2 = c + d\sqrt{3}$, where c and d are integers to be found. [4]

$2\sqrt{3} - 1 . (2\sqrt{3}+1)^2 = 0$ 3

$2\sqrt{3} - 1 \quad , 4(3) + \cancel{2\sqrt{3}} + \cancel{2\sqrt{3}}\ 4\sqrt{3} + (-1)$

$2\sqrt{3} - 1 \quad , 12 + 4\sqrt{3} + (-1)$

$2\sqrt{3} - 1 \quad , 11 + 4\sqrt{3}$

$2\sqrt{3}(11 + 4\sqrt{3}) - 1 \sqrt{11} (11 + 4\sqrt{3})$

$22\sqrt{3} + 8(3) - 11 - 4\sqrt{3}$

$18\sqrt{3} + 13$

$13 + 18\sqrt{3}$

Examiner comments

1 The candidate identifies the correct formula for calculating the area of a triangle from the information given and is awarded one mark.

2 The candidate is awarded a mark for a correct method to rationalise the surd.

Mark for (a) = 2 out of 4

3 Use of the cosine rule is needed but not seen in the working, so no marks can be awarded.

Mark for (b) = 0 out of 4

Total mark awarded = 2 out of 8

How the candidate could have improved their answer

- By writing the surd expression for the length AB in a bracket as it involves more than one term.
- **(a)** By not losing a factor of 2 in their working when expressing the length b as a surd.
- **(b)** By using the cosine rule.

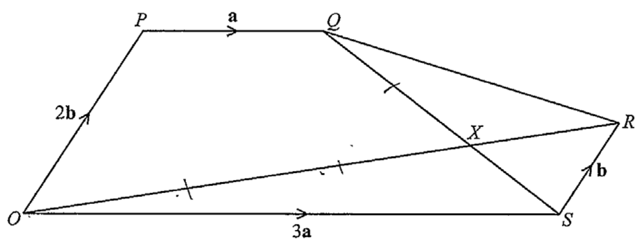
Common mistakes candidates made in this question

- Candidates frequently made algebraic and arithmetic errors in rearranging equations and in expanding brackets.
- Many candidates did not show the method for the rationalisation of the surd although the question stated a calculator must not be used.
- Some candidates did not use the sine rule and/or the cosine rule though it is possible, but much more complicated, to answer the question without using either.

Question 9

Example Candidate Response – high

9



In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

(a) Find $\overrightarrow{OQ}$ in terms of **a** and **b**. [1]

$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= 2\underline{b} + \underline{a}$

1

(b) Find $\overrightarrow{QS}$ in terms of **a** and **b**. [1]

$\overrightarrow{QS} = \overrightarrow{QO} + \overrightarrow{OS}$

$= -(2\underline{b} + \underline{a}) + 3\underline{a})$

$= -2\underline{b} - \underline{a} + 3\underline{a}$

$= 2\underline{a} - 2\underline{b}$

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and μ. [1]

$\overrightarrow{QX} = \mu \cdot (2\underline{a} - 2\underline{b})$

$\overrightarrow{OX} = \overrightarrow{OQ} + \overrightarrow{QX}$

$= 2\underline{b} + \underline{a} + \mu 2\underline{a} - \mu 2\underline{b}$

$= (2\mu + 1)a + (2 - 2\mu)b$

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and λ. [1]

$\overrightarrow{OR} = \overrightarrow{OS} + \overrightarrow{SR}$

$= 3\underline{a} + \underline{b}$

~~$\overrightarrow{QOR} =$~~

$\overrightarrow{OX} = \lambda(\overrightarrow{OR})$

$= \lambda(3\underline{a} + \underline{b})$

Examiner comments

1 The candidate's answers to the first five parts are correct.

Mark for (a) = 1 out of 1

Mark for (b) = 1 out of 1

Mark for (c) = 1 out of 1

Mark for (d) = 1 out of 1

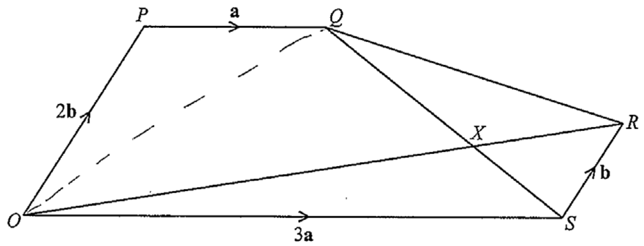
Example Candidate Response – high, continued	Examiner comments
(e) Find the value of λ and of μ. [3] $\vec{OX} = \vec{OX}$ $3\lambda a + 3\lambda b = (2\mu+1)a + (2-2\mu)b$ $\lambda = 2-2\mu$ $\frac{3\lambda}{3} = \frac{2\mu+1}{3}$ $3\lambda = 2\mu+1$ $3(2-2\mu) = 2\mu+1$ $6-6\mu = 2\mu+1$ $5 = 8\mu$ $\frac{5}{8} = \frac{8\mu}{8}$ $\mu = \frac{5}{8}$ $\lambda = 2-2\left(\frac{5}{8}\right) = 2-\frac{5}{4} = \frac{3}{4}$	Mark for (e) = 3 out of 3
(f) Find the value of $\frac{QX}{XS}$. [1] $\vec{QX} = \mu(2a-2b) = \frac{5a}{4} - \frac{5b}{4}$ $\lvert QX\rvert = \sqrt{\frac{25}{16}+\frac{25}{16}} = \sqrt{\frac{25}{8}}$ $\vec{XS} = \left(2\left(\frac{5}{8}\right)+1\right)a + \left(2-2\left(\frac{5}{8}\right)\right)b$ $\left(\frac{5}{4}+1\right)a + \left(2-\frac{5}{4}\right)b$ $= \frac{81}{16} + \frac{9}{16} = \sqrt{\frac{45}{8}}$ $= 0.745$ $\frac{QX}{XS} = \frac{2}{1} = 2$ 2	2 The candidate can not be awarded any marks for (f) despite the constants having been correctly evaluated in (e). Mark for (f) = 0 out of 1
(g) Find the value of $\frac{OR}{OX}$. $= \frac{4}{3}$ 3 [1]	3 The candidate gives a correct answer and is awarded a mark. Mark for (g) = 1 out of 1 **Total mark awarded = 8 out of 9**

How the candidate could have improved their answer

(f) The candidate could have used the given diagram and the value of μ to write down the required ratio. Since QX was $\frac{5}{8}$ QS, if QX was 5, XS was 3, giving the ratio $\frac{5}{3}$.

Example Candidate Response – middle

9



In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

(a) Find $\overrightarrow{OQ}$ in terms of **a** and **b**. [1]

$\overrightarrow{PQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$

$a = -2b + \overrightarrow{OQ}$

$\overrightarrow{OQ} = a + 2b$

(b) Find $\overrightarrow{QS}$ in terms of **a** and **b**. [1]

$\overrightarrow{QS} = -\overrightarrow{OQ} + \overrightarrow{OS}$

$= -a - 2b + 3a$

$\overrightarrow{QS} = 2a - 2b$

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and μ. [1]

$\overrightarrow{QX} = -\overrightarrow{OQ} + \overrightarrow{OX}$

$-a - 2b + \overrightarrow{OX} = \mu(2a - 2b)$

$\overrightarrow{OX} = \mu(2a - 2b) - (a + 2b)$

$\cancel{\overrightarrow{OX} = -\mu(2a-2b)}$ $(a+2b)$

$\cancel{\overrightarrow{OX} = \mu(2a-2b) - (a+2b)}$

$\overrightarrow{OX} = \mu(2a - 2b) - (a + 2b)$ 1

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and λ. [1]

$\overrightarrow{SR} = -\overrightarrow{OS} + \overrightarrow{OR}$

$b = -3a + \overrightarrow{OR}$

$\overrightarrow{OR} = 3a + b$

$\overrightarrow{OX} = \lambda(3a + b)$

Examiner comments

Mark for (a) = 1 out of 1

Mark for (b) = 1 out of 1

1 The candidate is awarded a mark each for (a), (b) and (d) but there is an incorrect sign in (c) so this is not awarded a mark.

Mark for (c) = 0 out of 1

Mark for (d) = 1 out of 1

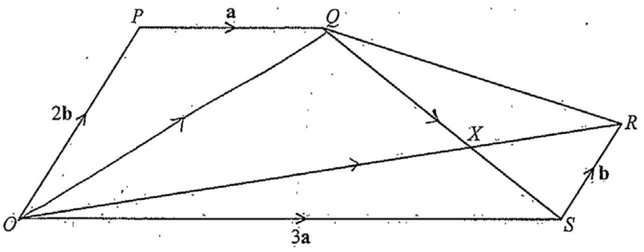
Example Candidate Response – middle, continued	Examiner comments
(e) Find the value of λ and of μ. [3] $\vec{OX} = \vec{OX}$ $-\mu(2a-2b) \cdot (a+2b) = \lambda(3a+b)$ $\mu(2a-2b) = \lambda(3a+b)(a+2b)$ $2a\mu - 2b\mu - a - 2b = 3\lambda a + \lambda b$ $a(2\mu-1) + b(-2\mu-2) = 3\lambda a + \lambda b$ $3\lambda = 2\mu - 1$ $+\ \lambda = -2\mu - 2$ $4\lambda = -3$ $\lambda = -\frac{3}{4}$ $\lambda = -2\mu - 2$ $-\frac{3}{4} = -2\mu - 2$ $\frac{\frac{2.5}{4}}{-2} = \mu$ $\mu = -\frac{5}{8}$ $\therefore \lambda = -\frac{3}{4}$; $\mu = -\frac{5}{8}$	**2** The candidate forms a pair of simultaneous equations in (e) and uses a correct method of solution so two marks are awarded. The values of λ and μ are incorrect so a third mark is not awarded. Mark for (e) = 2 out of 3
(f) Find the value of $\frac{QX}{XS}$. [1] $-\frac{5}{8} \times 2(a-b) - a - 2b$ $XS = -a - 2b + \frac{5}{4}b - \frac{5}{4}a - a - 2b$ $-\frac{13}{4}a - \frac{11}{4}b$ $QX = -\frac{5}{4}(a-b)$ $XS = -OX + OS$ $-\frac{3}{4}(3a+b) + 3a$ $-\frac{9a}{4} - \frac{3b}{4} + 3a$ $\frac{3a}{4} - \frac{3b}{4}$ $\frac{-\frac{5}{4}a + \frac{5}{4}b}{\frac{3}{4}}$ $\frac{\frac{3}{4}(a-b)}{-\frac{5}{4}(a-b)}$ $= \frac{3}{5}$	Mark for (f) = 0 out of 1 **3** The candidate's values of λ and μ are negative, meaning that no marks can be awarded for (f) or (g). Mark for (g) = 0 out of 1
(g) Find the value of $\frac{OR}{OX}$. [1] $OR = 3a + b$ $OX = -\frac{3}{4}(3a+b)$ $\frac{(3a+b)}{-\frac{3}{4}(3a+b)} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}$	**Total mark awarded = 5 out of 9**

How the candidate could have improved their answer

- **(c)** The candidate needed to avoid the sign error when rearranging the expression for $\vec{QX}$.
- **(e)** The candidate should have realised that negative values for λ and for μ meant that an error must have been made and they could have looked back to try and find their error.

Example Candidate Response – low

9



In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

(a) Find $\overrightarrow{OQ}$ in terms of **a** and **b**. [1]

$\overrightarrow{OQ} = 2b + a$

$= a + 2b$ (1)

(b) Find $\overrightarrow{QS}$ in terms of **a** and **b**. [1]

$\overrightarrow{QS} = -\overrightarrow{OQ} + 3a$

$\overrightarrow{QS} = -a - 2b + 3a$

$= 2a - 2b$

$= 2(a-b)$

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and μ. [1]

$\overset{QX}{\cancel{2a-2b}} = \mu(2a-2b)$ (2)

$\overrightarrow{QX} = 2\mu(a-b)$

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of **a**, **b** and λ. [1]

$\overrightarrow{OR} = 3a + b$

$\overrightarrow{OX} = \lambda(3a+b)$ (3)

Examiner comments

(1) The candidate gives correct answers for (a) and (b) which are awarded one mark each.

Mark for (a) = 1 out of 1

Mark for (b) = 1 out of 1

(2) The candidate gives an incorrect answer for the vector $\overrightarrow{OX}$, possibly misreading the question and instead gives an expression for the vector $\overrightarrow{OX}$.

Mark for (c) = 0 out of 1

(3) The candidate supplies a correct answer and is awarded one mark.

Mark for (d) = 1 out of 1

Example Candidate Response – low, continued	Examiner comments
(e) Find the value of λ and of μ. [3] $\overrightarrow{OX} = \lambda(3a+b)$ $\overrightarrow{QX} = 2\mu(a+b)$ 4 OX QQ QX $QX = -a-2b+\lambda(3a+b$ $3\lambda a + \lambda b =$ $2\mu a + \mu b$ $3\lambda a + \lambda b = 2\mu a + \mu b$ $3\lambda a - 2\mu a + \lambda b - \mu b = 0$ $a(3\lambda - 2\mu) + b(\lambda - \mu) = 0$ $\cancel{(a+b)}($ $-2b - a + 3\lambda a + \lambda = \lambda(3a+b)$	4 The candidate does not form a pair of linear equations by equating the terms in **a** and in **b** and no marks can be awarded. Mark for (e) = 0 out of 3
(f) Find the value of $\frac{QX}{XS}$. [1] $\frac{\mu(2a+2b)}{}$ $\frac{3}{4} = 3$ 5	5 The candidate has no value for λ or for μ so no mark can be awarded. Mark for (f) = 0 out of 1
(g) Find the value of $\frac{OR}{OX}$. [1] 4 6	6 Since the candidate has no value for λ or for μ no mark can be awarded. Mark for (g) = 0 out of 1 **Total mark awarded = 3 out of 9**

How the candidate could have improved their answer

- **(c)** The candidate needed to read the question more carefully as they gave an expression for the vector $\overrightarrow{QX}$ instead of $\overrightarrow{OX}$.
- **(e)** The candidate needed to form a pair of simultaneous equations by equating the terms in **a** and in **b** and solving them to find a value for each of λ and μ.

Common mistakes candidates made in this question

- **(c)** Some candidates gave an expression for $\overrightarrow{QX}$ rather than the required $\overrightarrow{OX}$.
- **(e)** Many candidates did not equate the terms in **a** and in **b** in order to form a pair of equations in λ and μ and then solve them to find their values.
- Sometimes candidates did not realise that λ and μ had to be positive and less than 1 for the values to be valid given the context of the question.

Question 10

Example Candidate Response – high	Examiner comments
10 The number, b, of bacteria in a sample is given by $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week. **(a)** Find the value of P and of Q. [4] $484.3 + \cancel{Qe^0}$ $15.7\,e^0$ $P + Qe^0 = 500$ $P + Q = 500$ $\quad \cancel{Q = 500}$ $\quad P = 500 - Q$ $P + Qe^2 = 600$ $500 - Q + Qe^2 = 600$ $Qe^2 - Q = 100$ $Q(e^2 - 1) = 100$ $Q = \dfrac{100}{e^2 - 1} = 15.7$ $P = 484.3$ $Q = 15.7$	Mark for (a) = 4 out of 4

Example Candidate Response – high, continued	Examiner comments
(b) Find the number of bacteria present after 2 weeks. [1] $484.3 + (15.7 \times e^4) = 1341.5$ (1342) **1**	**1** The candidate supplies fully correct answers to (a) and (c) but the integer value required for (b) is incorrect and is not awarded the mark. Mark for (b) = 0 out of 1
(c) Find the first week in which the number of bacteria is greater than 1 000 000. [3] $484.3 + (15.7 \times e^{2t}) = 1000\,000$ $e^{2t} = \dfrac{1000\,000 - 484.3}{\cancel{484}\ 15.7}$ $e^{2t} = 63663.42038$ $b^c = a$ ~~loga~~ $\ln 63663.42038 = t$ $2t = 11.06136543$ ~~t=NA~~ $t = 6$	Mark for (c) = 3 out of 3 **Total mark awarded = 7 out of 8**

How the candidate could have improved their answer

The candidate needed to use more accurate values for P and for Q in the calculation and should have rounded the answer 'down' to an integer value.

Example Candidate Response – middle

10 The number, b, of bacteria in a sample is given by $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week.

(a) Find the value of P and of Q. [4]

$\cancel{600 = P + Qe^{1}}$
$\cancel{500 = P + Q}$
$\cancel{100 = eQ}$
$\cancel{\therefore Q = 58.2}$
$\cancel{\therefore P = 500 - 58.2}$
$\cancel{= 441.8}$

$600 = P + Qe^{2}$
$500 = P + Q$
$100 = 6.39Q$
$\therefore Q = 15.7$
$\therefore P = 484.3$

1

Examiner comments

1 The candidate is awarded all four available marks for a correct answer.

Mark for (a) = 4 out of 4

Example Candidate Response – middle, continued	Examiner comments
(b) Find the number of bacteria present after 2 weeks. [1] $484.3 + 15.7 \times e^{4} \approx 1341.5$ 2	2 The candidate is awarded no marks as the value is incorrect. Mark for (b) = 0 out of 1
(c) Find the first week in which the number of bacteria is greater than 1 000 000. [3] $1000000 = \cancel{484.3+}\ 484.3 + 15.7 \times e^{x}$ $999515.7 \cancel{+15} = 15.7 \times e^{x}$ $\frac{999515.7}{15.7} = e^{x}$ $63663 = e^{x}$ $\log_e 63663 = x$ 3 $\therefore x = 11.$	3 The power of e is not $2t$ so the first mark is not awarded. A mark is awarded as logs are taken correctly in order to solve the equation, but the final value is incorrect. Mark for (c) = 1 out of 3 **Total mark awarded = 5 out of 8**

How the candidate could have improved their answer

- **(b)** The candidate should have used more accurate values for P and for Q in the calculation and should have rounded the answer ‘down’ to an integer value.
- **(c)** The candidate should have used the original given equation involving e^{2t} rather than replacing it with e^{x}.

Example Candidate Response – low	Examiner comments
10 The number, b, of bacteria in a sample is given by $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week. **(a)** Find the value of P and of Q. [4] $P = 500$ $600 = 500 + Qe^{2(1)}$ **1** $600 = 500 + Qe^{2 \cdot 1}$ $100 = Qe^{2}$ $\frac{100}{\cancel{7.389056099}} = Q\,\cancel{7.389056099}$ $Q = 13.53352832$ $Q \approx 13.534$	**1** The candidate attempts to create and solve a single equation containing P and Q. Two equations are required from the given information in the question, so no mark is awarded. Mark for (a) = 0 out of 4

Example Candidate Response – low, continued	Examiner comments
(b) Find the number of bacteria present after 2 weeks. [1] $b = 500 + 13.534e^{2(2)}$ $= 500 + 13.34e^{4}$ $\cancel{-500} = 738.90560909 + 500$ $b = 1238.90561$ $b \simeq 1238.0$ 2	2 An incorrect value, so the candidate is not awarded a mark Mark for (b) = 0 out of 1
(c) Find the first week in which the number of bacteria is greater than 1,000,000. [3] 12 $1000000 = 500 + 13.534e^{2t}$ $\dfrac{1000000 - 500}{13.534} = \dfrac{\cancel{13.534}e^{2t}}{\cancel{13.534}}$ $73818.3161 = e^{2t}$ 3 $\dfrac{\ln(73818.3161)}{2} = \dfrac{\cancel{2}t}{\cancel{2}}$ 4 $t = 5.604681083$	3 The candidate makes e^{2t} the subject of the formula and is awarded one mark. 4 A second mark is awarded for correctly taking the natural log of both sides and finding a value for t. Since the values used for P and for Q are incorrect, no mark can be awarded for the answer even if it had been rounded up to the next whole week. Mark for (c) = 2 out of 3 **Total mark awarded = 2 out of 8**

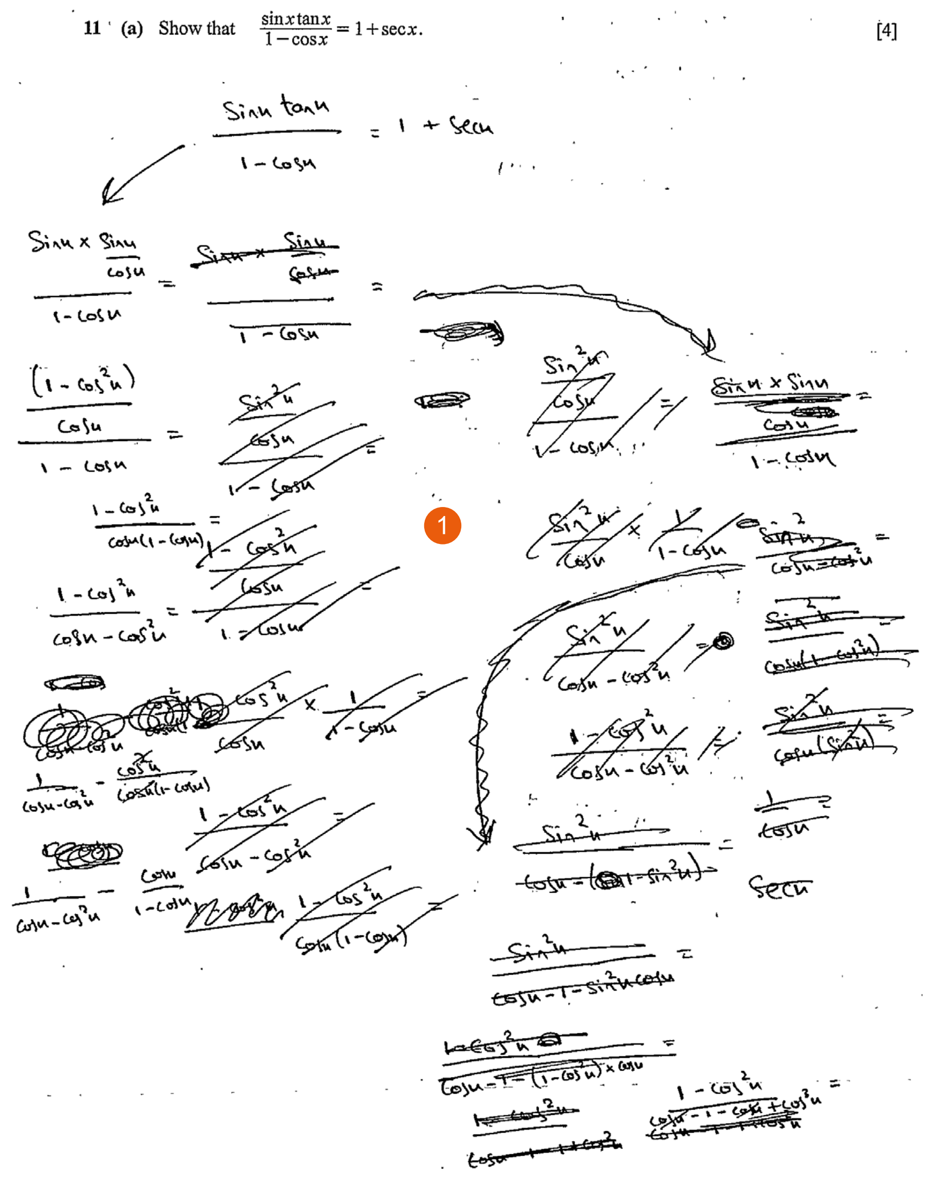
How the candidate could have improved their answer

- **(a)** The candidate should have used all the given information to form a pair of equations in P and Q and then solved them simultaneously to find the values of P and Q.
- **(c)** The candidate should have rounded their answer 'up' to the nearest integer.

Common mistakes candidates made in this question

- **(a)** Candidates often did not use all the given information to form a pair of equations which needed to be solved simultaneously to find the value of P and of Q.
- **(b)** Many candidates didn't use the values of P and Q to sufficient accuracy in the calculation.
- **(b)** Candidates often did not round the answer 'down' to an integer.
- **(c)** Candidates often did not round the answer 'up' to an integer.

Question 11

Example Candidate Response – high	Examiner comments
 11 (a) Show that $\frac{\sin x \tan x}{1-\cos x} = 1 + \sec x$. [4]	1 The candidate is awarded the first two method marks for correct trigonometric relationships. The next method mark is not obtained as the candidate does not attempt to factorise and as a result the final mark is also not awarded. Mark for (a) = 2 out of 4

Example Candidate Response – high, continued	Examiner comments
(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0° \leqslant x \leqslant 360°$. [6] $5\tan x - 3\cot x = 2\sec x \qquad 0° \le x \le 360°$ $\frac{5\sin x}{\cos x} - \frac{3}{\tan x} = \frac{2}{\cos x}$ $\frac{5\sin x}{\cos x} - \frac{3\cos x}{\sin x} = \frac{2}{\cos x}$ $\frac{5\sin x^{\times \sin x}}{\cos x^{\times \sin x}} - \frac{3\cos x^{\times \cos x}}{\sin x^{\times \cos x}} - \frac{2^{\times \sin x}}{\cos x^{\times \sin x}} = 0$ $\frac{5\sin^2 x - 3\cos^2 x - 2\sin x}{\cos x \sin x} = 0$ $5\sin^2 x - 3(1 - \sin^2 x) - 2\sin x = 0$ $5\sin^2 x - 3 + 3\sin^2 x - 2\sin x = 0$ $8\sin^2 x - 2\sin x - 3 = 0$ $\sin x = u$ $8u^2 - 2u - 3 = 0$ $u = \frac{3}{4}$ or $u = -\frac{1}{2}$ $\sin x = \frac{3}{4}$ or $\sin x = -\frac{1}{2}$ $x = 48.6$ or $x = 30$ $x = 48.6, 30, 180 - 48.6, 180 + 30, 360 - 30$ **2** $x = 48.6°, 30°, 131.4, 210°, 330°$	**2** The candidate gives a fully correct answer and is awarded all six marks. Mark for (b) = 6 out of 6 **Total mark awarded = 8 out of 10**

How the candidate could have improved their answer

The candidate should have factorised the numerator as the difference of two squares and then cancelled the common factor of $(1 - \cos x)$ leaving $\frac{(1 + \cos x)}{\cos x}$ which could be separated into $\frac{1}{\cos x} + \frac{\cos x}{\cos x}$ and hence the required result.

Example Candidate Response – middle	Examiner comments
11 (a) Show that $\frac{\sin x \tan x}{1-\cos x} = 1 + \sec x$. [4] $\frac{\sin x \tan x}{1-\cos x} = 1+\sec x$ $\frac{\sin x \tan x}{1-\cos x}$ $\frac{\sin x \times \frac{\sin x}{\cos x}}{1-\cos x}$ $\frac{\sin^2 x}{\cos x} \times \frac{1}{1-\cos x}$ $\frac{1-\cos^2 x}{\cos x} \times \frac{1}{1-\cos x}$ $\frac{1-\cos^2 x}{\cos x(1-\cos x)}$ $\frac{(1+\cos x)(1-\cancel{\cos x})}{\cos x\cancel{(1-\cos x)}}$ $\frac{1}{\cos x} + \frac{\cos x}{\cancel{\cos x}}$ $\frac{1}{\cos x} + 1$ $1 + \frac{1}{\cos x}$ $1 + \sec x$, hence shown 1	1 The candidate is awarded all four marks. Mark for (a) = 4 out of 4

Example Candidate Response – middle, continued	Examiner comments
(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0° \leqslant x \leqslant 360°$. [6] $5\tan x - 3\cot x = 2\sec x$ $5\left(\frac{\sin x}{\cos x}\right) - 3\left(\frac{\cos x}{\sin x}\right) = \frac{2}{\cos x}$ 2 $\frac{5\sin x}{\cos x} - \frac{3\cos x}{\sin x} = \frac{2}{\cos x}$ $\frac{5\sin x}{1} - \frac{3\cos^2 x}{\sin x} = 2$ $\frac{5\sin^2 x - 3\cos^2 x}{\sin x} = 2$ $5\sin^2 x - 3\cos^2 x = 2\sin x$	2 The candidate is awarded a mark for writing the trigonometric functions in terms of $\sin x$ and $\cos x$. However no further progress is made as the Pythagorean identity connecting $\sin x$ and $\cos x$ is not used in order to obtain a quadratic equation in just $\sin x$. Mark for (b) = 1 out of 6 **Total mark awarded = 5 out of 10**

How the candidate could have improved their answer

The candidate needed to replace $\cos 2x$ with $1 - \sin 2x$ to form a quadratic in $\sin x$ which could then have been solved.

Example Candidate Response – low	Examiner comments
11 (a) Show that $\frac{\sin x \tan x}{1-\cos x} = 1+\sec x$. [4]	

Example Candidate Response – low

$$\frac{\sin x \cdot \frac{\sin x}{\cos x}}{1-\cos x} \quad ①$$

$$= \frac{\sin^2 x}{\cos x} \times \frac{1-\cos x}{1}$$

$$= \frac{\sin^2 x - \sin^2 x \cos x}{\cos x}$$

$$= \frac{1-\cos^2 x - (1-\cos^2 x)\cos x}{\cos x}$$

$$= \frac{\cancel{1-\cos^2 x\,(1+\cos^2 x)}}{}$$

$$\frac{1-\cos^2 x - \cos x + \cos^3 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} \div \frac{1}{1-\cos x}$$

$$= \frac{\sin^2 x}{\cos x} \times (1-\cos x)$$

$$= \frac{\sin^2 x - \cos x \sin^2 x}{\cos}$$

$$= \cancel{\sin^2 x\,(} \quad ②$$

$$= (1-\cos^2 x) - \cos x(1-\cos^2 x)$$

$$= \frac{1-\cos^2 x - \cos x - \cos^3 x}{\cos}$$

$$=$$

$$1+\sec x = \frac{\cancel{\cos x}}{\cancel{\cos x}} + \frac{1}{\cos x} + \frac{\sin}{\sin} \quad ③$$

$$= \frac{\sin x + \cos x \sin x}{(\cos x)(\sin x)}$$

$$= \frac{\sin x}{(\cos x)(\sin x)} + \cos x \sin x$$

Examiner comments

① The candidate replaces $\tan x$ with $\frac{\sin x}{\cos x}$ and is awarded one mark.

② The candidate is awarded a second mark for using the correct Pythagorean identity. No further progress is made in the proof as the candidate has incorrectly moved the denominator to the numerator.

③ The candidate restarts by trying to work on the right-hand side of the given identity but makes no further progress.

Mark for (a) = 2 out of 4

Example Candidate Response – low, continued	Examiner comments
(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0° \leqslant x \leqslant 360°$. [6] $\frac{5\sin}{\cos} - \frac{3\cos}{\sin} = \frac{2}{\cos}$ 4 $\frac{5\sin^2 x - 3\cos^2 x}{(\cos x)(\sin x)} - \frac{2}{\cos} = 0$ $\frac{5\sin^2 x - 3\cos^2 x - 2\sin x}{(\cos x)(\sin x)} = 0$ $\frac{5\sin^2 x - 3(1 - \sin^2 x) - 2\sin x}{\cos x \sin x} = 0$ $\frac{5\sin^2 x - 3 + \sin^2 x - 2\sin x}{\cos x \sin x} = 0$ $\frac{6\sin^2 x - 2\sin x - 3}{\cos x \sin x} = 0$ $5\tan x - 3\cot x$ $\frac{5\sin x}{\cos x} - \frac{3\cos x}{\sin x}$ $\frac{5\sin^2 x - 3\cos^2 x}{(\cos)(\sin)}$ $\frac{5\sin^2 x - 3 + \sin^2 x}{(\cos x)(\sin x)}$ $= \frac{6\sin^2 x - 3}{(\cos x)(\sin x)}$	4 The candidate expresses $\tan x$, $\cot x$ and $\sec x$ correctly in terms of $\sin x$ and $\cos x$ and is awarded a mark, but there is not sufficient additional working to be awarded any further marks.

Example Candidate Response – low, continued	**Examiner comments**
8-(b) $BC^2 = AC^2 + AB^2 - 2(AC)(AB)(\cos A$ BLANK PAGE $= \left(\frac{-55}{6} + \frac{11\sqrt{3}}{6}\right)^2 + (2\sqrt{3}+1)^2 - 2\left(\frac{-55}{6} + \frac{11\sqrt{3}}{6}\right)(2\sqrt{3}+1)(\cos(30^\circ))$ $= \left(\frac{3025}{36} - \frac{605\sqrt{3}}{18} + \frac{121}{12}\right) + 13 + 4\sqrt{3} - \frac{11 - 99\sqrt{3}}{3} \times \frac{1}{2}$ $= \left(\frac{847}{9} - \frac{605\sqrt{3}}{18}\right) + 13 + 4\sqrt{3} - \frac{11 - 99\sqrt{3}}{6}$ $= \frac{1694 - 605\sqrt{3}}{18} + \frac{67 + 123\sqrt{3}}{6}$ $= \frac{5082 - 1815\sqrt{3} + 67 + 123\sqrt{3}}{18}$ $= \frac{-1692\sqrt{3} + 5149}{18}$ $= \frac{5149}{18} - 94\sqrt{3}$ $c = \frac{5149}{18}$ $d = 94$	Mark for (b) = 1 out of 6 **Total mark awarded = 3 out of 10**

How the candidate could have improved their answer

- **(a)** The candidate needed to multiply by $\frac{1}{1-\cos x}$ and not divide by it on their first line of working.
- **(b)** The candidate should have checked through the work they crossed out as except for an error when they multiplied out the bracket by 3, the work to that stage was correct. The next step should have been to remove the denominator as it is just the numerator which must equate to zero and then replace $\cos 2x$ with $1 - \sin 2x$ to form a quadratic in $\sin x$ which could then have been solved.

Common mistakes candidates made in this question

- Candidates often made algebraic and arithmetic errors.
- **(a)** Candidates often did not factorise $1 - \cos 2x$ as $(1 - \cos x)(1 + \cos x)$ so that the $(1 - \cos x)$ in the denominator could be cancelled.
- **(b)** Many candidates did not write one or more of $\tan$, $\cot$ and $\sec$ as a correct expression involving $\sin$ and/or $\cos$.
- Many candidates did not realise that the resulting equation involving $\sin$ and $\cos$ could be written as one just involving $\sin$ by using the appropriate Pythagorean identity.
- Often candidates did not attempt to solve the quadratic in $\sin x$ or did not realise it was a quadratic and then used an inappropriate method to solve the equation.

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