CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/41**

Paper 4 Mechanics **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Trapezium on the t-axis starting at (0, 0)	**B1**	
	Fully correct with correct labels	**B1**	With height 20, intersecting the t-axis at 60, horizontal line segment between 10 and 40. Axis need not be labelled with t and v but if they are they must be correct. If not labelled, then must assume t is on the horizontal axis.
		2	
1(b)	Area $= \frac{1}{2}(30+60)\times 20$ or Area $= \frac{1}{2}\times 10\times 20 + 30\times 20 + \frac{1}{2}\times 20\times 20$	**M1**	Must be considering the area of a trapezium. Allow a single slip in one term only. Must be adding all terms together if considering two triangles and a rectangle. Using *their* 20 from **(a)**. If no value for the height shown on the graph in **(a)**, then it must be correct.
	$= 900\,\text{m}$	**A1FT**	FT *their* 20 $(= 45\times \textit{their}\ 20)$ but A0 FT if using 2.
		2	

Question	Answer	Marks	Guidance
2(a)	$F = 20\sin 60$	**M1**	Attempt to resolve in y-direction; 2 terms; must be $20\cos 60$ or $20\sin 60$ and must be linked to F (can be implied by the correct answer seen only).
	$= 17.3\,\text{N}$	**A1**	AWRT 17.3 (17.320508…) or $10\sqrt{3}$.
		2	

Question	Answer	Marks	Guidance
2(b)	For resolving in any direction	***M1**	Correct number of terms; allow sin/cos mix; allow sign errors.
	(Horizontal component $= X = R\cos\theta) = \pm(20\cos 60)$ [± 10] (Vertical component $= Y = R\sin\theta) = \pm(20\sin 60 - 10)$ [$= \pm 7.3205\ldots$]	**A1**	For both correct.
	Magnitude = $\sqrt{(20\sin 60 - 10)^2 + (20\cos 60)^2}$ [=12.393136…]	**DM1**	OE – correct number of terms.
	Angle = $\tan^{-1}\left(\frac{20\sin 60 - 10}{20\cos 60}\right)$ [=36.206023…]	**DM1**	OE (e.g. reciprocal) - correct number of terms.
	Magnitude = 12.4 N and Direction = 36.2° above (positive) *x*-axis	**A1**	OE for direction e.g. 36.2° anticlockwise from (positive) *x*-direction, 36.2° above the horizontal. Possibly seen on a diagram. (Radians: 0.63191… to 3sf or better)
		5	

Question	Answer	Marks	Guidance
3	PE gained = $180000g \times 1500\sin 1.5$ [= 70677760.4…]	**B1**	$180000g \times 39.2654\ldots$
	$KE_{Initial} = \frac{1}{2} \times 180000 \times 45^2$ [= 182 250 000] $KE_{Final} = \frac{1}{2} \times 180000 \times 40^2$ [= 144 000 000]	**B1**	For initial KE or final KE (for reference: difference in KE is 38 250 000).
	For work energy equation: $WD = 180000g \times 1500\sin 1.5 + 12000000$ $-\left(\frac{1}{2} \times 180000 \times 45^2 - \frac{1}{2} \times 180000 \times 40^2\right)$	**M1**	Correct number of terms; dimensionally correct; allow sign errors and minor slip(s) in values; allow sin/cos mix on PE term. Work done $= \left(70677760.4\ldots + 12000000 - 38250000\right)$ $J = \left(70677.7\ldots + 12000 - 38250\right)$kJ
	= 44400 kJ [44427.7604…]	**A1**	Must be in kJ.
	Alternative Method for Question 3: Newton's second law and equations of motion		
	$a = -0.142$ [$= -0.141666\ldots$]	**(B1)**	Correct acceleration from $40^2 = 45^2 + 2a(1500)$. Allow AWRT –0.14 or exact $-\frac{17}{120}$.
	$1500(D - \frac{12\,000\,000}{1500} - 180000g\sin 1.5) = 180000 \times -\frac{17}{120} \times 1500$	**(M1)**	**M1** for applying Newton's second law parallel to the hill. Correct number of terms, allow sin/cos mix on weight component, dimensionally correct and multiplying both sides by 1500. Allow *their a* or *a* for acceleration (and minor slip(s) in values).
		(B1)	Correct weight component multiplied by 1500.
	$WD\ (=1500D) = 44400$ kJ [44427.7604…]	**(A1)**	Must be in kJ.
		4	

Question	Answer	Marks	Guidance
4	Driving force = $\frac{14000}{20}$ $[= 700]$	**B1**	OE, e.g. $20 \times DF = 14000$.
	Attempt at Newton's second law on car or trailer or system	***M1**	Correct number of relevant terms (e.g. correct masses); allow sign errors.
	$700 - 400 - T = 1700a$ $T - 150 = 300a$ $700 - 400 - 150 = (1700 + 300)a$	**A2**	A1 for one correct and A2 for any two correct. May have *DF* (or *their DF*) for 700 but A0 if using 14000 as *DF* without first stating *DF*. Must have same T if using first two equations for A2.
	Solving for a or T	**DM1**	From equations with the correct number of relevant terms.
	Acceleration = $\frac{3}{40}$ $= 0.075\,\text{m}\,\text{s}^{-2}$		For both correct answers.
	Tension = 172.5 N	**A1**	Condone 173.
		6	

Question	Answer	Marks	Guidance
5(a)	$a = 2.08 \quad [2.07911\ldots]$	**B1**	From $mg\sin 12 = ma$. Allow exact (e.g. $a = g\sin 12$).
	$v^2 = 5^2 + 2a \times 60$	**M1**	For use of $v^2 = u^2 + 2as$ with $u = 5$ and $s = 60$. Allow sign errors but a must be either $g\sin 12$ or $g\cos 12$ only.
	Speed = $16.6\,\text{m s}^{-1}$ [16.567861…]	**A1**	AWRT 16.6
	Alternative Method for Q5(a)		
	For attempt at work energy equation	**(M1)**	3 terms, dimensionally correct. Allow sign errors; allow sin/cos mix on PE term – condone m missing from all terms. Must be a weight component.
	$\frac{1}{2}mv^2 = \frac{1}{2}m \times 5^2 + mg \times 60\sin 12$	**(A1)**	Correct equation. (for reference: $60\sin 12 = 12.4747\ldots$)
	Speed = $16.6\,\text{m s}^{-1}$ [16.567861…]	**(A1)**	AWRT 16.6
		3	

Question	Answer	Marks	Guidance
5(b)	$R = mg\cos 12$	**B1**	Resolving correctly perpendicular to the plane.
	$mg\sin 12 - F = ma$	***M1**	Use of Newton's second law, correct number of terms; allow sign errors; allow sin/cos mix (must be a weight component).
	For use of $F = 0.03R$ to get equation in a (and m) only	**DM1**	Where R is a component of weight only (dimensionally correct but allow sin/cos mix). $\left[\Rightarrow a = 1.79\ [1.78567\ldots.]\right]$
	$60 = 5t + \frac{1}{2}at^2$ and solve for t	**DM1**	Dependent on previous two M marks. For use of $s = ut + \frac{1}{2}at^2$ with $s = 60$, $u = 5$ and *their* a or other complete method to find positive value(s) of t.
	Time = 5.86 s [5.86260…]	**A1**	AWRT 5.86 (from using $a = 1.79$ or better). AWRT 5.85 (from using $a = 1.8$). AWRT 5.87 (from correct working).

Question	Answer	Marks	Guidance
5(b)	**Alternative method for Question 5(b): Using energy**		
	$R = mg\cos 12$	**(B1)**	Resolving correctly perpendicular to the plane.
	$\frac{1}{2}mv^2 - \frac{1}{2}m \times 5^2 = 60 \times mg\sin 12 - 60 \times F$	**(*M1)**	Use of work-energy principle, correctly number of relevant terms; allow sign errors; allow sin/cos mix on PE term (must be a weight component).
	For use of $F = 0.03R$ to get equation in v (and m) only	**(DM1)**	Where R is a component of weight only (dimensionally correct but allow sin/cos mix) $\left[\Rightarrow v = 15.5\ [15.46870\ldots.]\right]$.
	$60 = \frac{1}{2}(5+v)t$ and solve for t	**(DM1)**	Dependent on previous two M marks. Use of $s = \frac{1}{2}(u+v)t$ with $s = 60$, $u = 5$ and *their* v or other complete method to find positive value(s) of t.
	Time = 5.86 s [5.86260…]	**(A1)**	
		5	

Question	Answer	Marks	Guidance
6(a)	For an attempt at differentiation	***M1**	Decrease power by 1 and a change in coefficient in at least one term (which must be the same term). Therefore, must see $\frac{\mathrm{d}}{\mathrm{d}t}(kt^{\frac{1}{2}}) = ct^{-\frac{1}{2}}$ with $c \neq k$.
	$a = \frac{1}{2}kt^{\frac{1}{2}-1} - 2t^{1-1} + 0 = \frac{1}{2}kt^{-\frac{1}{2}} - 2$	**A1**	May be unsimplified.
	$a = 0 \Rightarrow \frac{1}{2}kt^{-\frac{1}{2}} - 2 = 0 \left[t = \frac{k^2}{16} \right]$ or $k = 4t^{\frac{1}{2}}$	**DM1**	Equate a to 0 and attempt to solve for t or k. Must be of the correct form, e.g. $t = \lambda k^2$ or $k = \delta t^{\frac{1}{2}}$ (possibly implied by forming a correct equation in k).
	$(v =) k\left(\frac{k^2}{16}\right)^{\frac{1}{2}} - 2\frac{k^2}{16} - 8 = 4.5$	**M1**	Or complete method to find k or an equation involving k, e.g. $4t^{\frac{1}{2}} \times t^{\frac{1}{2}} - 2t - 8 = 4.5 \Rightarrow t = 6.25$ therefore $k = 4(6.25)^{\frac{1}{2}}$. Dependent on both previous M marks.
	$\Rightarrow \frac{k^2}{8} = 12.5 \quad \Rightarrow k = 10$	**A1**	AG - Allow verification. Any errors seen is A0.
	Alternative method for last two marks of Question 6(a): Completing the square method		
	(Let $x = t^{\frac{1}{2}}$) $v = -2\left(x^2 - \frac{k}{2}x + 4\right) = -2\left(x^2 - \frac{k}{2}x + \frac{k}{16}^2\right) + \frac{k^2}{8} - 8$	**(M1)**	Dependent on both previous M marks.
	so $\frac{k^2}{8} - 8 = 12.5$ so $k = 10$	**(A1)**	AG
		5	

Question	Answer	Marks	Guidance
6(b)(i)	$10\times1^{\frac{1}{2}}-2\times1-8=0$ and $10\times16^{\frac{1}{2}}-2\times16-8=0$ Or equate $v=0$ and solve for t to get $t=1$ and $t=16$.	**B1**	AG Any errors seen is B0.
		1	
6(b)(ii)	For integration (do not penalise missing c)	***M1**	Increase power by 1 and a change in coefficient in at least one term (which must be the same term).
	$(s=)\frac{10}{\frac{3}{2}}t^{\frac{3}{2}}-\frac{2t^2}{2}-8t\ [+c]=\frac{20}{3}t^{\frac{3}{2}}-t^2-8t\ [+c]$ or $(s=)\frac{k}{\frac{3}{2}}t^{\frac{3}{2}}-\frac{2t^2}{2}-8t\ [+c]=\frac{2k}{3}t^{\frac{3}{2}}-t^2-8t\ [+c]$	**A1**	Allow unsimplified; allow with k not substituted.
	$\left(\frac{20}{3}1^{\frac{3}{2}}-1^2-8\times1\right)-0\left[=-\frac{7}{3}\right]$ and $\left(\frac{20}{3}16^{\frac{3}{2}}-16^2-8\times16\right)-\left(\frac{20}{3}1^{\frac{3}{2}}-1^2-8\times1\right)=$ $\left[\frac{128}{3}--\frac{7}{3}=45\right]$	**DM1**	Attempt at correct use of limits 0 and 1 or 1 and 16 – allow a single slip.
	For both set of limits applied correctly	**DM1**	Allow at most one slip in both.
	$\frac{142}{3}$ m	**A1**	Allow 47.3 or better (47.3333…) **SC** if no integration shown: **B1** for $\pm\frac{7}{3}$ or 45, **B1** for $\frac{142}{3}$ (so 2 marks max).
		5	

Question	Answer	Marks	Guidance
7(a)	$v^2 = 25^2 - 2g \times 20$	**M1**	For use of $v^2 = u^2 + 2as$ or equivalent to get an equation in v only with $u = 25,\ s = 20$ and $a = \pm g$.
	$\Rightarrow$ speed = $15\ \text{m s}^{-1}$	**A1**	AG Allow verification – at least one intermediate step from equation of motion to given result. Any errors seen is A0.
	Alternative method for Question 7(a):		
	$0.2g \times 20 = \frac{1}{2} \times 0.2 \times 25^2 - \frac{1}{2} \times 0.2 \times v^2$	**(M1)**	Attempt at energy with $m = 0.2,\ h = 20,\ u = 25$; correct number of terms, allow sign errors.
	$\Rightarrow$ speed = $15\ \text{m s}^{-1}$	**(A1)**	AG Allow verification - at least one intermediate step from conservation of energy to given result. Any errors seen is A0.
		2	
7(b)	$0.2 \times 15 - 0.1 \times 20 = 0 + 0.1v$ or $0.2 \times 15 - 0.1 \times 20 = 0 - 0.1V$	**M1**	OE Attempt at conservation of momentum; 3 non-zero terms; allow sign errors – use of 25 is M0.
	$v = 10\ \text{m s}^{-1}$ upwards	**A1**	Must have direction (possibly seen on diagram). If using mgv for momentum, then M1 A0 max.
		2	

Question	Answer	Marks	Guidance
7(c)	Speed of P at impact is $20\,\text{m}\,\text{s}^{-1}$	**B1**	From $v^2 = 0 + 2 \times g \times 20$ (OE) - possibly implied by speed of P after impact being stated at 10.
	Time to when P reaches ground = 2 s	**B1**	From $20 = 0 + \frac{1}{2} \times g \times t^2$ (OE).
	$s_P = 10t + \frac{1}{2} \times -g \times t^2$	**M1**	Distance travelled by P after impact with the ground. Must be using *their* 10 (speed of P after impact with the ground) and $a = \pm g$.
	$s_Q = 10t + \frac{1}{2} \times g \times t^2$	**M1**	Distance travelled by Q after P's impact with the ground. Must be using their ±10 (the speed/vel. of Q after *their* 2 s $(= \lvert '10' + (-g) \times 2 \rvert$ where '10' is the value from **(b)**) and $a = \pm g$.
	$s_P + s_Q = 20 \Rightarrow t = 1$	**A1**	Time after P hits the ground to next collision.
	Height = 5 m	**A1**	CWO

Question	Answer	Marks	Guidance
7(c)	**Alternative Method for last 4 marks:**		
	$s_Q = 10t + \frac{1}{2} \times -g \times t^2$	**(M1)**	Expression for the displacement of Q after first impact of P and Q. Must be using *their* 10 (from **(b)**) and $a = \pm g$.
	$s_P = 10 \times (t-2) + \frac{1}{2} \times -g \times (t-2)^2$	**(M1)**	Expression for the displacement of P (for values of $t \geqslant 2$) measured from point of first collision between P and Q. Must be using *their* 2 (time for P to reach ground), *their* 10 (speed of P after impact with the ground), $a = \pm g$.
	$s_Q + 20 = s_P \Rightarrow t = 3$	**(A1)**	Correct time between collisions of P and Q.
	Height = 5 m	**(A1)**	CWO
		6	