Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*3911748467*

**FURTHER MATHEMATICS** **9231/31**

Paper 3 Further Mechanics **May/June 2024**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

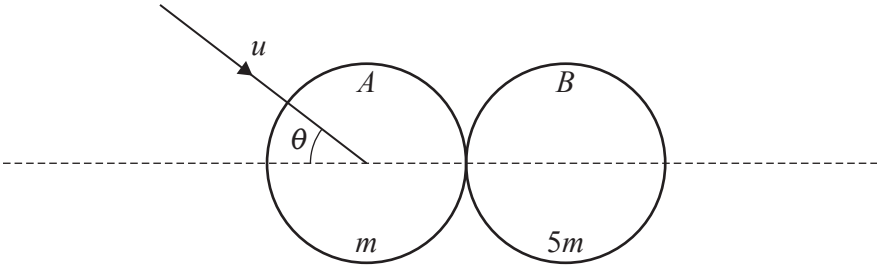
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

**1**




Two smooth uniform spheres $A$ and $B$ of equal radii have masses $m$ and $5m$ respectively. Sphere $A$ is moving on a smooth horizontal surface with speed $u$ when it collides with sphere $B$ which is at rest on the surface. Immediately before the collision, $A$'s direction of motion makes an angle of $\theta$ with the line of centres. After the collision, the kinetic energies of $A$ and $B$ are equal. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find the value of $\tan\theta$. [6]

**2** The points $A$ and $B$ are at the same horizontal level a distance $4a$ apart. The ends of a light elastic string, of natural length $4a$ and modulus of elasticity $\lambda$, are attached to $A$ and $B$. A particle $P$ of mass $m$ is attached to the midpoint of the string. The system is in equilibrium with $P$ at a distance $\frac{3}{2}a$ below $M$, the midpoint of $AB$.

**(a)** Find $\lambda$ in terms of $m$ and $g$. [3]

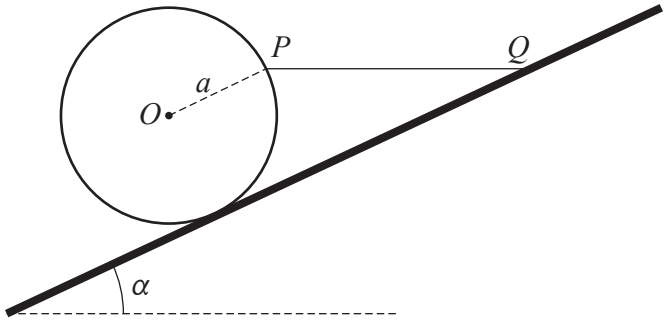
The particle $P$ is pulled down vertically and released from rest at a distance $\frac{8}{3}a$ below $M$.

**(b)** Find, in terms of $a$ and $g$, the speed of $P$ as it passes through $M$ in the subsequent motion. [4]

**3** At time $t = 0$ seconds, a particle $P$ is projected with speed $u\,\text{m s}^{-1}$ at an angle 60° above the horizontal from a point $O$. In the subsequent motion $P$ moves freely under gravity. The direction of motion of $P$ when $t = 5$ is perpendicular to its direction of motion when $t = 15$.

Find the value of $u$. [5]

**4**




A ring of weight $W$, with radius $a$ and centre $O$, is at rest on a rough surface that is inclined to the horizontal at an angle $\alpha$ where $\tan\alpha = \frac{1}{2}$. The plane of the ring is perpendicular to the inclined surface and parallel to a line of greatest slope of the surface. The point $P$ on the circumference of the ring is such that $OP$ is parallel to the surface.

A light inextensible string is attached to $P$ and to the point $Q$, which is on the surface, such that $PQ$ is horizontal (see diagram). The points $O$, $P$ and $Q$ are in the same vertical plane. The system is in limiting equilibrium and the coefficient of friction between the ring and the surface is $\mu$.

**(a)** Find, in terms of $W$, the tension in the string $PQ$. [4]

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**(b)** Find the value of $\mu$. [3]

**5** Two particles $A$ and $B$ of masses $m$ and $km$ respectively are connected by a light inextensible string of length $a$. The particles are placed on a rough horizontal circular turntable with the string taut and lying along a radius of the turntable. Particle $A$ is at a distance $a$ from the centre of the turntable and particle $B$ is at a distance $2a$ from the centre of the turntable. The coefficient of friction between each particle and the turntable is $\frac{1}{5}$.

When the turntable is made to rotate with angular speed $\frac{2}{5}\sqrt{\frac{g}{a}}$, the system is in limiting equilibrium.

**(a)** Find the tension in the string, in terms of $m$ and $g$. [4]

**(b)** Find the value of $k$. [3]

**6** A particle $P$ of mass 2 kg moving on a horizontal straight line has displacement $x$ m from a fixed point $O$ on the line and velocity $v\,\text{m s}^{-1}$ at time $t$ s. The only horizontal force acting on $P$ has magnitude $\frac{1}{10}(2v-1)^2\text{e}^{-t}$ N and acts towards $O$. When $t = 0$, $x = 1$ and $v = 3$.

**(a)** Find an expression for $v$ in terms of $t$. [5]

**(b)** Find an expression for $x$ in terms of $t$. [4]

**7** A smooth sphere with centre $O$ and of radius $a$ is fixed to a horizontal plane. A particle $P$ of mass $m$ is projected horizontally from the highest point of the sphere with speed $u$, so that it begins to move along the surface of the sphere. The particle $P$ loses contact with the sphere at the point $Q$ on the sphere, where $OQ$ makes an angle $\theta$ with the upward vertical through $O$.

**(a)** Show that $\cos\theta = \dfrac{u^2 + 2ag}{3ag}$. [4]

It is given that $\cos\theta = \frac{5}{6}$.

**(b)** Find, in terms of $a$ and $g$, an expression for the vertical component of the velocity of $P$ just before it hits the horizontal plane to which the sphere is fixed. [3]

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**(c)** Find an expression for the time taken by $P$ to fall from $Q$ to the plane. Give your answer in the form $k\sqrt{\frac{a}{g}}$, stating the value of $k$ correct to 3 significant figures. [2]

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## Additional page

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