



Cambridge International AS & A Level

CANDIDATE NAME



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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]

Dotted lines for writing the answer.

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2 The curve C has parametric equations

$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leq \frac{3}{5}.$$

The length of C is denoted by s .

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t} dt$. [4]

Dotted lines for writing the answer.

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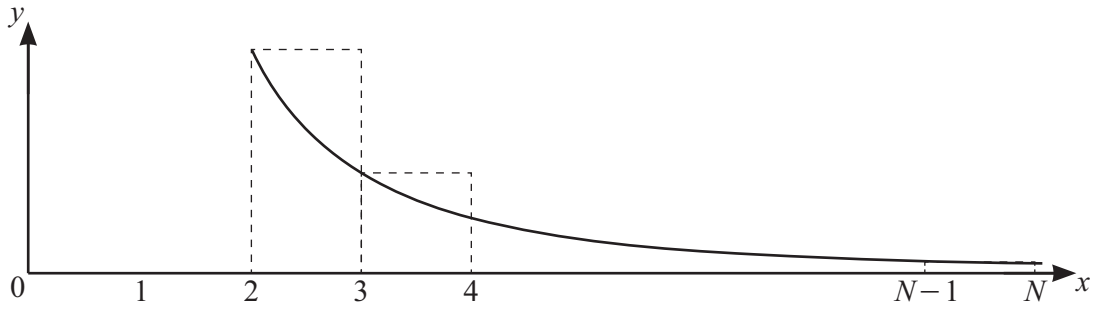
- (b) By finding the Maclaurin's series for $\sqrt{\cosh 2t}$ up to and including the term in t^2 , deduce an approximation to s . [5]

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The diagram shows the curve with equation $y = x^{-2}$ for $2 \leq x \leq N$ together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{1}{r^2} > \frac{3}{2} - \frac{1}{N} + \frac{1}{N^2}. \quad [5]$$

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(b) Use a similar method to find, in terms of N , an upper bound for $\sum_{r=1}^N \frac{1}{r^2}$. [3]

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(c) Deduce lower and upper bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. [2]

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5 (a) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t. \quad [7]$$

Lined area for writing the solution to the differential equation, consisting of multiple horizontal dotted lines.

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(b) Show that, for large positive values of t and for any initial conditions,

$$x \approx R \sin(t - \phi),$$

where the constants R and ϕ are to be determined.

[3]

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6 (a) Show that $\sum_{r=1}^n z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}$, for $z^2 \neq z^{-2}$. [2]

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(b) By letting $z = \cos \theta + i \sin \theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^n \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2 \sin 2\theta}. \quad [5]$$

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Handwriting practice area with horizontal dotted lines.

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7 (a) Show that

dx (x/2 sqrt(x^2-9) - 9/2 cosh^-1(x/3)) = sqrt(x^2-9). [3]

Dotted lines for working out part (a)

(b) Find the solution of the differential equation

x dy/dx - y = x^2 sqrt(x^2-9),

given that y = 1 when x = 3. Give your answer in the form y = f(x). [9]

Dotted lines for working out part (b)

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8 The planes Π_1 and Π_2 do not intersect and are both perpendicular to $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The line l intersects Π_1 at the point $(1, 6, 0)$ and intersects Π_2 at the point $(3, -6, 0)$.

(a) Find Cartesian equations of Π_1 and Π_2 . [3]

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(b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + \lambda\mathbf{b}$, where \mathbf{a} and \mathbf{b} are vectors to be determined, and hence show that for points on l , $\frac{1}{2}x + \frac{1}{12}y = 1$ and $z = 0$. [2]

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The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \frac{1}{2} & \frac{1}{12} & 0 \end{pmatrix}.$$

- (c) Show that the characteristic equation of \mathbf{A} is $-\lambda^3 + 3\lambda^2 + \frac{7}{4}\lambda = 0$ and hence find the eigenvalues of \mathbf{A} . [3]

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