* 0019655537601 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 8 8 2 6 4 0 1 9 1 *

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the exact value of $\int_{2}^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}}\,dx$. [5]

2 The curve C has parametric equations

$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leqslant \tfrac{3}{5}.$$

The length of C is denoted by s.

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t}\,\mathrm{d}t$. [4]

(b) By finding the Maclaurin's series for $\sqrt{\cosh 2t}$ up to and including the term in t^2, deduce an approximation to s. [5]

3 The curve C has equation

$$x^3 + 2xy + 8y^3 = -12.$$

(a) Show that, at the point $(-2, -1)$ on C, $\frac{dy}{dx} = -\frac{1}{2}$. [3]

(b) Find the value of $\dfrac{d^2y}{dx^2}$ at the point $(-2,-1)$. [5]

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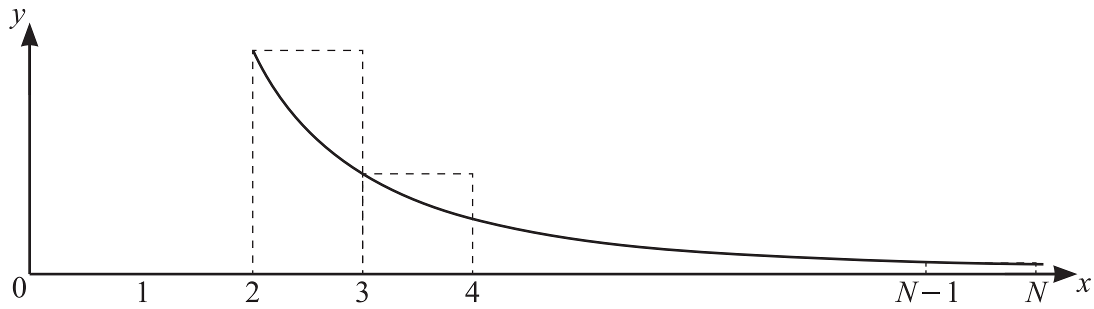

The diagram shows the curve with equation $y = x^{-2}$ for $2 \leqslant x \leqslant N$ together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{N} \frac{1}{r^2} > \frac{3}{2} - \frac{1}{N} + \frac{1}{N^2}.$$ [5]

(b) Use a similar method to find, in terms of N, an upper bound for $\sum_{r=1}^{N} \frac{1}{r^2}$. [3]

(c) Deduce lower and upper bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. [2]

5 **(a)** Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338\sin t.$$ [7]

(b) Show that, for large positive values of t and for any initial conditions,

$$x \approx R\sin(t-\phi),$$

where the constants R and ϕ are to be determined. [3]

6 (a) Show that $\sum_{r=1}^{n} z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}$, for $z^2 \neq z^{-2}$. [2]

(b) By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2\sin 2\theta}.$$ [5]

7 **(a)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{2}\sqrt{x^2-9}-\frac{9}{2}\cosh^{-1}\frac{x}{3}\right)=\sqrt{x^2-9}.$$ [3]

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(b) Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-y=x^2\sqrt{x^2-9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = \mathrm{f}(x)$. [9]

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8 The planes Π_1 and Π_2 do not intersect and are both perpendicular to $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. The line l intersects Π_1 at the point $(1,6,0)$ and intersects Π_2 at the point $(3,-6,0)$.

(a) Find Cartesian equations of Π_1 and Π_2. [3]

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(b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a}+\lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be determined, and hence show that for points on l, $\frac{1}{2}x+\frac{1}{12}y = 1$ and $z = 0$. [2]

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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \frac{1}{2} & \frac{1}{12} & 0 \end{pmatrix}.$$

(c) Show that the characteristic equation of **A** is $-\lambda^3 + 3\lambda^2 + \frac{7}{4}\lambda = 0$ and hence find the eigenvalues of **A**. [3]

(d) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^n = \mathbf{PDP}^{-1}$, where n is a positive integer. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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