Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

6809632015

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 The cubic equation $2x^3 + x^2 - px - 5 = 0$, where p is a positive constant, has roots α, β, γ.

(a) State, in terms of p, the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. [1]

.....

.....

.....

.....

.....

(b) Find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Deduce a cubic equation whose roots are $\alpha\beta, \beta\gamma, \alpha\gamma$. [1]

.....

.....

.....

.....

.....

.....

.....

(d) Given that $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$, find the value of p. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 Prove by mathematical induction that $6^{4n} + 38^n - 2$ is divisible by 74 for all positive integers n. [6]

[Turn over

3 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{N} r(r+1)(3r+4) = \tfrac{1}{12}N(N+1)(N+2)(9N+19).$$ [3]

(b) Express $\frac{3r+4}{r(r+1)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{N} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$$

in terms of N. [4]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$. [1]

4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) The triangle *ABC* in the *x*-*y* plane is transformed by **M** onto triangle *DEF*.

Given that the area of triangle *DEF* is $28\,\text{cm}^2$, find the area of triangle *ABC*. [2]

.....

.....

.....

.....

.....

.....

[Turn over

5 The points A, B, C have position vectors

$$2\mathbf{i}+2\mathbf{j}+4\mathbf{k}, \qquad 2\mathbf{i}+4\mathbf{j}-\mathbf{k}, \qquad -3\mathbf{i}-3\mathbf{j}+4\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

The point D has position vector $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$.

(b) Find the perpendicular distance from D to the plane ABC. [2]

.....

.....

.....

.....

.....

(c) Find the shortest distance between the lines AB and CD. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 The curve C has equation $y = \dfrac{x^2 + ax + 1}{x + 2}$, where $a > \frac{5}{2}$.

(a) Find the equations of the asymptotes of C. [3]

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that C has no stationary points. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Sketch C, stating the coordinates of the point of intersection with the y-axis and labelling the asymptotes. [3]

.....

(d) **(i)** Sketch the curve with equation $y = \left|\dfrac{x^2 + ax + 1}{x + 2}\right|$. [2]

(ii) On your sketch in part **(i)**, draw the line $y = a$. [1]

(iii) It is given that $\left|\dfrac{x^2 + ax + 1}{x + 2}\right| < a$ for $-5 - \sqrt{14} < x < -3$ and $-5 + \sqrt{14} < x < 3$.

Find the value of a. [2]

.....

.....

.....

.....

.....

.....

.....

7 The curve C has polar equation $r^2 = (\pi - \theta)\tan^{-1}(\pi - \theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

.....

(b) Using the substitution $u = \pi - \theta$, or otherwise, find the area of the region enclosed by C and the initial line. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Show that, at the point of C furthest from the initial line,

$$2(\pi-\theta)\tan^{-1}(\pi-\theta)\cot\theta-\frac{\pi-\theta}{1+(\pi-\theta)^2}-\tan^{-1}(\pi-\theta)=0$$

and verify that this equation has a root for θ between 1.2 and 1.3. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....