Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3119764393

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (AP) 331331

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The functions f and g are defined as follows, for all real values of x.

$$\mathrm{f}(x) = 2\sin x + 3\cos x$$

$$\mathrm{g}(x) = \mathrm{e}^{3x} - 1$$

(a) Find fg(0). [2]

(b) Find gg(x). [1]

(c) Solve the equation $\mathrm{g}^{-1}(x) = \frac{1}{3}\ln 5$. [3]

2 Find the values of k for which the curve $y = x^2 + kx + (4k - 15)$ is completely above the x-axis. [4]

3 **(a)** Solve the following simultaneous equations.

$$3\log_2 x + 2\log_2 y = 24$$

$$5\log_2 x - 3\log_2 y = 2$$ [5]

(b) Solve the equation $\dfrac{2^{t+4}}{2^{1-2t}} = 512$. [4]

4 Find the exact value of $\displaystyle\int_3^5 \frac{(x-1)^2}{x^3}\,dx$. [6]

5 The curved surface area of a cylinder with radius r and height h is $2\pi rh$.

A closed cylinder has radius r cm and volume $1000\,\text{cm}^3$.

(a) Show that the total surface area of the cylinder is $2\pi r^2 + \dfrac{2000}{r}\,\text{cm}^2$. [3]

(b) Find the value of r which makes this area a minimum. You should show that your value of r gives a minimum for this area. [5]

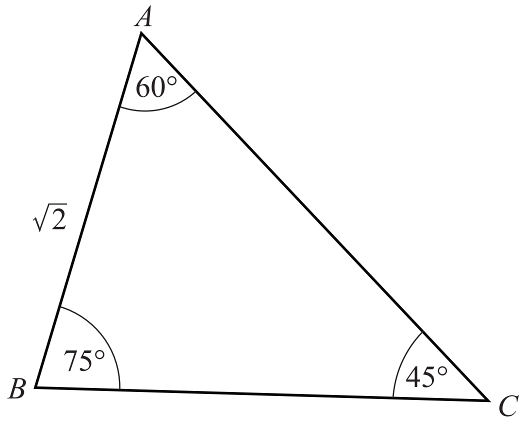
6 A particle travels in a straight line. Its displacement, s metres, from the origin, at time t seconds, where $t > 2$, is given by $s = \ln(4t^2 - 5) - t$.

(a) Find expressions for the velocity, $v\,\text{ms}^{-1}$, and acceleration, $a\,\text{ms}^{-2}$, of the particle. [4]

(b) Find the time when the particle is at rest. [3]

(c) Find the acceleration at this time. [2]

7 DO NOT USE A CALCULATOR IN THIS QUESTION.



You may use the following trigonometrical ratios.

$\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 60° = \frac{1}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 60° = \sqrt{3}$, $\tan 45° = 1$

(a) Given that the area of triangle ABC is $\frac{3+\sqrt{3}}{4}$, show that $\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$. [5]

(b) Hence find the exact length of AC. [2]

8 **(a)** Show that $\dfrac{\sin x}{\tan x - 1} - \dfrac{\cos x}{\tan x + 1} = \dfrac{\cos x}{\sin^2 x - \cos^2 x}$. [5]

(b) Hence solve the equation $\dfrac{\sin x}{\tan x - 1} - \dfrac{\cos x}{\tan x + 1} = 1$ for $0° < x < 360°$. [5]

9 A curve has equation $y = xe^{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the normal to the curve at $x = 1$. [4]

(c) Use your answer to **part (a)** to find the exact value of $\int_0^2 2x\mathrm{e}^{2x}\,\mathrm{d}x$. [5]

10 **(a)** In an arithmetic progression the 5th term is 11. The 7th term is three times the 2nd term. Find the 1st term and the common difference. [4]

(b) A different arithmetic progression (AP) and a geometric progression (GP) have the following properties.

- The 1st terms of the AP and GP are both 3.
- The 2nd term of the AP is the same as the 3rd term of the GP.
- The 6th term of the AP is the same as the 5th term of the GP.
- The common ratio of the GP is greater than 1.

Find the common difference of the AP and the common ratio of the GP. [6]

BLANK PAGE