Cambridge O Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

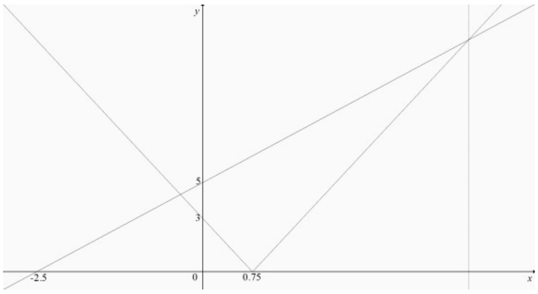
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

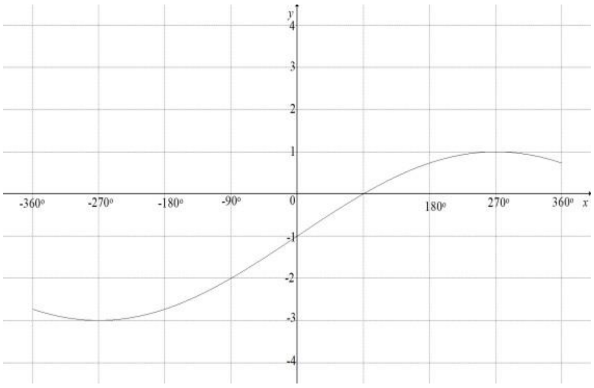
B Mark for a correct result or statement independent of Method marks.

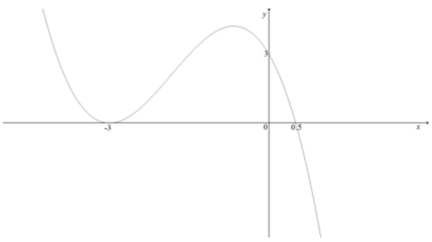
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		**3**	**B1** for a V shaped graph with a vertex on the positive *x*-axis. **B1** for 0.75 and 3 marked correctly and **dependent** on first B1 **B1** for a straight line passing through **–**2.5 and 5 marked correctly, axis with a gradient such that there are two points of intersection. The second point of intersection may be implied.
1(b)	$4x-3<2x+5$ so $x<4$	**B1**	
	$2x+5>-4x+3$ so $x>-\frac{1}{3}$	**B1**	**nfww**
	$-\frac{1}{3}<x<4$	**B1**	Dependent on both B1 **SC2** for the values $-\frac{1}{3}$ and 4 without any or with wrong inequality signs **nfww**
	Alternative		
	$3x^2-11x-4<0$ or $=0$	**(M1)**	For squaring each side of the inequality and forming a 3-term quadratic. Allow multiples.
	$-\frac{1}{3}$, 4	**(A1)**	Critical values
	$-\frac{1}{3}<x<4$	**(A1)**	
2	Mid-point $\left(\frac{3}{2},-\frac{5}{6}\right)$	**B1**	Do not allow if unsimplified
	Gradient $=-\frac{1}{3}$	**B1**	Allow unsimplified
	$k+\frac{5}{6}=3\times\left(2-\frac{3}{2}\right)$ oe	**M1**	For attempt at perpendicular bisector **Must** be with *their* perpendicular gradient and *their* mid-point
	$k=\frac{2}{3}$	**A1**	

Question	Answer	Marks	Guidance
3		4	**B1** for a correct shape, starting in approximately correct places between -2 and -3 and finishing in approximately correct places between 0 and 1, having an amplitude of 2 and crossing the *x*-axis only once, on the positive *x*-axis. **B1** for a correct shape and $(0,\ -1)$ **B1** for a correct shape and a max and a min in approximately correct places. $\left(270^\circ, 1\right)$ and $\left(-270^\circ, -3\right)$ **B1** for a correct shape crosses at $\left(90^\circ, 0\right)$
4(a)	$\mathrm{P}\left(-\frac{1}{2}\right): -\frac{a}{8}+\frac{b}{4}-\frac{3}{2}+2=0$	**M1**	Allow one arithmetic error. **Must** be equated to 0 soi Allow unsimplified
	$\mathrm{P}(-1): -a+b-3+2=-6$	**M1**	Allow unsimplified **Must** be equated to -6 soi
	$-a+2b+4=0$ oe $-a+b+5=0$ oe	**A1**	For both allow unsimplified
	$a=6,\quad b=1$	**2**	**M1 dep** on at least one previous M1 for attempt to solve *their* simultaneous equations to find at least one of *their* unknowns. **A1** for both.
4(b)	$(2x+1)(3x^2-x+2)$	**2**	**M1** for correct attempt to obtain 2 terms of the quadratic for *their* P(*x*). Must divide by $(2x+1)$ **A1** for correct quadratic $(3x^2-x+2)$
	For $3x^2-x+2,$ the dicriminant is <0 so only one real root of $-\frac{1}{2}$ oe	**B1**	Must have a valid attempt to evaluate the discriminant.
5(a)(i)	30 240	**B1**	
5(a)(ii)	720	**B1**	the number of passwords with no symbols. Not part of a product
	29 520	**B1**	
5(a)(iii)	$(6\times5)\times6\times(4\times3)=2160$ oe	**2**	**B1** for (6×5) and (4×3) soi

Question	Answer	Marks	Guidance
5(b)	1 of each and 6 police officers = 20	**B1**	For ${}^4C_1 \times {}^5C_1 \times {}^6C_6$ must be evaluated, could be implied by a correct total
	2 of each and 4 police officers = 900	**B1**	For ${}^4C_2 \times {}^5C_2 \times {}^6C_4$ must be evaluated, could be implied by a correct total
	3 of each and 2 police officers = 600	**B1**	For ${}^4C_3 \times {}^5C_3 \times {}^6C_2$ must be evaluated, could be implied by a correct total
	4 of each and no police officers = 5	**B1**	For ${}^4C_4 \times {}^5C_4$ must be evaluated, could be implied by a correct total
	Total = 1525	**B1**	
6(a)	$\text{q}'(x) = -\frac{1}{3}(2(2x-1)(x+3) + 2(x+3)^2)$oe	**2**	**M1** for attempt to differentiate, allow one arithmetic slip. **A1** – allow unsimplified.
	$\left[\text{q}'(x) = -\frac{2}{3}\right](3x^2 + 11x + 6) = 0$ $x = -3$ and $x = -\frac{2}{3}$	**2**	**Dep M1** for equating *their* $\text{q}'(x)$ to zero and attempt to solve *their* 3-term quadratic to get two solutions for $x = \ldots$ **A1** for both x values correct **nfww**
6(b)		**3**	**B1** for correct cubic shape with maximum point in correct quadrant. **B1** for correct cubic shape touching at $(-3,\ 0)$ and passing through $(0.5, 0)$, intercepts must be marked. **B1** for correct cubic shape passing through $(0,\ 3)$ intercept must be marked.
6(c)	$k < 0$	**B1**	Condone $y < 0$
	$x = -\frac{2}{3},\ y = \frac{343}{81}$ or $y = 4.23$	**M1**	For finding the value of y at *their* max point. If incorrect must see substitution of *their* $x = -\frac{2}{3}$ **nfww**
	$k > \frac{343}{81}$ or $k > 4.23$	**A1**	Condone $y > \frac{343}{81}$ or $y > 4.23$

Question	Answer	Marks	Guidance
7	$6\left(x^{\frac{1}{3}}\right)^2 - x^{\frac{1}{3}} - 2 = 0$ or $6m^2 - m - 2 = 0$ where $m = x^{\frac{1}{3}}$ oe	**B1**	
	$x^{\frac{1}{3}} = \frac{2}{3}$, $x^{\frac{1}{3}} = -\frac{1}{2}$ oe	**M1**	For attempt to solve 3-term quadratic equation in the form $6m^2 \pm m \pm 2 = 0$ and obtain $x^{\frac{1}{3}} = ...$ or $m = ...$ from correct work only
	$x = \frac{8}{27}$, $x = -\frac{1}{8}$	**2**	**Dep M1** for dealing with the power of $\frac{1}{3}$ correctly at least once. **A1** for both
8	$(2x)^2 = 256x^{16}$ soi $n = 8$	**B1**	
	$\binom{8}{1}(2x^2)^7 \times \left(-\frac{1}{4x}\right) = ax^{13}$ oe leading to $a = -256$	**2**	**M1** for attempt at 2nd term with *their n* to find *a*, need to see one step to evaluate. Allow a sign error in simplifying but not missing in $-\frac{1}{4x}$
	$\binom{8}{2}(2x^2)^6\left(-\frac{1}{4x}\right)^2 = bx^c$ leading to $b = 112$	**2**	**M1** for attempt at 3rd term with *their n* to find *b*, need to see one step to evaluate. Allow a sign error but not missing in $\left(-\frac{1}{4x}\right)^2$
	$c = 10$	**B1**	Can be seen by observation.

Question	Answer	Marks	Guidance
9	$\left[\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right]\dfrac{\frac{5}{3}(5x+2)^{-\frac{2}{3}}\times(x-1)^2-2(5x+2)^{\frac{1}{3}}(x-1)}{(x-1)^4}$ or $\left[\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right]=\dfrac{5}{3}(5x+2)^{-\frac{2}{3}}\times(x-1)^{-2}+(5x+2)^{\frac{1}{3}}\times-2\times(x-1)^{-3}$	3	**B1** for $\dfrac{5}{3}(5x+2)^{-\frac{2}{3}}$ **M1** for attempt at differentiation of a quotient or product. **A1** all other terms correct.
	$\dfrac{(5x+2)^{-\frac{2}{3}}}{3(x-1)^3}(5x-5-30x-12)$	M1	**Dep M1** for attempt to simplify by factorising $(5x+2)^{-\frac{2}{3}}$ or $(x-1)$ **nfww to the given form**, allow one arithmetic slip and/or one sign slip.
	$\dfrac{-(25x+17)}{3(x-1)^3(5x+2)^{\frac{2}{3}}}$	A1	Must be in correct form.

Question	Answer	Marks	Guidance
10	$40+20\theta=65$	***M1**	
	$\theta=1.25$	**A1**	
	$\sin\left(\frac{their\ \theta}{2}\right)=\frac{\frac{1}{2}AB}{20}$ $AB=23.4$ or $\frac{1}{2}AB=11.7$	**2**	**Dep M1** for an attempt to find AB or $\frac{1}{2}AB$
	Either $\tan\left(\frac{their\ \theta}{2}\right)=\frac{\text{height of triangle } ACB}{their\ \frac{1}{2}AB}$ Height of triangle = 8.44 Area of triangle = 98.8	**3**	**DepM1** for a correct attempt to find the height of the triangle **M1** for attempt to find the area of the triangle using *their* height and *their AB* **A1** must be at least 3 significant figures.
	Or $\cos\left(\frac{their\ \theta}{2}\right)=\frac{their\ \frac{1}{2}AB}{AC}$ $AC=14.4$ Area of triangle $=\frac{1}{2}\times their\ AB\times their\ AC\times\sin\left(\frac{\theta}{2}\right)$ Area of triangle = 98.8	**(3)**	**DepM1** for a correct attempt to find *CA* **M1** for attempt to find the area of the triangle using the sine rule with *their CA*. **A1** must be at least 3 significant figures.
	Area of the segment $=\frac{1}{2}\times 20^2\times(1.25-\sin 1.25)$ Area of the segment = 60.2	**B1**	
	Area = 38.6	**A1**	

Question	Answer	Marks	Guidance
10	**Alternative 1**		
	$40+20\theta=65$	**(*M1)**	
	$\theta=1.25$	**(A1)**	
	$\tan\left(\frac{their\ \theta}{2}\right)=\frac{AC}{20}$ oe soi AC =14.43	**(2)**	**DepM1** for a correct attempt to find the *AC*
	Area of triangle *ACO* = $\frac{1}{2}\times 20\times 14.43=144.3$	**(2)**	**M1** for a correct attempt to find the area of the triangle using *their AC*
	Area of the sector = 250	**(B1)**	
	Area of half shaded region = (144.3 $-$ 125) $\times 2$	**(M1)**	Dependent on a valid method for finding triangle *ACO* . Allow use of 144
	Area = 38.6	**(A1)**	
	Alternative 2		
	$40+20\theta=65$	**(*M1)**	
	$\theta=1.25$	**(A1)**	
	$\sin\left(\frac{their\ \theta}{2}\right)=\frac{\frac{1}{2}AB}{20}$ $AB=23.4$	**(2)**	**Dep M1** for an attempt to find *AB* or $\frac{1}{2}AB$
	$\tan\left(\frac{their\ \theta}{2}\right)=\frac{AC}{20}$ oe soi AC =14.43	**(2)**	**DepM1** for a correct attempt to find *AC*
	Shaded area $=14.4\times 20-\frac{1}{2}\times 20^2\times\frac{5}{4}$ Area = 38.6	**(3)**	**M1** for area of Kite **B1** for area of sector

Question	Answer	Marks	Guidance
10	**Alternative 3**		
	$40+20\theta=65$	**(*M1)**	
	$\theta=1.25$	**(A1)**	
	$\tan\left(\frac{their\ \theta}{2}\right)=\frac{AC}{20}$ oe soi AC =14.43	**(2)**	**DepM1** for a correct attempt to find *AC*
	Area of triangle *AOB* $=\frac{1}{2}\times 20^2\times\sin\ (their\ \theta)$ $=189.[7969...]$	**(M1)**	
	Area of triangle *ACB* $=\frac{1}{2}\times their\ AC\times their\ AB\times\sin\ \frac{\theta}{2}$ $=98.8$	**(M1)**	for a correct attempt to find the area of the triangle using *their AC* and *their AB*
	Area of the sector = 250	**(B1)**	
	Area of half shaded region = area of triangle *ACB* + area of triangle *AOB* − area of sector = 189.8 + 98.8 – 250	**(M1)**	
	Area = 38.6	**(A1)**	

Question	Answer	Marks	Guidance
10	**Alternative 4**		
	$40+20\theta=65$	**(*M1)**	
	$\theta=1.25$	**(A1)**	
	$\sin\left(\frac{their\ \theta}{2}\right)=\frac{\frac{1}{2}AB}{20}$ $AB=23.4$ or $\frac{1}{2}AB=11.7$	**(2)**	**Dep M1** for an attempt to find AB or $\frac{1}{2}AB$
	$\tan\left(\frac{their\ \theta}{2}\right)=\frac{\text{height of triangle } ACB}{their\frac{1}{2}AB}$ Height of triangle = 8.44	**(M1)**	**DepM1** for a correct attempt to find the height of the triangle
	Height of triangle *ABO* $=\sqrt{20^2-\left(\frac{1}{2}AB\right)^2}$ $=16.22$	**(M1)**	for a correct attempt to find to find the height of the triangle
	Area of the sector = 250	**(B1)**	
	Area of kite $=\frac{1}{2}\times23.4\times(16.22+8.44)$ $=288.5$	**(M1)**	
	Area = 288.5 $-$250 =38.5	**(A1)**	
11(a)	$\overrightarrow{XY}=-\overrightarrow{OX}+\mathbf{a}+\frac{1}{3}\overrightarrow{AB}$ oe soi or $\overrightarrow{XY}=\overrightarrow{XB}-\frac{2}{3}\overrightarrow{AB}$ oe soi	**M1**	
	$\overrightarrow{XY}=-\frac{4}{5}\mathbf{b}+\mathbf{a}+\frac{1}{3}(\mathbf{b}-\mathbf{a})$ oe soi or $\overrightarrow{XY}=\frac{1}{5}\mathbf{b}+\frac{2}{3}(\mathbf{a}-\mathbf{b})$ oe soi	**M1**	For $\pm\frac{1}{3}(\mathbf{b}-\mathbf{a})$ or $\pm\frac{4}{5}\mathbf{b}$ For $\pm\frac{1}{5}\mathbf{b}$ or $\pm\frac{2}{3}(\mathbf{a}-\mathbf{b})$
	$\overrightarrow{XY}=\frac{2}{3}\mathbf{a}-\frac{7}{15}\mathbf{b}$ cao	**A1**	**AG**
11(b)	$\overrightarrow{YZ}=\lambda\left(\frac{2}{3}\mathbf{a}-\frac{7}{15}\mathbf{b}\right)$ cao	**B1**	

Question	Answer	Marks	Guidance
11(c)	$\overrightarrow{YZ} = \mu\mathbf{a} - \frac{1}{3}\overrightarrow{AB}$ oe soi	**M1**	
	$\overrightarrow{YZ} = \mu\mathbf{a} - \frac{1}{3}(\mathbf{b} - \mathbf{a})$ oe	**A1**	Allow unsimplified ISW from correct answer
11(d)	$\mu\mathbf{a} - \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \lambda\left(\frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}\right)$ soi	**M1**	For equating *their* (b) and *their* (c) **and** attempt to equate coefficients of **a** or **b** at least once.
	$\lambda = \frac{5}{7}$	**A1**	**nfww**
	$\mu = \frac{1}{7}$	**A1**	**nfww**
12	$\sin\left(\frac{2x}{3} - \frac{\pi}{3}\right) = \pm\frac{\sqrt{3}}{2}$ or $\tan\left(\frac{2x}{3} - \frac{\pi}{3}\right) = \pm\sqrt{3}$	**B1**	Allow if $\pm$ is missing
	$x = \pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$	**4**	**M1dep** on **B1** for obtaining $\frac{2x}{3} - \frac{\pi}{3} = \frac{\pi}{3}$ or any valid value **A1** for one correct solution **A1** for a 2nd correct solution **A1** for a 3rd and 4th correct solutions and no extras in the range