Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*1394054656*

**FURTHER MATHEMATICS** **9231/32**

Paper 3 Further Mechanics **October/November 2023**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

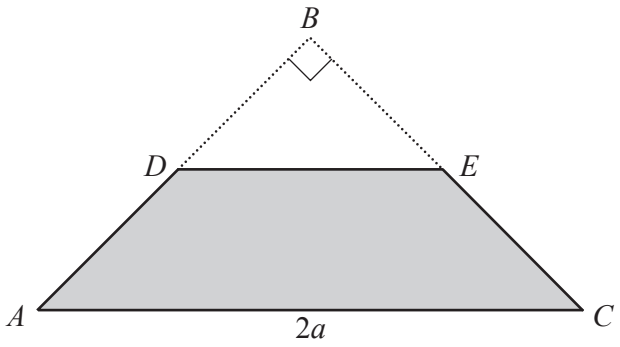
**1** One end of a light inextensible string of length $a$ is attached to a fixed point $O$. The other end of the string is attached to a particle of mass $m$. The string is taut and makes an angle $\theta$ with the downward vertical through $O$, where $\cos\theta = \frac{2}{3}$. The particle moves in a horizontal circle with speed $v$.

Find $v$ in terms of $a$ and $g$. [4]

**2** A particle $P$ of mass 0.5 kg moves in a straight line. At time $t$ s the velocity of $P$ is $v\,\text{m}\,\text{s}^{-1}$ and its displacement from a fixed point $O$ on the line is $x$ m. The only forces acting on $P$ are a force of magnitude $\frac{150}{(x+1)^2}$ N in the direction of increasing displacement and a resistive force of magnitude $\frac{450}{(x+1)^3}$ N. When $t = 0$, $x = 0$ and $v = 20$.

Find $v$ in terms of $x$, giving your answer in the form $v = \frac{Ax+B}{(x+1)}$, where $A$ and $B$ are constants to be determined. [6]

**3**




A uniform lamina is in the form of an isosceles triangle $ABC$ in which $AC = 2a$ and angle $ABC = 90°$. The point $D$ on $AB$ is such that the ratio $DB : AB = 1 : k$. The point $E$ on $CB$ is such that $DE$ is parallel to $AC$. The triangle $DBE$ is removed from the lamina (see diagram).

**(a)** Find, in terms of $k$, the distance of the centre of mass of the remaining lamina $ADEC$ from the midpoint of $AC$. [4]

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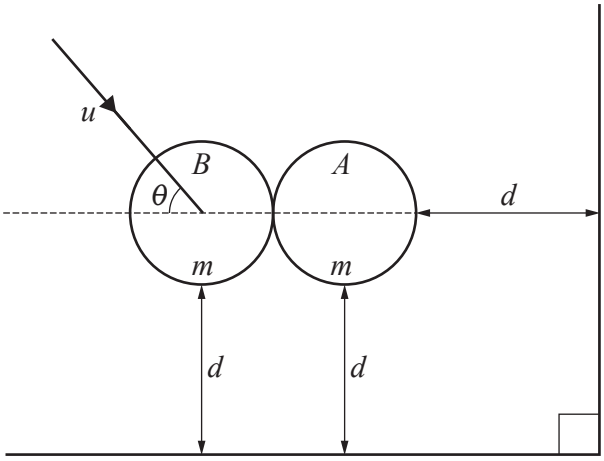
When the lamina $ADEC$ is freely suspended from the vertex $A$, the edge $AC$ makes an angle $\theta$ with the downward vertical, where $\tan\theta = \frac{5}{18}$.

**(b)** Find the value of $k$. [3]

**4**

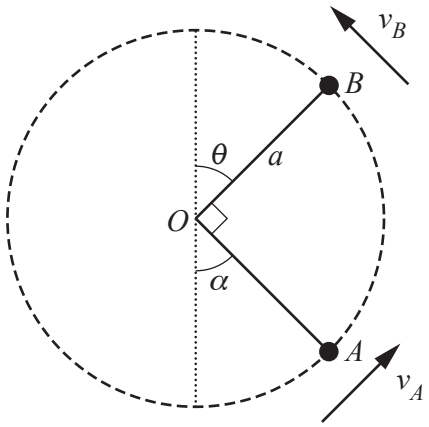



Two smooth vertical walls meet at right angles. The smooth sphere $A$, with mass $m$, is at rest on a smooth horizontal surface and is at a distance $d$ from each wall. An identical smooth sphere $B$ is moving on the horizontal surface with speed $u$ at an angle $\theta$ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.

**(a)** Find the value of $\tan\theta$. [4]

**(b)** Find the percentage loss in the total kinetic energy of the spheres as a result of this collision. [3]

**5**




A bead of mass $m$ moves on a smooth circular wire, with centre $O$ and radius $a$, in a vertical plane. The bead has speed $v_A$ when it is at the point $A$ where $OA$ makes an angle $\alpha$ with the downward vertical through $O$, and $\cos\alpha = \frac{3}{5}$. Subsequently the bead has speed $v_B$ at the point $B$, where $OB$ makes an angle $\theta$ with the upward vertical through $O$. Angle $AOB$ is a right angle (see diagram). The reaction of the wire on the bead at $B$ is in the direction $OB$ and has magnitude equal to $\frac{1}{6}$ of the magnitude of the reaction when the bead is at $A$.

**(a)** Find, in terms of $m$ and $g$, the magnitude of the reaction at $B$. [6]

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**(b)** Given that $v_A = \sqrt{kag}$, find the value of $k$. [2]

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**6** A particle $P$ is projected with speed $u$ at an angle $\alpha$ above the horizontal from a point $O$ on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of $P$ from $O$ at a subsequent time $t$ are denoted by $x$ and $y$ respectively.

**(a)** Derive the equation of the trajectory of $P$ in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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During its flight, $P$ must clear an obstacle of height $h$ m that is at a horizontal distance of 32 m from the point of projection. When $u = 40\sqrt{2}\,\text{m s}^{-1}$, $P$ just clears the obstacle. When $u = 40\,\text{m s}^{-1}$, $P$ only achieves 80% of the height required to clear the obstacle.

**(b)** Find the two possible values of $h$. [6]

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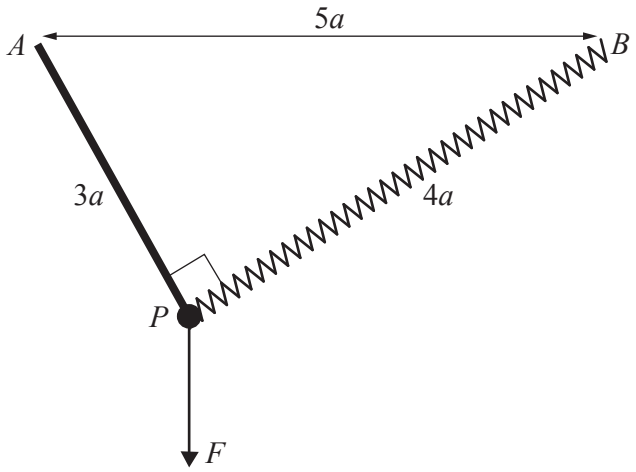


A particle $P$ of mass $m$ is attached to one end of a light rod of length $3a$. The other end of the rod is able to pivot smoothly about the fixed point $A$. The particle is also attached to one end of a light spring of natural length $a$ and modulus of elasticity $kmg$. The other end of the spring is attached to a fixed point $B$. The points $A$ and $B$ are in a horizontal line, a distance $5a$ apart, and these two points and the rod are in a vertical plane.

Initially, $P$ is held in equilibrium by a vertical force $F$ with the stretched length of the spring equal to $4a$ (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.

**(a)** Find the value of $k$. [5]

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**(b)** Find $F$ in terms of $m$ and $g$. [2]

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**(c)** Find, in terms of $m$ and $g$, the tension in the rod immediately before it is released. [2]

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## Additional page

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