



Cambridge International AS & A Level

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MATHEMATICS

9709/23

Paper 2 Pure Mathematics 2

May/June 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

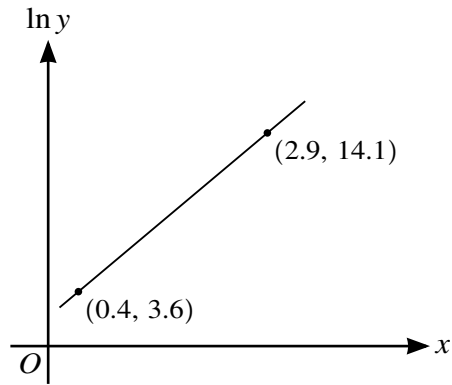
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

2



The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points $(0.4, 3.6)$ and $(2.9, 14.1)$, as shown in the diagram.

Find the values of A and B correct to 3 significant figures.

[5]

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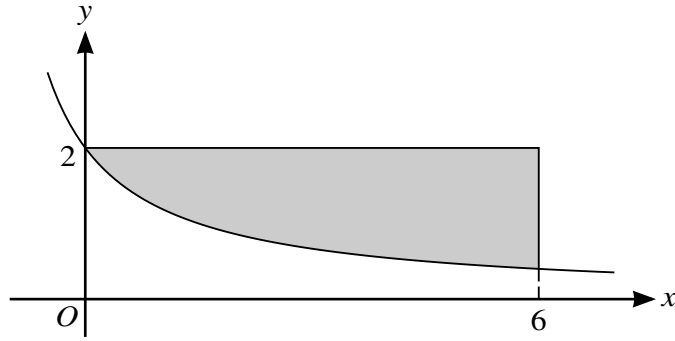
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The diagram shows part of the curve $y = \frac{6}{2x + 3}$. The shaded region is bounded by the curve and the lines $x = 6$ and $y = 2$.

Find the exact area of the shaded region, giving your answer in the form $a - \ln b$, where a and b are integers. [5]

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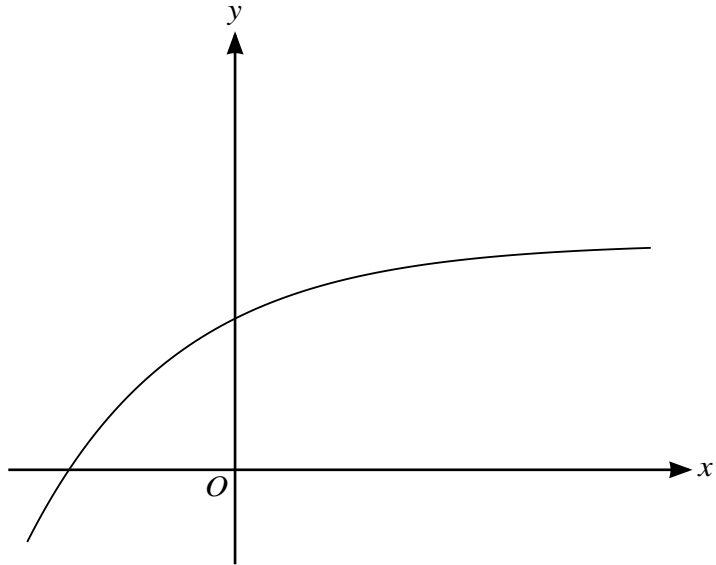
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4 (a)



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β , where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

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(c) Use the iterative formula $x_{n+1} = \frac{1}{5}(7 - e^{-\frac{1}{2}x_n})$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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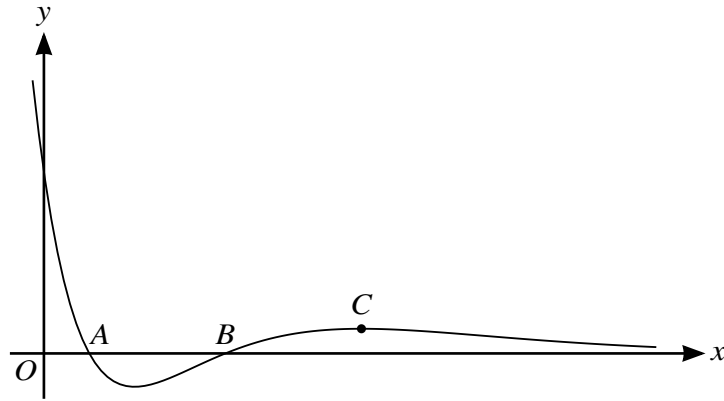
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The diagram shows the curve with equation $y = e^{-\frac{1}{2}x}(x^2 - 5x + 4)$. The curve crosses the x -axis at the points A and B , and has a maximum at the point C .

(a) Find the exact gradient of the curve at B . [5]

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- 6 (a) Show that $4 \sin\left(\theta + \frac{1}{3}\pi\right) \cos\left(\theta - \frac{1}{3}\pi\right) \equiv \sqrt{3} + 2 \sin 2\theta$. [4]

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- (b) Find the exact value of $4 \sin \frac{17}{24}\pi \cos \frac{1}{24}\pi$. [2]

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7 A curve has parametric equations

$$x = \frac{2t + 3}{t + 2}, \quad y = t^2 + at + 1,$$

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1 = 0. \quad [4]$$

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(b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a .

[2]

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(c) Hence show that P is the only point on the curve at which the gradient is 1.

[3]

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Additional Page

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Lined area consisting of multiple rows of dotted lines for writing answers.

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