Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

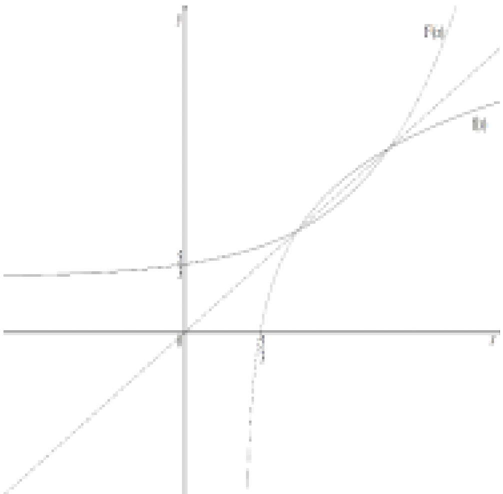
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1	$a = 4$	**B1**	
	$b = \frac{3}{8}$ oe	**B1**	
	$c = -2$	**B1**	
2	$(x =)\ \frac{4 \pm \sqrt{16 + 12(2+\sqrt{5})(2-\sqrt{5})}}{2(2+\sqrt{5})}$ oe with simplification to $\frac{4 \pm \sqrt{16-k}}{2(2+\sqrt{5})}$	**M1**	For attempt to equate to zero and use quadratic formula, must see substitution and $\frac{4 \pm \sqrt{16-k}}{2(2+\sqrt{5})}$
	$\frac{4 \pm 2}{2(2+\sqrt{5})}$ or exact equivalent	**2**	**A1** for one exact solution
	$\frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ or $\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**M1**	For evidence of rationalisation and evaluation
	$-6+3\sqrt{5}$ and $-2+\sqrt{5}$	**A1**	
	Alternative $\left((2+\sqrt{5})x-3\right)\left(x+(2-\sqrt{5})\right)$	**(B2)**	
	$x = -2+\sqrt{5}$	**(B1)**	**Dep** on previous **B2**
	$\frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ leading to $x = -6+3\sqrt{5}$	**(2)**	**M1** for attempt at rationalisation and evaluation
3(a)	$\pm 3(x+2)(x-1)(x-4)$	**3**	**B1** for 3 soi **B1** for $\pm$ **B1** for $(x+2)(x-1)(x-4)$

Question	Answer	Marks	Guidance
3(b)	$5x-2*4x+1$ leading to critical value 3	**B1**	* can be $\leqslant$, =, $\geqslant$
	$5x-2*-4x-1$ oe	**M1**	* can be $\leqslant$, =, $\geqslant$
	leading to critical value $\frac{1}{9}$	**A1**	
	$\frac{1}{9}\leqslant x\leqslant 3$	**A1**	Mark final answer
	Alternative $9x^2-28x+3*0$	**(M1)**	Squaring both sides of the inequality and collecting terms, allow one sign error. * can be $\leqslant$, =, $\geqslant$
	$(9x-1)(x-3)*0$	**(M1)**	**Dep** for attempt to find two critical values * can be $\leqslant$, =, $\geqslant$
	Critical values $\frac{1}{9}$ and 3	**(A1)**	
	$\frac{1}{9}\leqslant x\leqslant 3$	**(A1)**	Mark final answer
4(a)	$r\theta=12$ soi	**B1**	
	$\frac{1}{2}r^2\theta=57.6$ soi	**B1**	
	$r=9.6$ oe nfww	**B1**	
	$\theta=1.25$ oe nfww	**B1**	
4(b)	$AC=28.89$	**B1**	
	Shaded area = $\left(\frac{1}{2}\times 28.89\times 9.6\right)-57.6$ soi	**M1**	Using *their AC*
	81.1	**A1**	
	Alternative $OC=30.45$	**(B1)**	
	Shaded area = $\left(\frac{1}{2}\times 30.45\times 9.6\times\sin 1.25\right)-57.6$ soi	**(M1)**	Using *their OC*
	81.1	**(A1)**	

Question	Answer	Marks	Guidance
5(a)	$6p^{\frac{2}{3}} - 13p^{\frac{1}{3}} - 5(=0)$ soi	**B1**	May introduce *their* own variable e.g. x
	$\left(\left(2p^{\frac{1}{3}} - 5\right)\left(3p^{\frac{1}{3}} + 1\right) = 0 \text{ oe}\right)$ $p^{\frac{1}{3}} = \frac{5}{2}$ $p^{\frac{1}{3}} = -\frac{1}{3}$	**M1**	For attempt to solve quadratic equation to obtain $p^{\frac{1}{3}} = ..$ or e.g. $x = \ldots$
	$\frac{125}{8}$ or 15.625 or $15\frac{5}{8}$	**A1**	Must be simplified and exact
	$-\frac{1}{27}$	**A1**	Must be simplified and exact
5(b)	$\lg(2x+5)^2$	**B1**	
	$\lg\frac{(2x+5)^2}{x+2}$	**B1**	**Dep** on first **B1**, must be a correct statement
	$1 = \lg 10$ soi	**B1**	
	$\frac{(2x+5)^2}{(x+2)} = k$ oe	**M1**	Dep on second **B** mark For correct attempt to obtain a quadratic equation with no log terms, where k = 1 or 10
	$4x^2 + 10x + 5 = 0$ $x = \frac{-5 \pm \sqrt{5}}{4}$ or exact equivalent	**2**	**M1** for attempt to solve *their* quadratic to obtain $x = \ldots$, implied by decimals of -1.8 or -0.69 or better **A1** for both, **A0** if one is discarded
6(a)	A correct equation in terms of x and y only	**B1**	No inverse trig functions
	$y = (x-4)^2 - 3$ or $y = x^2 - 8x + 13$	**B1**	
6(b)	$\sin\left(2\phi + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2}$ soi	**B1**	May be implied by one correct solution
	$-\frac{5\pi}{24}, -\frac{\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ with no extra solutions within the range	**4**	**M1** for explicitly correct order of operations from *their* $\left(2\phi + \frac{3\pi}{4}\right) = k$, or may be implied by one correct solution **A1** for two correct solutions **A1** for a third correct solution **A1** for a further solution with no extra solutions in the range

Question	Answer	Marks	Guidance
7(a)	${}^{14}C_2 \times {}^{12}C_3 \times {}^{9}C_4$ oe, soi 2 522 520	3	**B1** for a product of 3 combinations (ignore combinations that are equal to 1), one of which must be in the form ${}^{14}C_k$ where $k = 2, 3, 4, 5, 9, 10, 11, 12$ **B2** for a correct product of combinations
7(b)(i)	136 080	**B1**	
7(b)(ii)	15 120	**B1**	
7(b)(iii)	38 640	3	**B1** for ${}^{8}P_4$ or 1680 or $(8\times7\times6\times5)$ **B2** for $8\times{}^{8}P_4$ (13 440) oe or $15\times{}^{8}P_4$ (25 200) oe
8(a)	$(a =)\ \frac{4}{3}$ or $1.\dot{3}$	**B1**	Allow a recurring decimal Must not be an inequality in terms of *a* Allow $x > \frac{4}{3}$
8(b)	$f \in \mathbb{R}$ or $-\infty < f < \infty$ or $\mathbb{R}$	**B1**	Allow *y* or $f(x)$ but not *x*.
8(c)		4	**B1** for a correct shape for $y = f(x)$ in quadrants 1 and 4 **B1** for $\left(\frac{5}{3}, 0\right)$, must have a correct shape in either quadrant 1 or quadrant 4 **B1** for $y = f^{-1}(x)$, must be a correct shape in quadrants 1 and 2 and intersect twice. **B1** for $\left(0, \frac{5}{3}\right)$, must have a reasonable shape for $y = f^{-1}(x)$ in either the first quadrant or the second quadrant
8(d)(i)	$g(g(x)) = 4x - 9$	**B1**	Must be simplified
8(d)(ii)	$fg(g(x)) = 2\ln(12x - 31)$	**M1**	allow unsimplified, using *their* answer to **(i)**
	$2\ln(12x - 31) = 4$ $x = \frac{e^2 + 31}{12}$	2	**Dep M1** for correct order of operations to solve *their* equation, to get as far as *x* = … Implied by decimal answer of awrt 3.2 **A1** Must be exact form.

Question	Answer	Marks	Guidance
9	$\frac{(2x+6)}{3} = 3 + \frac{4}{2x+1}$ $4x^2 - 4x - 15 (= 0)$	2	**M1** for equating the line and curve and obtaining a 3 term quadratic expression in terms of x.
	$x = \frac{5}{2}$	A1	For x-coordinate of the point of intersection.
	Either $\int \left(3 + \frac{4}{2x+1}\right) \mathrm{d}x = 3x + 2\ln(2x+1)$	2	**M1** for attempt to integrate with one term correct
	$\left[3x + 2\ln(2x+1)\right]_0^{\frac{5}{2}} = \frac{15}{2} + 2\ln 6$	2	**Dep M1** for using *their* x-coordinate of C in *their* integral. Must have a term in $\ln(2x+1)$ Allow for awrt 11.1. **A1** Must be exact but allow unsimplified
	Area of trapezium $\frac{1}{2}\left(2 + \frac{11}{3}\right) \times \frac{5}{2}$ or $\left[\frac{x^2}{3} + 2x\right]_0^{their\ \frac{5}{2}}$ or $\left[\frac{(2x+6)^2}{12}\right]_0^{their\ \frac{5}{2}} = \frac{85}{12}$	2	**M1** for attempt at the trapezium, must have at least one side correct. If using integration, the integral must be correct using *their* $\frac{5}{2}$
	Shaded area = $2\ln 6 + \frac{5}{12}$ or $\ln 36 + \frac{5}{12}$ or $\ln 6^2 + \frac{5}{12}$	A1	
	Or $\int \left\lvert 1 + \frac{4}{2x+1} - \frac{2}{3}x \right\rvert \mathrm{d}x$ $x + 2\ln(2x+1) - \frac{x^2}{3}$ or $-x - 2\ln(2x+1) + \frac{x^2}{3}$	(5)	**M2** for attempt to subtract and integrate with at least one term correct, allow x terms considered separately. If subtraction is reversed allow accuracy marks. Separate x terms should be considered as one term for A marks. **A1** for one term only correct **A2** for two terms only correct
	$\left[x + 2\ln(2x+1) - \frac{x^2}{3}\right]_0^{\frac{5}{2}} = \frac{5}{2} + 2\ln 6 - \frac{25}{12}$ or $\left[\frac{x^2}{3} - x - 2\ln(2x+1)\right]_0^{\frac{5}{2}} = \frac{25}{12} - \frac{5}{2} - 2\ln 6$	(M1)	**Dep M1** for using *their* x-coordinate of the point of intersection in *their* integral, must have a term in $\ln(2x+1)$ Allow for awrt ± 4 as appropriate

Question	Answer	Marks	Guidance
9	Shaded area = $2\ln 6+\frac{5}{12}$ or $\ln 36+\frac{5}{12}$ or $\ln 6^2+\frac{5}{12}$	**(A1)**	
	Or $\int\left\|3+\frac{4}{2x+1}-\frac{11}{3}\right\|\,\mathrm{d}x$ $2\ln(2x+1)-\frac{2x}{3}$	**(3)**	**M1** for attempt to subtract and integrate with at least one term correct, allow x terms considered separately. Separate x terms should be considered as one term for A marks. **A1** for one term only correct
	$\left[2\ln(2x+1)-\frac{2x}{3}\right]_0^{\frac{5}{2}}=2\ln 6-\frac{5}{3}$	**(M1)**	**Dep M1** for using *their* x-coordinate of the point of intersection in *their* integral, must have a term in $\ln(2x+1)$
	Area of triangle $=\frac{1}{2}\times\left(\frac{11}{3}-2\right)\times\frac{5}{2}$ $=\frac{25}{12}$	**(2)**	**M1** for attempt at the area of the triangle
	Shaded area = $2\ln 6+\frac{5}{12}$ or $\ln 36+\frac{5}{12}$ or $\ln 6^2+\frac{5}{12}$	**(A1)**	
	Alternative $3y^2-14y+11(=0)$	**(2)**	**M1** for a correct attempt to equate the line and curve and obtain a 3 term quadratic expression in terms of y.
	$y=\frac{11}{3}$	**(A1)**	
	$\int\left(\frac{2}{y-3}-\frac{1}{2}\right)\mathrm{d}y=2\ln(y-3)-\frac{1}{2}y$	**(2)**	**M1** for attempt to integrate with one term correct
	$\left[2\ln(y-3)-\frac{1}{2}y\right]_{\frac{11}{3}}^{7}=2\ln 6-\frac{5}{3}$	**(2)**	**Dep M1** for using *their* y-coordinate of C in *their* integral. Allow for awrt 1.92 **A1** Must be exact
	Area of triangle $\frac{1}{2}\left(\frac{11}{3}-2\right)\times\frac{5}{2}$ or $\left[\frac{3y^2}{4}-3y\right]_2^{\text{their }\frac{11}{3}}$ $=\frac{25}{12}$	**(2)**	**M1** for attempt at the triangle, must have at least one side correct. If using integration, the integral must be correct using *their* $\frac{11}{3}$
	Shaded area = $2\ln 6+\frac{5}{12}$ or $\ln 36+\frac{5}{12}$ or $\ln 6^2+\frac{5}{12}$	**(A1)**	

Question	Answer	Marks	Guidance
10(a)(i)	$\frac{n}{2}(2x+1)(3n-1)$	2	**B1** for $\frac{n}{2}(2(2x+1)+3(n-1)(2x+1))$ or $\frac{n}{2}(2(2x+1)+(6x+3)(n-1))$ oe
10(a)(ii)	$\frac{n}{2}(2x+1)(3n-1)=(54n+37)(2x+1)$ $3n^2-109n-74=0$	M1	For equating *their* answer to **(a)** to $(54n+37)(2x+1)$ and attempt to solve a 3-term quadratic equation in n to obtain $n = ...$
	37 only	A1	
10(a)(iii)	$1017.5=(54(\textit{their } n)+37)(2x+1)$	M1	For attempt to solve to obtain a value for x. n must be a positive integer
	$x=-\frac{1}{4}$	A1	
10(b)	$(2y+1)(3(2y+1))^{n-1}=$ $4(2y+1)(3(2y+1))^{n+1}$ Or $(3(2y+1))^{n-1}=4(3(2y+1))^{n+1}$ Or $(2y+1)r^{n-1}=4(2y+1)r^{n+1}$ Or $ar^{n-1}=4ar^{n+1}$	B1	Award when a correct statement is first seen
	Either $(2y+1)^2=\frac{1}{36}$ oe or $(6y+3)^2=\frac{1}{4}$ oe or $r^2=\frac{1}{4}$ oe	M1	**Either M1** for an equation of the form $(2y+1)^2=k$ or $(6y+3)^2=m$ where k and m are numerical and not zero (may be expanded) with no terms in n **Or M1** for $r^2=p$, where p is numerical and not zero
	$2y+1=\pm\frac{1}{6}$ or $6y+3=\pm\frac{1}{2}$	A1	
	$-\frac{5}{12}$, $-\frac{7}{12}$ and no others	A1	For both
10(c)	$-1<2\sin^2\theta<1$ oe soi or $0<2\sin^2\theta<1$ soi	B1	Allow $2\sin^2\theta<1$ May be implied by $\theta<\frac{\pi}{4}$
	$\theta<\frac{\pi}{4}$ or $\theta<0.785$	B1	
	$0<\theta<\frac{\pi}{4}$ Or $0<\theta<0.785$ or better	B1	