Cambridge Assessment
International Education

Example Candidate Responses – Paper 1

Cambridge IGCSE™
Additional Mathematics 0606

Cambridge O Level
Additional Mathematics 4037

For examination from 2020

Cambridge Pathway

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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge IGCSE / O Level Additional Mathematics 0606 / 4037, and to show how different levels of candidates' performance (high, middle and low) relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen from the November 2020 exam series to exemplify a range of answers.

For each question, the response is annotated with a clear explanation of where and why marks were awarded or omitted. This is followed by examiner comments on how the answer could have been improved. In this way, it is possible for you to understand what candidates have done to gain their marks and what they could do to improve their answers. There is also a list of common mistakes candidates made in their answers for each question.

This document provides illustrative examples of candidate work with examiner commentary. These help teachers to assess the standard required to achieve marks beyond the guidance of the mark scheme. Therefore, in some circumstances, such as where exact answers are required, there will not be much comment.

The questions and mark schemes used here are available to download from the School Support Hub. These files are:

0606 November 2020 Question Paper 12

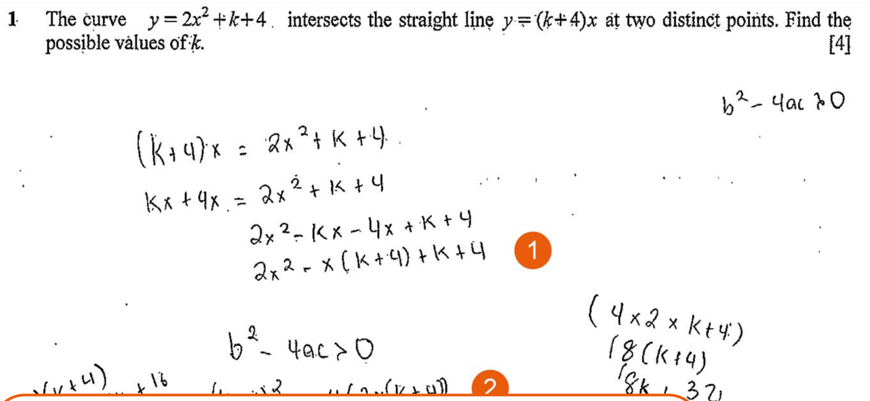
0606 November 2020 Mark Scheme 12

Past exam resources and other teaching and learning resources are available on the School Support Hub:

www.cambridgeinternational.org/support

How to use this booklet

This booklet goes through the paper one question at a time, showing you the high-, middle- and low-level response for each question. The candidate answers are set in a table. In the left-hand column are the candidate answers, and in the right-hand column are the Examiner comments.

Example Candidate Response – high	Examiner comments
 1 The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k+4)x$ at two distinct points. Find the possible values of k. [4] $b^2 - 4ac > 0$ $(k+4)x = 2x^2 + k + 4$ $kx + 4x = 2x^2 + k + 4$ $2x^2 - kx - 4x + k + 4$ $2x^2 - x(k+4) + k + 4$ ① $b^2 - 4ac > 0$ $(4 \times 2 \times k + 4)$ $(8(k+4)$ $8k + 32$	① The candidate equates the equation of the curve and the equation of the straight line and re-arranges them to obtain the required form of a quadratic with the terms

Answers are by real candidates in exam conditions. These show you the types of answers for each level. Discuss and analyse the answers with your learners in the classroom to improve their skills.

Examiner comments are alongside the answers. These explain where and why marks were awarded. This helps you to interpret the standard of Cambridge exams so you can help your learners to refine their exam technique.

How the candidate could have improved their answer

Having obtained the critical values, the candidate should have checked that any value between the critical values satisfied the condition that the discriminant was greater than zero. A substitution of zero, for example, would have alerted the candidate that the range they had given was incorrect.

This section explains how the candidate could have improved each answer. This helps you to interpret the standard of Cambridge exams and helps your learners to refine their exam technique.

Common mistakes candidates made in this question

- Sign errors were common and usually occurred when like terms were being collected. Once critical values were obtained, errors involving the range that these critical values produce, were also common. A quick check using values other than the critical values could have helped candidates to identify errors.
- Candidates sometimes wrote disjoint inequalities in an incorrect continuous form as shown in the low response. The expectation is that they should be written as two separate inequalities.

Often candidates were not awarded marks because they misread or misinterpreted the questions.

Lists the common mistakes candidates made in answering each question. This will help your learners to avoid these mistakes and give them the best chance of achieving the available marks.

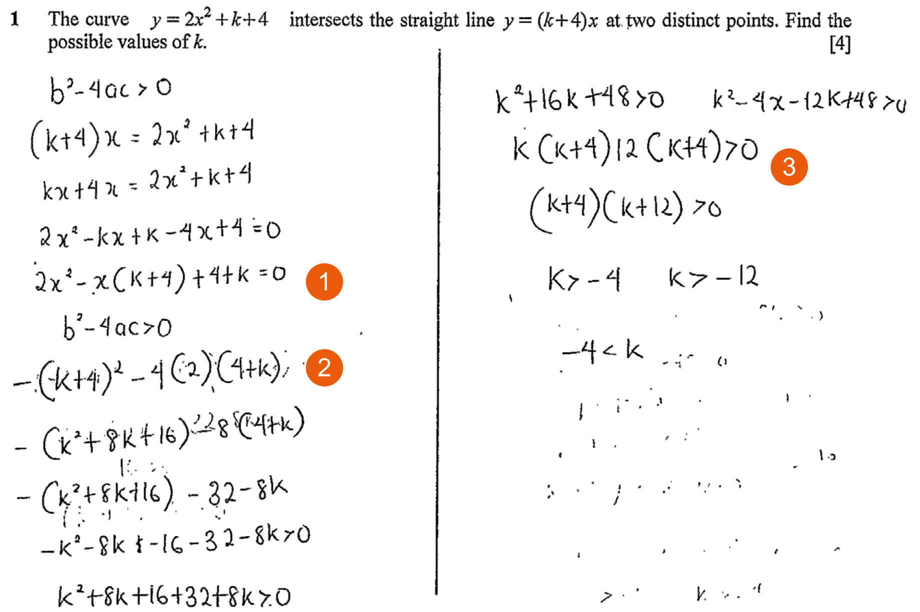
Question 1

Example Candidate Response – high	Examiner comments
1 The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k+4)x$ at two distinct points. Find the possible values of k. [4] $b^2 - 4ac > 0$ $(k+4)x = 2x^2 + k + 4$ $kx + 4x = 2x^2 + k + 4$ $2x^2 - kx - 4x + k + 4$ $2x^2 - x(k+4) + k + 4$ ① $(4 \times 2 \times k+4)$ $(8(k+4)$ $(8k + 32)$ $b^2 - 4ac > 0$ $(k+4)^2 - 4(2 \times (k+4))$ ② $(k+4)(k+4)$ $k^2 + 4k + 4k + 16$ $k^2 + 8k + 16$ $k^2 + 8k + 16 - 8k - 32 > 0$ $k^2 - 16 > 0$, $\sqrt{k^2}$ $\sqrt{16}$ $k > 4$ $k < -4$ $k > 4$ $-4 < k < 4$ ③	① The candidate equates the equation of the curve and the equation of the straight line and re-arranges them to obtain the required form of a quadratic with the terms collected correctly. One mark is awarded. ② The candidate finds the discriminant of the quadratic correctly. One mark is awarded. ③ The candidate obtains the correct critical values and is awarded one mark. However, they then give the set of values between the critical values instead of those outside which are needed to satisfy the discriminant being greater then zero. **Total mark awarded = 3 out of 4**

How the candidate could have improved their answer

Having obtained the critical values, the candidate should have checked that any value between the critical values satisfied the condition that the discriminant was greater than zero. A substitution of zero, for example, would have alerted the candidate that the range they had given was incorrect.

Example Candidate Response – middle



1 The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k+4)x$ at two distinct points. Find the possible values of k. [4]

$b^2 - 4ac > 0$

$(k+4)x = 2x^2 + k + 4$

$kx + 4x = 2x^2 + k + 4$

$2x^2 - kx + k - 4x + 4 = 0$

$2x^2 - x(k+4) + 4 + k = 0$ ①

$b^2 - 4ac > 0$

$-(k+4)^2 - 4(2)(4+k)$ ②

$-(k^2 + 8k + 16) - 8(4+k)$

$-(k^2 + 8k + 16) - 32 - 8k$

$-k^2 - 8k - 16 - 32 - 8k > 0$

$k^2 + 8k + 16 + 32 + 8k > 0$

$k^2 + 16k + 48 > 0$ $\quad k^2 - 4x - 12k + 48 > 0$

$k(k+4)12(k+4) > 0$ ③

$(k+4)(k+12) > 0$

$k > -4 \quad k > -12$

$-4 < k$

Examiner comments

① The candidate equates the equations for the curve and the straight line and forms a quadratic with the terms collected correctly. One mark is awarded.

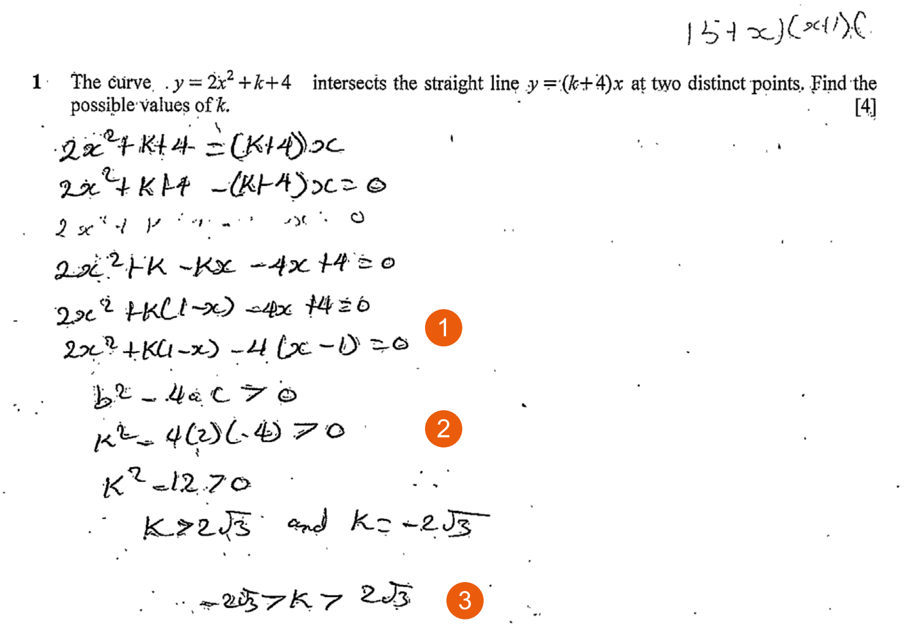
② The condition for the discriminant is quoted correctly, but when applied to the quadratic equation previously obtained, a sign error is made. The candidate makes a correct attempt to solve their quadratic equation to obtain two critical values and is awarded a mark for the method.

③ As the critical values are incorrect, no further marks can be awarded.

Total mark awarded = 2 out of 4

How the candidate could have improved their answer

If the candidate had made more careful use of brackets, they could have avoided misapplication of the negative coefficient of *x* in the discriminant.

Example Candidate Response – low	Examiner comments
 $15+x)(x+1)($ **1** The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k+4)x$ at two distinct points. Find the possible values of k. [4] $2x^2 + k + 4 = (k+4)x$ $2x^2 + k + 4 - (k+4)x = 0$ [illegible] $2x^2 + k - kx - 4x + 4 = 0$ $2x^2 + k(1-x) - 4x + 4 = 0$ ① $2x^2 + k(1-x) - 4(x-1) = 0$ $b^2 - 4ac > 0$ $k^2 - 4(2)(-4) > 0$ ② $k^2 - 12 > 0$ $k > 2\sqrt{3}$ and $k = -2\sqrt{3}$ $-2\sqrt{3} > k > 2\sqrt{3}$ ③	① The candidate equates the equations of the curve and the straight line and attempts to re-arrange into a form to which the discriminant can be applied. This is not done correctly. ② The candidate attempts to use the discriminant on their quadratic to obtain the two critical values and is awarded a mark. ③ As the resulting values are incorrect, no further marks are awarded. **Total mark awarded =** **1 out of 4**

How the candidate could have improved their answer

The candidate should have checked carefully that the quadratic equation obtained was in the correct form with terms collected correctly.

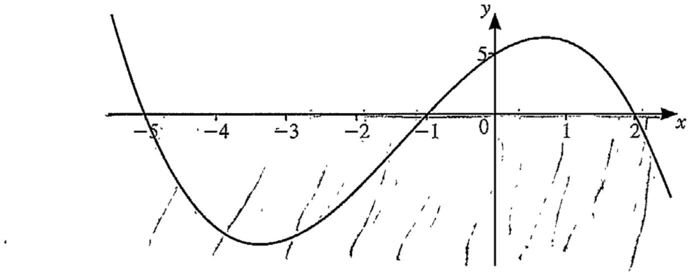
Common mistakes candidates made in this question

- Sign errors were common and usually occurred when like terms were being collected. Once critical values were obtained, errors involving the range that these critical values produce, were also common. A quick check using values other than the critical values could have helped candidates to identify errors.
- Candidates sometimes wrote disjoint inequalities in an incorrect continuous form as shown in the low response. The expectation is that they should be written as two separate inequalities.

Question 2

Example Candidate Response – high

2



The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial.

(a) Find $f(x)$. [3]

roots

$+\frac{1}{2}(x+5)(x+1)(x-2)$

$f(x) = -\frac{1}{2}(x+5)(x+1)(x-2)$ ①

(b) Write down the values of x such that $f(x) \leqslant 0$. [2]

~~$-5 < -1 < 2$~~

$x < 2$

$-5 < x < -1$

②

Examiner comments

① The candidate originally puts a positive sign in the first line of their answer, but this is corrected on the next line and the candidate supplies a correct final answer.

Mark for (a) = 3 out of 3

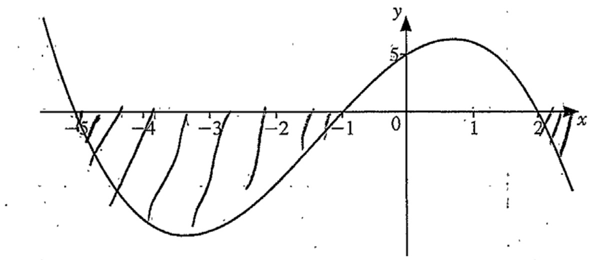
② The candidate's first range of values for x is incorrect, but the second set of values given is correct and is awarded one mark.

Mark for (b) = 1 out of 2

Total mark awarded = 4 out of 5

How the candidate could have improved their answer

More accurate shading in **(b)** would have helped the candidate to visualise the situation.

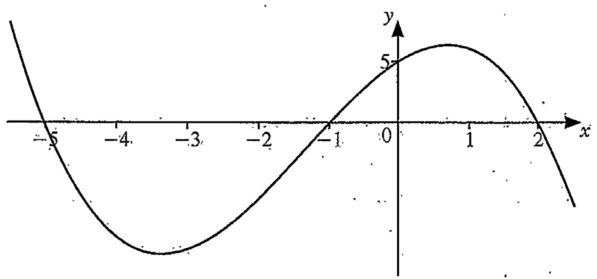
Example Candidate Response – middle	Examiner comments
2  The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial. **(a)** Find $f(x)$. [3] ~~$y = p(a-x)(b-x)(c-x)$~~ $y = 5(-5-x)(-1-x)(2-x)$ $y = 5($ ~~$y = p(a-1)(b-1)(c-1)$~~ $y = 5(-5-1)(-1-1)(2-1)$ $y = 5(-6)(-2)(1)$ $y = 60$ $f(x) = 5(x+5)(x+1)(x-2) \longrightarrow$ $y = p(x-a)(x-b)(x-c)$ $y = 5(x+5)(x+1)(x-2)$ $y = 5(x+5)(x+1)(x-2)$ ① **(b)** Write down the values of x such that $f(x) < 0$. [2] $f(x) < 0$ $x = -5$ or $x = -1$ or $x = 2 \longrightarrow$ $-5 < x < -1$ or $x > 2$ ②	① The candidate recognises the correct three factors and this is awarded one mark. However there is no recognition that the curve is a negative cubic and the candidate does not use the intercept on the y-axis correctly. Mark for (a) = 1 out of 3 ② The candidate's accurate shading enables them to identify the correct ranges and write them in the correct form. Two marks are awarded. Mark for (b) = 2 out of 2 **Total mark awarded = 3 out of 5**

How the candidate could have improved their answer

The candidate needed to identify the shape of the cubic graph as having a negative coefficient with the factors, and that this coefficient could be calculated by using the product of 5, 1 and -2 and by making a comparison with the intercept on the y -axis.

Example Candidate Response – low

2



The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial.

(a) Find $f(x)$. [3]

$f(x) = (5+x)(x+1)(x-2)$ ①

$= (5x+5+x^2+x)(x-2)$

$= (6x+5+x^2)(x-2)$

$= 6x^2-12x+5x-10+x^3-2x^2$

$4x^2-7x-10+x^3$

$x^3+4x^2-7x-10$

(b) Write down the values of x such that $f(x) < 0$. [2]

Values of x are −5 and −1 ②

Examiner comments

① The candidate identifies the three correct factors and is awarded one mark. There is no recognition that the cubic is negative and the intercept on the y-axis is not used. Expansion of the factors is unnecessary.

Mark for (a) = 1 out of 3

② Although the candidate identifies two of the critical values, there is no attempt to give either of the two ranges. No marks are awarded.

Mark for (b) = 0 out of 2

Total mark awarded =
1 out of 5

How the candidate could have improved their answer

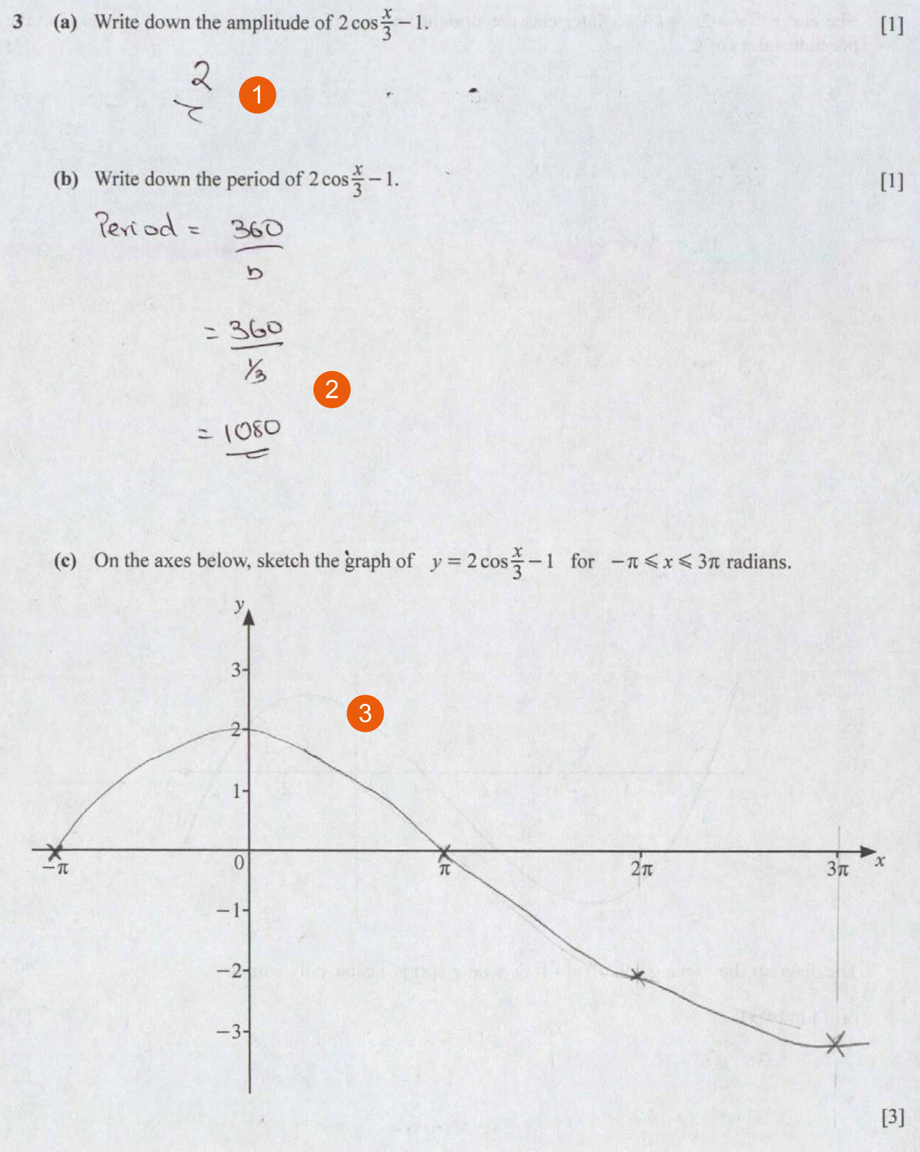
- The candidate needed to identify the shape of the cubic graph as having a negative coefficient with the factors, and that this coefficient could be calculated by using the product of 5, 1 and -2 and by making a comparison with the intercept on the y-axis.
- Suitable shading would have helped the candidate visualise the situation.

Common mistakes candidates made in this question

- **(a)** Many candidates considered the cubic polynomial to be a product of three linear factors only, not considering the basic shape of the curve and the intercept on the y-axis.
- **(b)** Interpretation of the demand of the question led many candidates to write down the critical values only, making no attempt at inequalities.

Question 3

Example Candidate Response – high



3 (a) Write down the amplitude of $2\cos\frac{x}{3}-1$. [1]

2 ①

(b) Write down the period of $2\cos\frac{x}{3}-1$. [1]

Period = $\frac{360}{b}$

= $\frac{360}{1/3}$ ②

= 1080

(c) On the axes below, sketch the graph of $y = 2\cos\frac{x}{3} - 1$ for $-\pi \leqslant x \leqslant 3\pi$ radians.

③

[3]

Examiner comments

① The candidate gives the correct amplitude.

Mark for (a) = 1 out of 1

② The candidate identifies the correct period which they choose to give in degrees.

Mark for (b) = 1 out of 1

③ The candidate's curve passes through the correct starting and finishing points and it has the correct shape. However the intercept on the y-axis is incorrect. Two marks are awarded.

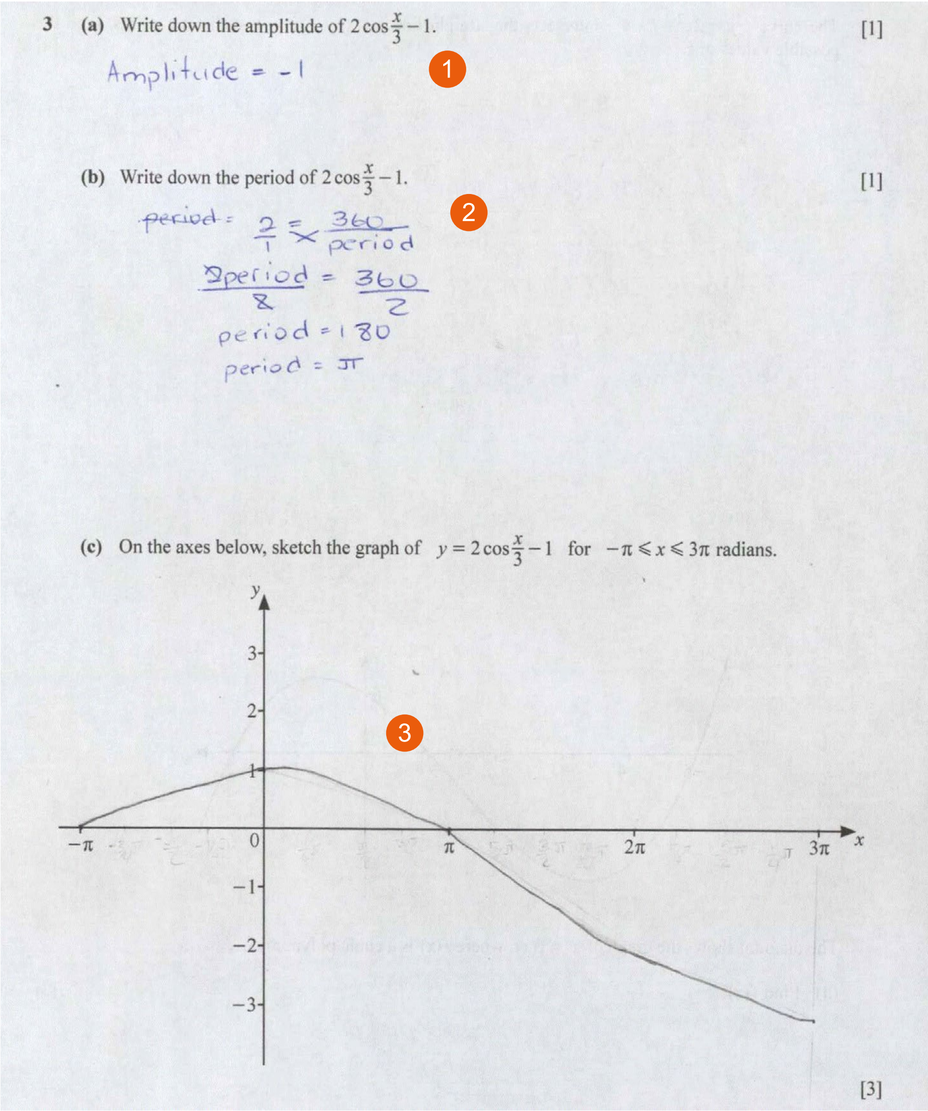
Mark for (c) = 2 out of 3

Total mark awarded = 4 out of 5

How the candidate could have improved their answer

The intercept on the y-axis needed to be checked. The candidate appeared to make a calculation error.

Example Candidate Response – middle



3 (a) Write down the amplitude of $2\cos\frac{x}{3}-1$. [1]

Amplitude = -1

①

(b) Write down the period of $2\cos\frac{x}{3}-1$. [1]

②

period: $\frac{2}{1} \times \frac{360}{period}$

$\frac{2period}{2} = \frac{360}{2}$

period = 180

period = π

(c) On the axes below, sketch the graph of $y = 2\cos\frac{x}{3} - 1$ for $-\pi \leqslant x \leqslant 3\pi$ radians.

[3]

Examiner comments

① The candidate supplies an incorrect answer.

Mark for (a) = 0 out of 1

② The candidate supplies an incorrect answer.

Mark for (b) = 0 out of 1

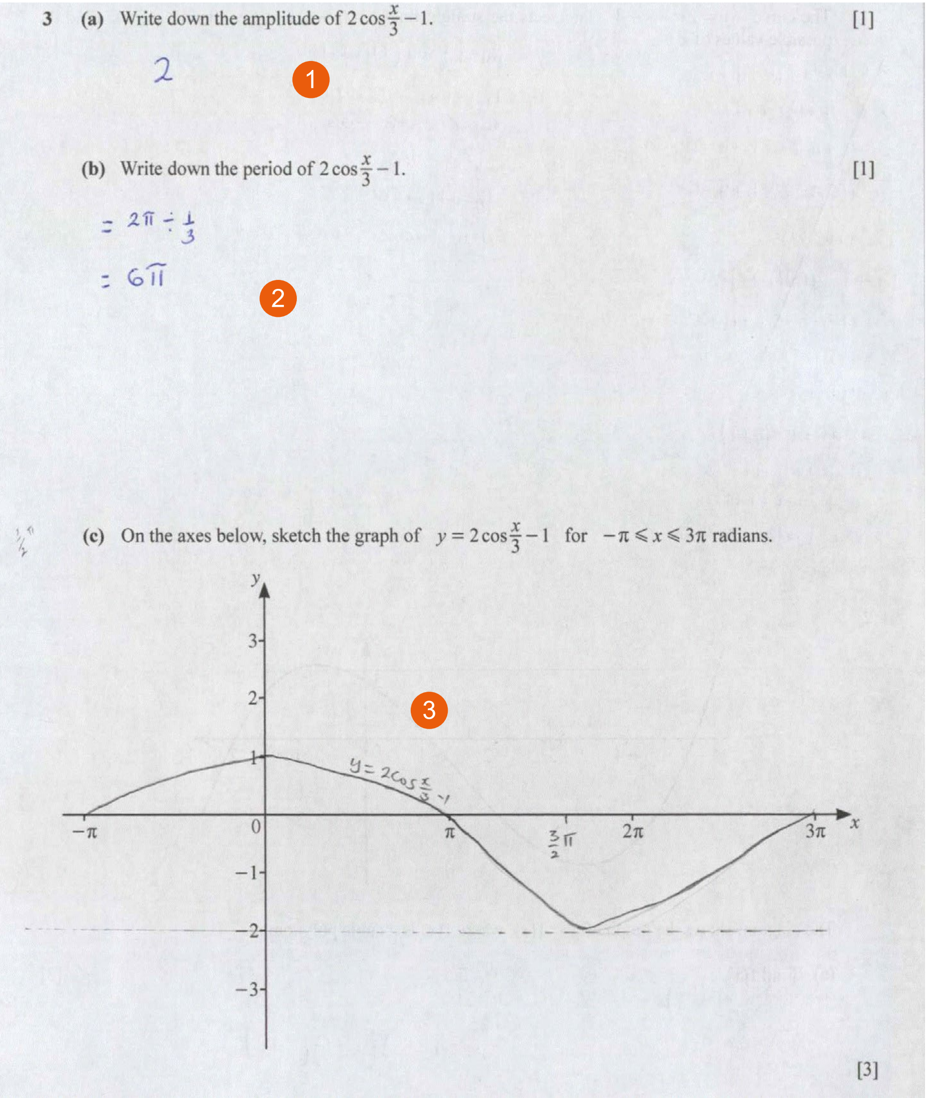
③ The shape of the candidate's curve and all the relevant points are correct. Three marks are awarded.

Mark for (c) = 3 out of 3

Total mark awarded = 3 out of 5

How the candidate could have improved their answer

The candidate required a better understanding of the key features of trigonometric graphs. They mistakenly thought that the intercept related to the amplitude and the coefficient of the trigonometric term related to the period, when in fact the opposite was true.

Example Candidate Response – low	Examiner comments
 3 **(a)** Write down the amplitude of $2\cos\frac{x}{3}-1$. [1] 2 ①	① The candidate gives a correct amplitude. Mark for (a) = 1 out of 1
(b) Write down the period of $2\cos\frac{x}{3}-1$. [1] $= 2\pi \div \frac{1}{3}$ $= 6\pi$ ②	② The candidate gives a correct period and chooses to express this in radians. Mark for (b) = 1 out of 1
(c) On the axes below, sketch the graph of $y = 2\cos\frac{x}{3} - 1$ for $-\pi \leqslant x \leqslant 3\pi$ radians. [3]	③ The candidate's graph has an incorrect shape and an end point that is incorrect. The curve intersects the x-axis at more than the two correct points. No marks can be awarded. Mark for (c) = 0 out of 3 **Total mark awarded = 2 out of 5**

How the candidate could have improved their answer

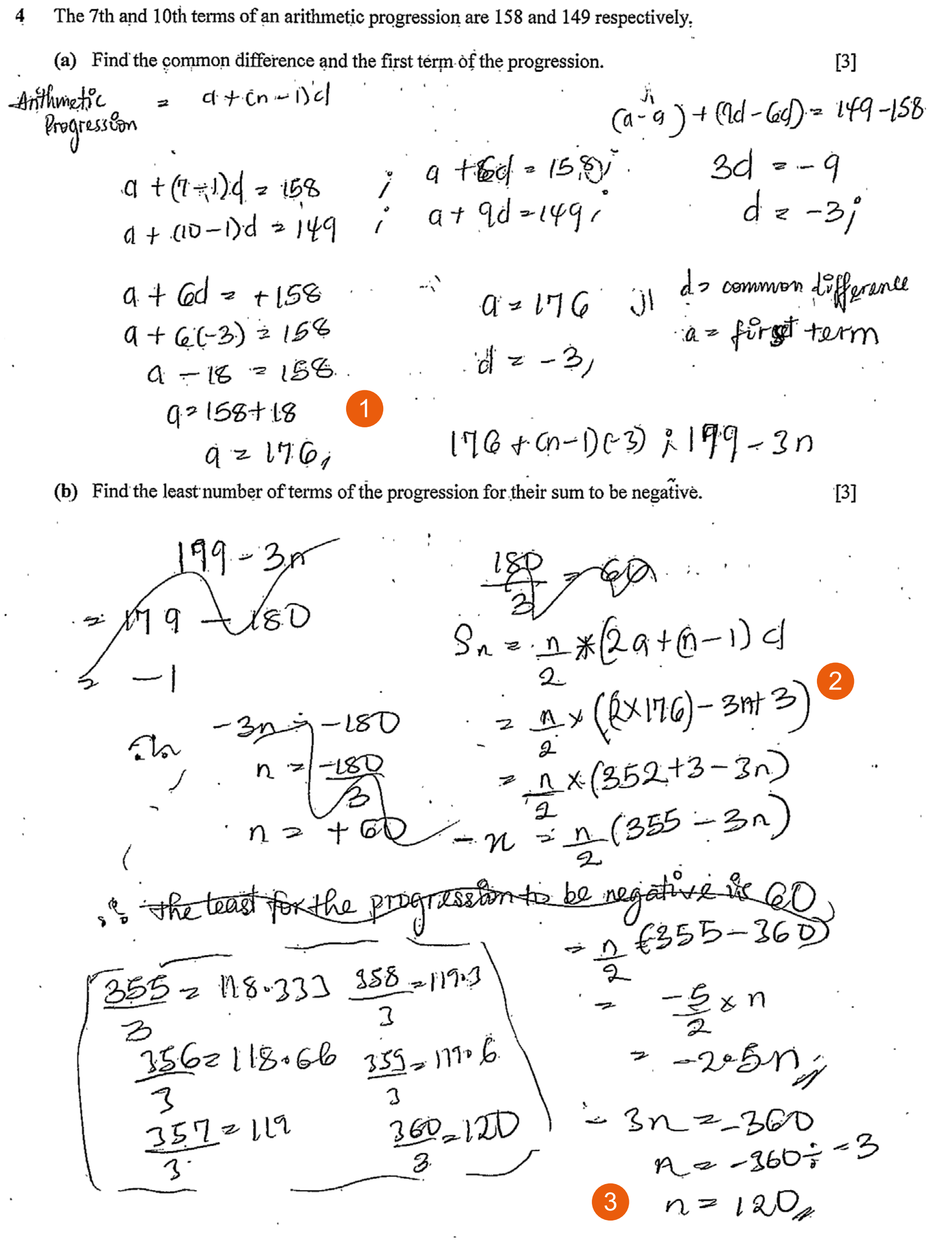
Substituting the x values of $-\pi$, 0, π, 2π and 3π into the function to find the corresponding y values and plotting these points correctly would have shown that the curve was not being sketched over a complete period.

Common mistakes candidates made in this question

- Candidates occasionally confused the relationships between a given trigonometric function and its amplitude and period.
- The first two parts of the question were intended to help with the sketching of the trigonometric function in the third part. Many candidates did not check their graph with the answers they had obtained in the first two parts of the question.

Question 4

Example Candidate Response – high



4 The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

(a) Find the common difference and the first term of the progression. [3]

Arithmetic Progression = $a+(n-1)d$

$a+(7-1)d = 158$; $a+6d = 158$;

$a+(10-1)d = 149$; $a+9d = 149$;

$(a-a)+(9d-6d) = 149-158$

$3d = -9$

$d = -3$;

$a+6d = +158$

$a+6(-3) = 158$

$a-18 = 158$

$a = 158+18$

$a = 176$,

∴ $a = 176$; $d = -3$,

d = common difference

a = first term

①

$176+(n-1)(-3)$; $199-3n$

(b) Find the least number of terms of the progression for their sum to be negative. [3]

~~$199-3n$~~

~~$= 179 - 180$~~

~~$= -1$~~

~~$-3n = -180$~~

~~$n = \frac{-180}{3}$~~

~~$n = +60$~~

~~$\frac{180}{3} = 60$~~

$S_n = \frac{n}{2} * (2a+(n-1)d)$

②

$= \frac{n}{2} \times ((2\times 176)-3n+3)$

$= \frac{n}{2} \times (352+3-3n)$

$= \frac{n}{2}(355-3n)$

~~∴ the least for the progression to be negative is 60~~

$= \frac{n}{2}(355-360)$

$= -\frac{5}{2} \times n$

$= -2.5n$

$-3n = -360$

$n = -360 \div -3$

③ $n = 120$

$\frac{355}{3} = 118.333$ $\frac{358}{3} = 119.3$

$\frac{356}{3} = 118.66$ $\frac{359}{3} = 119.6$

$\frac{357}{3} = 119$ $\frac{360}{3} = 120$

Examiner comments

① The candidate forms the correct equations and solves them to give the correct first term and common difference.

Mark for (a) = 3 out of 3

② A correct sum formula is used with the correct first term, common difference and n terms, which is awarded one mark.

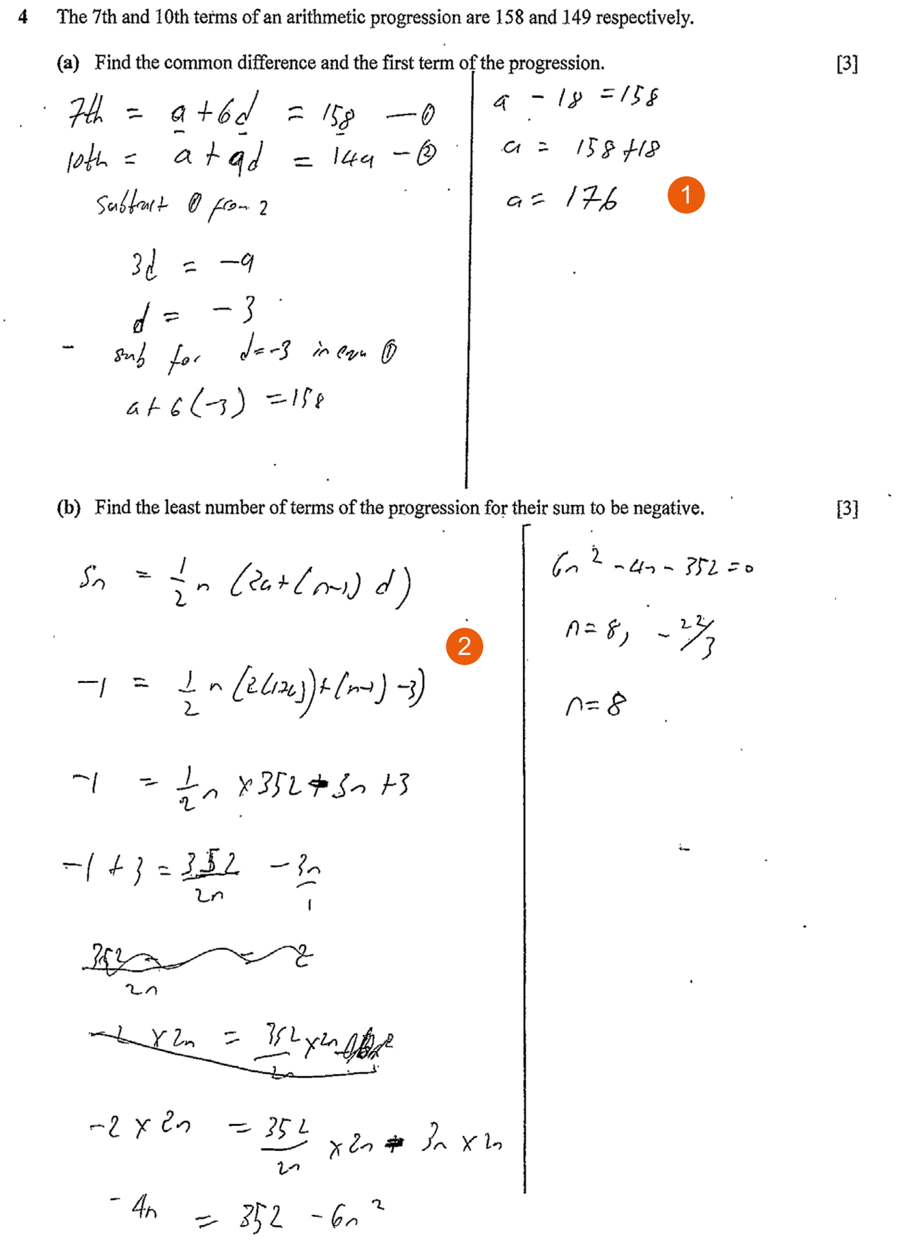
③ The answer of 118.333 is correct and is awarded one mark, but it is not rounded up to the nearest integer.

Mark for (b) = 2 out of 3

Total mark awarded = 5 out of 6

How the candidate could have improved their answer

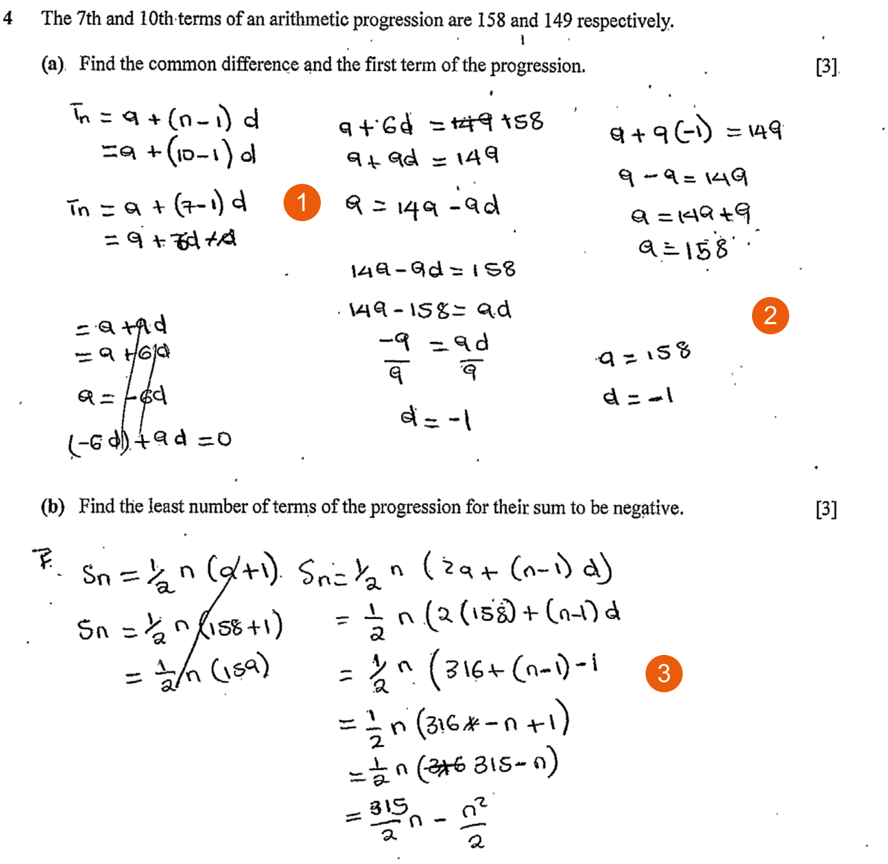
Interpretation of the word ‘least’ in the stem of the question should have led the candidate to assume that the sum was less than zero and hence form an inequality which could be solved.

Example Candidate Response – middle	Examiner comments
 **4** The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively. **(a)** Find the common difference and the first term of the progression. [3] 7th = $a + 6d = 158$ — ① 10th = $a + 9d = 149$ — ② subtract ① from 2 $3d = -9$ $d = -3$ sub for $d = -3$ in eqn ① $a + 6(-3) = 158$ $a - 18 = 158$ $a = 158 + 18$ $a = 176$ ①	① The candidate forms two correct equations and solves them to obtain the correct first term and common difference. Full marks are awarded. Mark for (a) = 3 out of 3
(b) Find the least number of terms of the progression for their sum to be negative. [3] $S_n = \frac{1}{2}n(2a + (n-1)d)$ ② $-1 = \frac{1}{2}n(2(176)) + (n-1) - 3)$ $-1 = \frac{1}{2}n \times 352 + 3n + 3$ $-1 + 3 = \frac{352}{2n} - \frac{3n}{1}$ ~~$\frac{352}{2n}$ 2~~ ~~$-2 \times 2n = \frac{352 \times 2n}{2n}$~~ $-2 \times 2n = \frac{352}{2n} \times 2n + 3n \times 2n$ $-4n = 352 - 6n^2$ $6n^2 - 4n - 352 = 0$ $n = 8, -\frac{22}{3}$ $n = 8$	② The candidate uses a correct sum formula with the correct first term, common difference and *n* terms. One mark is awarded. Although equating the sum to zero would result in a linear equation which is easier to solve, the candidate equates the sum to –1 but does not solve the resulting quadratic correctly, so no further marks are awarded. Mark for (b) = 1 out of 3 **Total mark awarded = 4 out of 6**

How the candidate could have improved their answer

Interpretation of the word ‘least’ in the stem of the question should have led the candidate to assume that the sum was less than zero and hence form an inequality or an equation which could be solved. Equating the sum to –1 and solving was acceptable as long as the candidate checked that the solution they obtained was the first one to give a sum less than zero.

Example Candidate Response – low



4 The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

(a) Find the common difference and the first term of the progression. [3]

$T_n = a + (n-1)d$

$= a + (10-1)d$

$T_n = a + (7-1)d$

$= a + \cancel{7d} + \cancel{d}$

$= a + \cancel{9}d$

$= a + \cancel{6}a$

$a = \cancel{-6}d$

$(-6d) + 9d = 0$

①

$a + 6d = \cancel{149}\ 158$

$a + 9d = 149$

$a = 149 - 9d$

$149 - 9d = 158$

$149 - 158 = 9d$

$\frac{-9}{9} = \frac{9d}{9}$

$d = -1$

$a + 9(-1) = 149$

$a - 9 = 149$

$a = 149 + 9$

$a = 158$

②

$a = 158$

$d = -1$

(b) Find the least number of terms of the progression for their sum to be negative. [3]

F. $S_n = \frac{1}{2}n(\cancel{a}+1)$

$S_n = \frac{1}{2}n(158+1)$

$= \frac{1}{2}n(159)$

$S_n = \frac{1}{2}n(2a + (n-1)d)$

$= \frac{1}{2}n(2(158) + (n-1)d$

$= \frac{1}{2}n(316 + (n-1) - 1$

③

$= \frac{1}{2}n(316\cancel{*} - n + 1)$

$= \frac{1}{2}n(\cancel{316}\ 315 - n)$

$= \frac{315}{2}n - \frac{n^2}{2}$

Examiner comments

① The candidate forms two correct equations using the given information. One mark is awarded

② An error is made by the candidate in solving the equations, so no further marks can be awarded.

Mark for (a) = 1 out of 3

③ A correct sum formula is used using *n* terms and the values obtained in (a) for which a method mark is awarded.

The candidate does not attempt to do any further work.

Mark for (b) = 1 out of 3

Total mark awarded = 2 out of 6

How the candidate could have improved their answer

- **(a)** The candidate would have benefited from checking that their solutions satisfied the original equations. This would have alerted them to an error.
- **(b)** Interpretation of the word ‘least’ in the stem of the second part of the question should have led the candidate to assume that the sum was less than zero and hence form either an inequality or equation which could be solved.

Common mistakes candidates made in this question

- **(a)** Most candidates were able to obtain two correct equations but occasionally made errors in solution.
- **(b)** It was essential that the word ‘least’ was interpreted correctly. Many candidates did use a correct sum formula but did not form an equation or inequality using this formula and zero. Candidates also needed to appreciate that the answer would be an integer.
- **(b)** Trial and improvement methods and use of -1 rather than zero were acceptable, but candidates needed to show that their solution was the first solution that gave a sum less than zero. For those candidates who used this method, many did not do this and were unable to gain full marks.

Question 5

Example Candidate Response – high	Examiner comments
5 Find the coefficient of x^2 in the expansion of $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5$. [5] $(x+\frac{3}{x})^5$ $= a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\binom{n}{3}a^{n-3}b^3+\binom{n}{4}a^{n-4}b^4+\binom{n}{5}a^{n-5}b^5$ $= x^5+\binom{5}{1}x^{\overset{4}{5-1}}\cdot\frac{2}{x}+\binom{5}{2}x^{\overset{3}{5-2}}\cdot\left(\frac{2}{x}\right)^2+\binom{5}{3}x^{\overset{2}{5-3}}\cdot\left(\frac{2}{x}\right)^3+\binom{5}{4}x^{\overset{1}{5-4}}\cdot\left(\frac{2}{x}\right)^4+\binom{5}{5}x^{5-5}\cdot\left(\frac{2}{x}\right)^5$	
① $= x^5+10x^3+40x^2+80/x+80/x^3+35/x^5$ $= \left(x-\frac{3}{x}\right)\left(x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{35}{x^5}\right)$ $= x^6+10x^4+40x^2+80+\frac{80}{x^2}+\frac{3}{x^4}-3x^4-30x^2+120-\frac{240}{x^2}+\frac{240}{x^4}+\frac{105}{x^6}$ $= 40x^2+\frac{80}{x^2}-30x^2-\frac{240}{1x^2}$ $= 10x^2+\frac{80}{1}x^2-\frac{240}{1}x^2$	① The first three terms of the expansion are correctly expanded by the candidate.
② $= 330x^2$ $= 330$	② The candidate uses a correct method to find the term in x^2 which is awarded one mark, but the candidate then mistakenly includes terms with x^2 in the denominator to get to their final answer. No further marks are awarded. **Total mark awarded = 4 out of 5**

How the candidate could have improved their answer

The candidate needed to take greater care over the terms involving x^2 in the denominator as these were misread or misinterpreted and the final accuracy mark was not awarded.

Example Candidate Response – middle

5 Find the coefficient of x^2 in the expansion of $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5$. [5]

$\left(x+\frac{2}{x}\right)^5$

$5C_0(x)^5\left(\frac{2}{x}\right)^0 + 5C_1(x)^4\left(\frac{2}{x}\right)^1 + 5C_2(x)^3\left(\frac{2}{x}\right)^2 + 5C_3(x)^2\left(\frac{2}{x}\right)^3 + 5C_4(x)^1\left(\frac{2}{x}\right)^4 + 5C_5(x)^0\left(\frac{2}{x}\right)^5$

$1 \times x^5 \times 1 + 5 \times x^4 \times \frac{2}{x} + 10 \times x^3 \times \frac{4}{x^2} + 10 \times x^2 \times \frac{8}{x^3} + 5 \times x \times \frac{16}{x^4} + 1 \times 1 \times \frac{32}{x^5}$

$x^5 + \frac{10x^4}{x} + \frac{40x^3}{x^2} + \frac{80x^2}{x^3} + \frac{80x}{x^4} + \frac{32}{x^5}$

~~$x^5 + 10x^3 + 80x$~~

① $\left(x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}\right)\left(x - \frac{3}{x}\right)$

$x^6 + 10x^4 + 40x^2 + 80 + \frac{80}{x^2} + \frac{32}{x^4}) - 3x^4 + 30x^2 + 120 - \frac{240}{x^2}$

$40x^2 + \frac{80}{x^2} - \frac{240}{x^2}$ ②

$\underline{\underline{-120}}$

Examiner comments

① The candidate expands the expression correctly for the first three terms.

② Although there is evidence of both of the required x^2 terms, the candidate does not use one of them in their answer. No further marks are awarded.

Total mark awarded = 3 out of 5

How the candidate could have improved their answer

The candidate only identified and used one term in x^2 even though the second term in x^2 had been calculated. Incorrect terms were also included in the final answer.

Example Candidate Response – low	Examiner comments
5 Find the coefficient of x^2 in the expansion of $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5$. [5] $\left(x+\frac{2}{x}\right)^5$ $5C_0(x)^5\left(\frac{2}{x}\right)^0 + 5C_1(x)^4\left(\frac{2}{x}\right)^1 + 5C_2(x)^3\left(\frac{2}{x}\right)^2$ $x^5 + 5x^4\left(\frac{2}{x}\right) + 10(x^3)\left(\frac{4}{x^2}\right)$ ① $x^5 + \frac{5x^2(2)}{x} + \frac{40x^3}{x^2}$ $x^5 + \cancel{8x}\ \frac{10x^2}{\cancel{x}} + \frac{40x^3}{x^2}$ $\left(x-\frac{3}{x}\right)\left(x^5 + 10x + 40x\right)$ ② $x(10x) + 40x(x)$ $\quad\left(x-\frac{3}{x}\right)\left(x^5 + \frac{10x^2}{x} + \frac{40x^3}{x^2}\right)$ ③ $10x^2 + 40x^2$ $\quad -\frac{12x}{x}$ $= 50x^2$ $= 50$	① The candidate correctly expresses the first three terms of the expansion in their unsimplified form. One mark is awarded. ② The candidate's simplification of the second term contains an error, but the third term is correct and one mark is awarded. ③ Although the candidate multiplies their expansion by x they do not also multiply it by the $-\frac{3}{x}$. **Total mark awarded = 2 out of 5**

How the candidate could have improved their answer

The candidate miscopied their second term in line three. They should have been alerted to an error when they obtained two terms both in x.

Common mistakes candidates made in this question

Candidates often thought that terms with x^2 in the denominator also needed to be included.

Question 6

Example Candidate Response – high

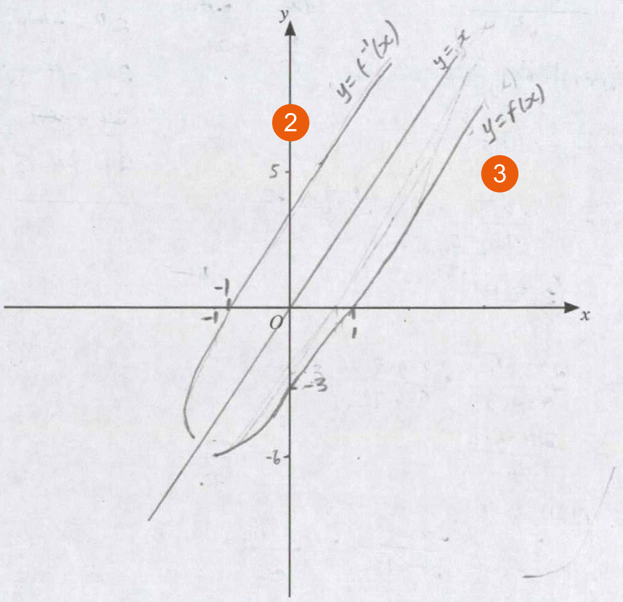
6 $f(x) = x^2 + 2x - 3$ for $x \geqslant -1$

(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse. [1]

Due to the restriction, $f(x)$ is now a one to one function.

①

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.



[4]

Examiner comments

① The candidate provides the correct justification that the function is a one-one function and is awarded the mark.

Mark for (a) = 1 out of 1

② The inverse function is a reflection in the line $y = x$ of their first graph and is awarded one mark. However the intercepts on the axes are incorrect.

③ The original function is sketched correctly with the correct intercepts marked on the axes. Two marks are awarded.

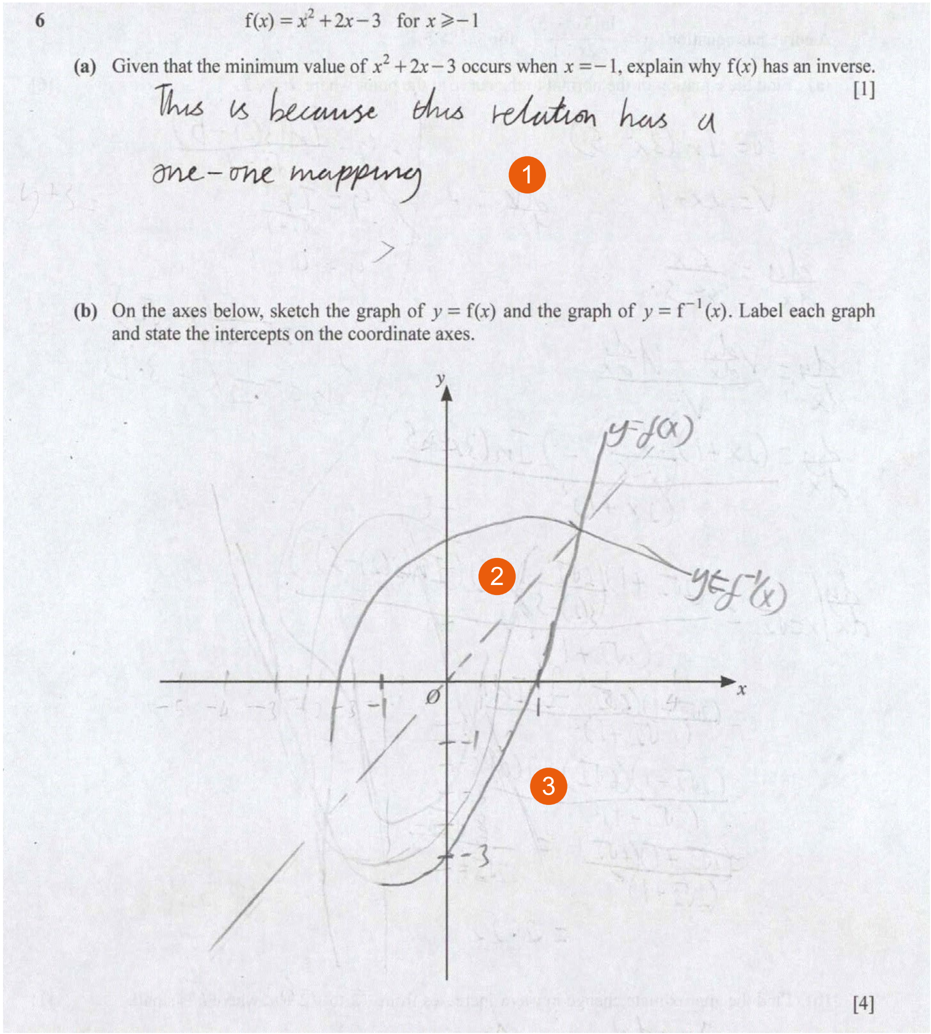
Mark for (b) = 3 out of 4

Total mark awarded = 4 out of 5

How the candidate could have improved their answer

The candidate needed to appreciate that when the original function was reflected in the line $y = x$, the coordinates of the intercepts need to be reflected similarly.

Example Candidate Response – middle



6 $\quad f(x) = x^2 + 2x - 3 \quad \text{for } x \geqslant -1$

(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse. [1]

This is because this relation has a one-one mapping ①

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.

[4]

Examiner comments

① The candidate identifies that the function is a one-one function but provides no justification.

Mark for (a) = 0 out of 1

② Although a straight line is drawn, it is not marked as $y = x$ and the curve produced by the candidate is not sufficiently accurate to be classed as a reflection of the original. There is no indication of where the inverse intercepts on the axes.

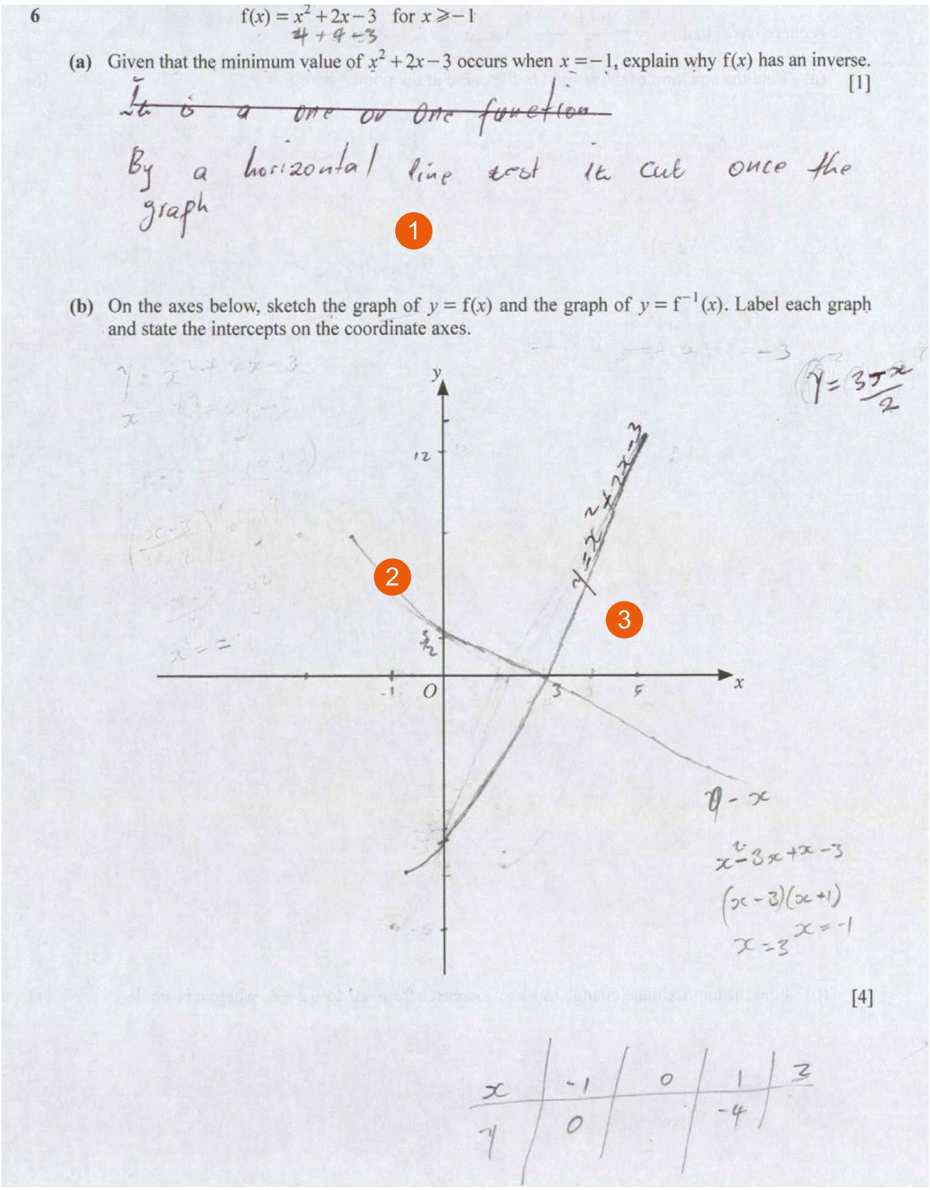
③ The candidate's sketch of the original function is correct. The correct intercepts with both axes are given. Two marks are awarded.

Mark for (b) = 2 out of 4

Total mark awarded = 2 out of 5

How the candidate could have improved their answer

- A better attempt at sketching a reflection in the line $y = x$ of the original function would have been beneficial to the candidate.
- The candidate needed to appreciate that when the original function was reflected in the line $y = x$, the coordinates of the intercepts needed to be reflected similarly.

Example Candidate Response – low	Examiner comments
 **6** $f(x) = x^2 + 2x - 3$ for $x \geqslant -1$ $4 + 4 - 3$ **(a)** Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why f(x) has an inverse. [1] ~~It is a one or one function~~ By a horizontal line test it cut once the graph ① **(b)** On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes. $y = 3 + \frac{x}{2}$ y x 12 -1 O 3 5 $\frac{5}{2}$ $y = x^2 + 2x - 3$ ② ③ $y - x$ $x^2 - 3x + x - 3$ $(x - 3)(x + 1)$ $x = 3$ $x = -1$ [4] x | -1 | 0 | 1 | 3 y | 0 | | -4 |	① The candidate does not provide a sufficient explanation or a statement that the function is one-one. Mark for (a) = 0 out of 1 ② The candidate makes an incorrect attempt at sketching the inverse function. The intercepts with the axes are incorrect. ③ The candidate's sketch has the correct shape for the original function which is awarded one mark, however the intercepts on the axes are incorrect. Mark for (b) = 1 out of 4 **Total mark awarded = 1 out of 5**

How the candidate could have improved their answer

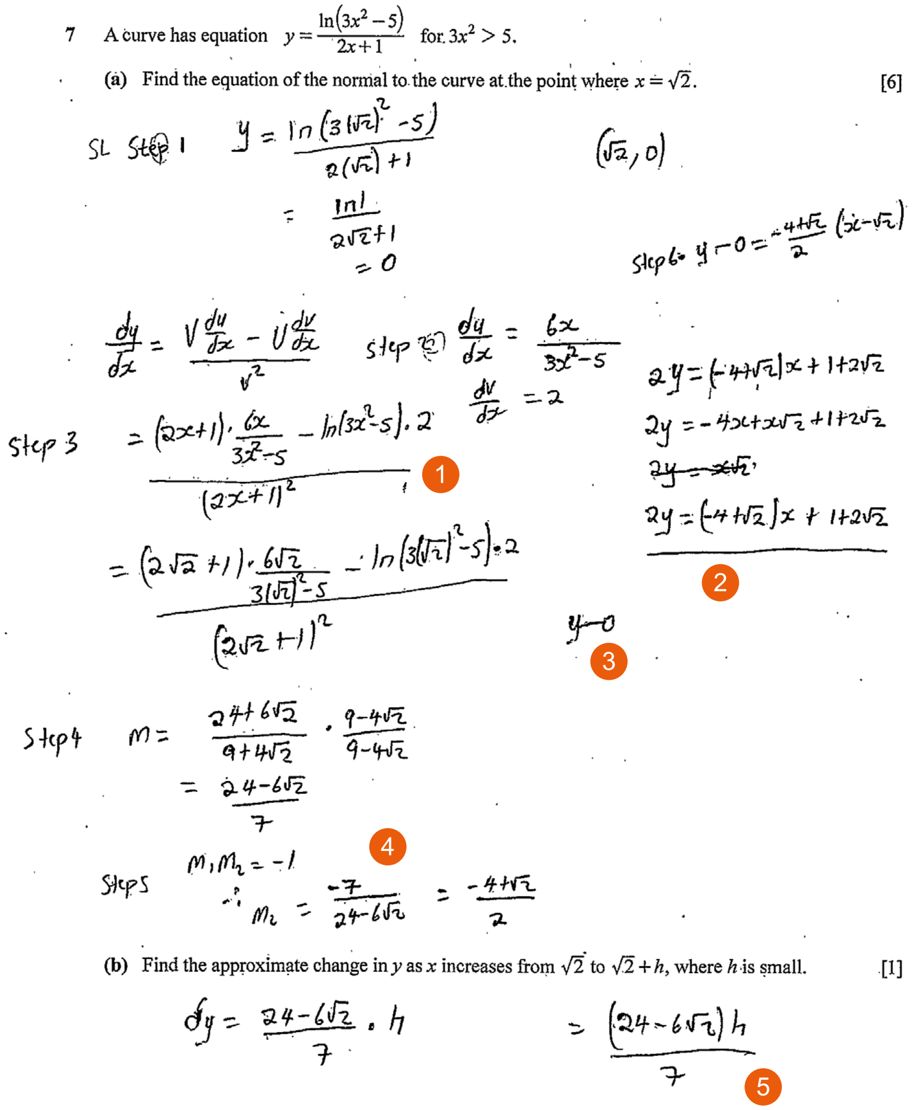
The candidate needed to be more aware of the relationships between functions and their inverses and the conditions for which a function has an inverse.

Common mistakes candidates made in this question

- **(a)** Many candidates stated correctly that the function was a one-one function but provided no justification. It was essential that the restricted domain be mentioned.
- **(b)** A common error was to sketch the function without the restricted domain. Part **(a)** was intended to alert candidates to the fact that $x \geqslant -1$ and that this would need to be taken into account in any sketches.

Question 7

Example Candidate Response – high



7 A curve has equation $y = \dfrac{\ln(3x^2 - 5)}{2x+1}$ for $3x^2 > 5$.

(a) Find the equation of the normal to the curve at the point where $x = \sqrt{2}$. [6]

SL Step 1 $y = \dfrac{\ln(3(\sqrt{2})^2 - 5)}{2(\sqrt{2})+1}$ $(\sqrt{2}, 0)$

$= \dfrac{\ln 1}{2\sqrt{2}+1}$

$= 0$

Step 6 $y - 0 = \dfrac{-4+\sqrt{2}}{2}(x - \sqrt{2})$

$\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ step 2 $\dfrac{du}{dx} = \dfrac{6x}{3x^2-5}$ $\dfrac{dv}{dx} = 2$

$2y = (-4+\sqrt{2})x + 1+2\sqrt{2}$

$2y = -4x + x\sqrt{2} + 1 + 2\sqrt{2}$

~~$2y = x\sqrt{2}$~~

$2y = (-4+\sqrt{2})x + 1+2\sqrt{2}$ ②

Step 3 $= \dfrac{(2x+1)\cdot\frac{6x}{3x^2-5} - \ln(3x^2-5)\cdot 2}{(2x+1)^2}$ ①

$= \dfrac{(2\sqrt{2}+1)\cdot\frac{6\sqrt{2}}{3(\sqrt{2})^2-5} - \ln(3(\sqrt{2})^2-5)\cdot 2}{(2\sqrt{2}+1)^2}$

$y - 0$ ③

Step 4 $m = \dfrac{24+6\sqrt{2}}{9+4\sqrt{2}} \cdot \dfrac{9-4\sqrt{2}}{9-4\sqrt{2}}$

$= \dfrac{24-6\sqrt{2}}{7}$ ④

Step 5 $m_1 m_2 = -1$

$\therefore m_2 = \dfrac{-7}{24-6\sqrt{2}} = \dfrac{-4+\sqrt{2}}{2}$

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2}+h$, where h is small. [1]

$\delta y = \dfrac{24-6\sqrt{2}}{7} \cdot h$ $= \dfrac{(24-6\sqrt{2})h}{7}$ ⑤

Examiner comments

① The candidate supplies a correct expression for the differentiation of the quotient including the correct differential of the $\ln$ term. Three marks are awarded.

② The candidate attempts to find the equation of the normal using a correct method which is awarded one mark, but an earlier arithmetic error means the answer is incorrect.

③ The candidate indicates the correct value of y at the given point and this is awarded one mark.

④ The candidate correctly evaluates the gradient at the given point and uses this to attempt to find the gradient of the normal.

Mark for (a) = 5 out of 6

⑤ The candidate correctly applies approximate change.

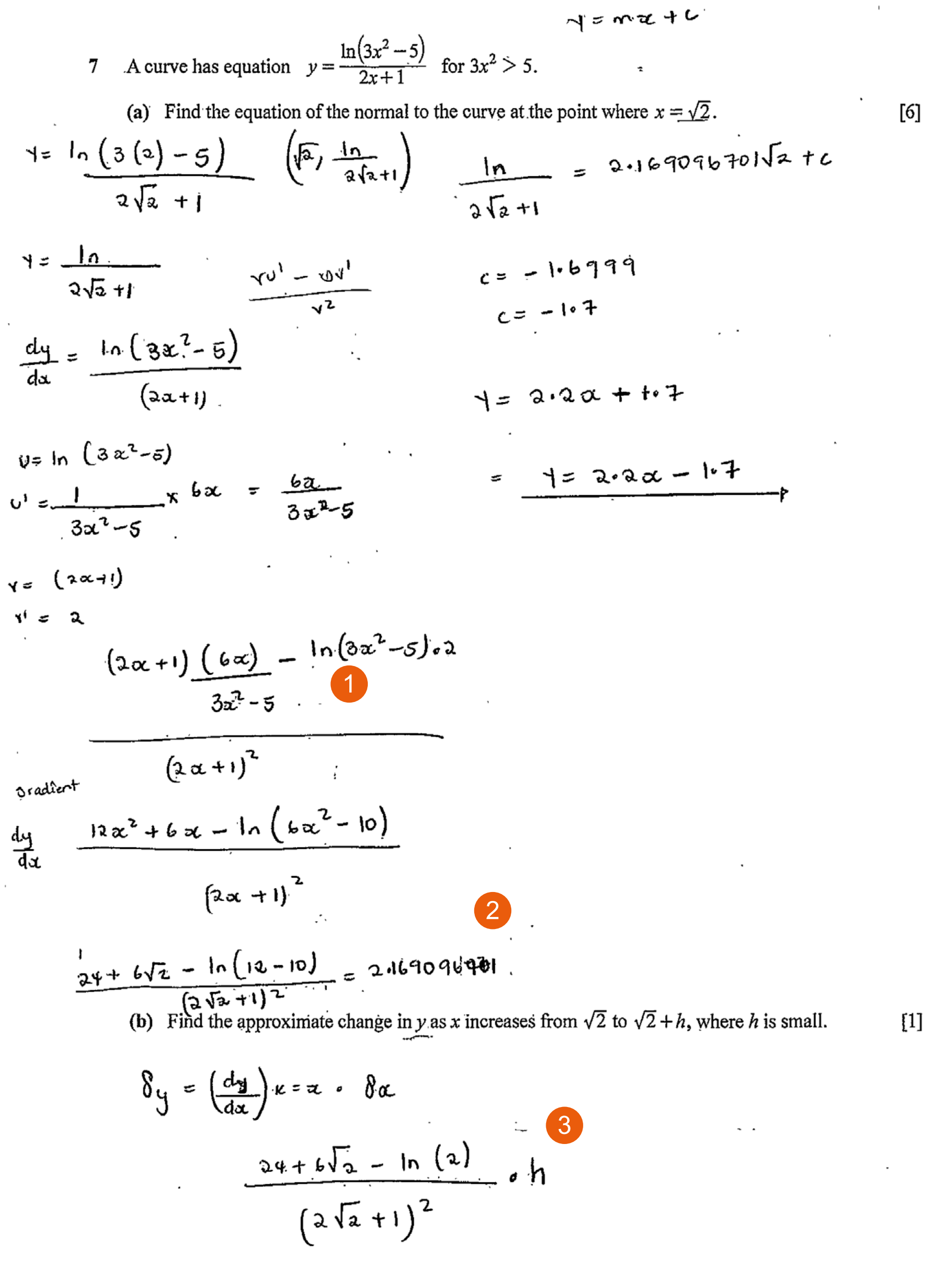
Mark for (b) = 1 out of 1

Total mark awarded = 6 out of 7

How the candidate could have improved their answer

The candidate made an error when calculating the gradient of the normal which could have been avoided by using brackets.

Example Candidate Response – middle



$y = mx + c$

7 A curve has equation $y = \frac{\ln(3x^2 - 5)}{2x+1}$ for $3x^2 > 5$.

(a) Find the equation of the normal to the curve at the point where $x = \sqrt{2}$. [6]

$y = \frac{\ln(3(2) - 5)}{2\sqrt{2} + 1}$ $\left(\sqrt{2}, \frac{\ln}{2\sqrt{2}+1}\right)$

$y = \frac{\ln}{2\sqrt{2}+1}$ $\frac{vu' - uv'}{v^2}$

$\frac{dy}{dx} = \frac{\ln(3x^2 - 5)}{(2x+1)}$

$u = \ln(3x^2 - 5)$

$u' = \frac{1}{3x^2 - 5} \times 6x = \frac{6x}{3x^2 - 5}$

$v = (2x+1)$

$v' = 2$

$\frac{(2x+1)\frac{(6x)}{3x^2-5} - \ln(3x^2-5) \cdot 2}{(2x+1)^2}$ ①

gradient

$\frac{dy}{dx}$ $\frac{12x^2 + 6x - \ln(6x^2 - 10)}{(2x+1)^2}$ ②

$\frac{24 + 6\sqrt{2} - \ln(12 - 10)}{(2\sqrt{2}+1)^2} = 2.169096701$

$\frac{\ln}{2\sqrt{2}+1} = 2.169096701\sqrt{2} + c$

$c = -1.6999$

$c = -1.7$

$y = 2.2x + 1.7$

$= \underline{y = 2.2x - 1.7}$

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2} + h$, where h is small. [1]

$\delta y = \left(\frac{dy}{dx}\right)_{x = x} \cdot \delta x$ ③

$\frac{24 + 6\sqrt{2} - \ln(2)}{(2\sqrt{2}+1)^2} \cdot h$

Examiner comments

① The candidate produces a correct expression for the differentiation of the quotient including the differentiation of the ln term.

② Although the candidate attempts to find the gradient of the curve at the given point, there is no attempt to use it to find the gradient of the normal. The value of y at the given point is incorrect. No further marks are awarded.

Mark for (a) = 3 out of 6

③ The candidate uses their gradient from (a) correctly to find the approximate change. Although the gradient is incorrect, a ‘follow through’ mark is awarded.

Mark for (b) = 1 out of 1

Total mark awarded = 4 out of 7

How the candidate could have improved their answer

- The candidate needed to read the question carefully as it appeared that they attempted to find the equation of the tangent rather than the normal.
- Working needed to be set out more clearly after the differentiation as there appeared to be no attempt at finding the value of y. The candidate needed to check their working as they incorrectly evaluated the second term in the numerator of the derivative.

Example Candidate Response – low

7 A curve has equation $y = \frac{\ln(3x^2-5)}{2x+1}$ for $3x^2 > 5$.

(a) Find the equation of the normal to the curve at the point where $x = \sqrt{2}$. [6]

$$\frac{dy}{dx} = \frac{Vdu - Udv}{V^2}$$

$$= \frac{2x+1 \cdot \left(\frac{6x}{3x^2-5}\right) - \ln(3x^2-5) \cdot 2}{(2x+1)^2}$$ ①

$$= \frac{\frac{6x^2+6x}{3x^2-5} - 2\ln(3x^2-5)}{(2x-1)^2}$$

$$\frac{\frac{6x^2+6 - 6x^2-10(\ln(3x-5)}{3x^2-5}}{(2x-1)^2}$$

$$\frac{-4\ln 3x - 5}{3x^2-5} \cdot (2x-1)^2 = 0$$ ②

$$\frac{\cancel{-4\ln 3(\sqrt{2})-5}\ -4\ln(-5+3\sqrt{2})}{3(\sqrt{2})-5} \cdot 2(\sqrt{2})-1 = 0$$

$$\frac{= 3.59}{-5+3\sqrt{2}} \cdot -1+2\sqrt{2}$$

$$= -1.91$$

$U = \ln(3x^2-5)$

$dU = \frac{1}{3x^2-5} \cdot 6x$

$= \frac{6x}{3x^2-5}$

$V = 2x+1$

$dV = 2$

$$\frac{-4\ln 3x-5}{\cancel{(3x^2-5)}} \cdot \frac{(\cancel{2x}-1)^2}{1}$$

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2}+h$, where h is small. [1]

$$\delta y = \left.\frac{dy}{dx}\right|_{x=k} \delta x$$

$$\cancel{\Rightarrow}\ \underline{\underline{\delta y = 1.91h}}$$ ③

Examiner comments

① The candidate produces a correct expression for the differentiation of the quotient including the ln term. Three marks are awarded.

② The candidate does not use the given value of x to find the gradient at that point. There is no attempt to find the value of y at the given point and no attempt at either the gradient or the equation of the normal. No further marks are awarded.

Mark for (a) = 3 out of 6

③ A mark is awarded if the candidate uses their tangent gradient from (a) correctly. In this case, the candidate omits the negative sign.

Mark for (b) = 0 out of 1

Total mark awarded = 3 out of 7

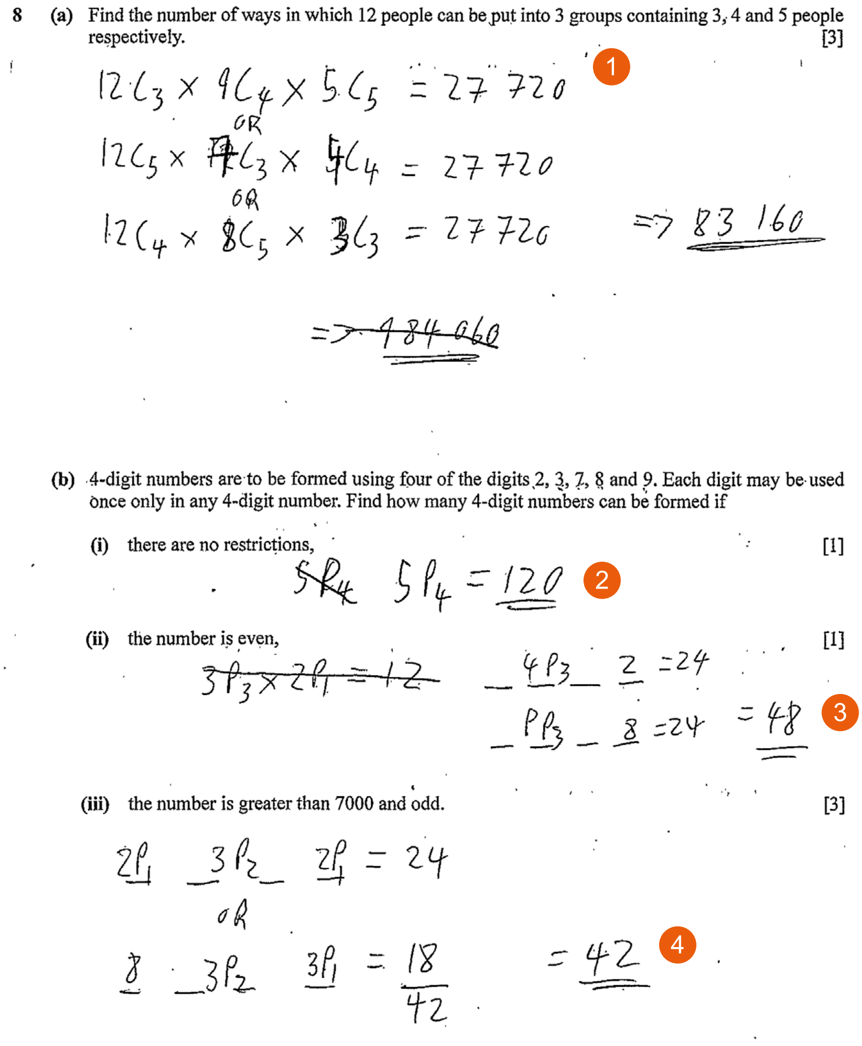
How the candidate could have improved their answer

The candidate needed to decide exactly what process was needed to solve the problem. They correctly identified that they needed to differentiate, but were not able to progress further.

Common mistakes candidates made in this question

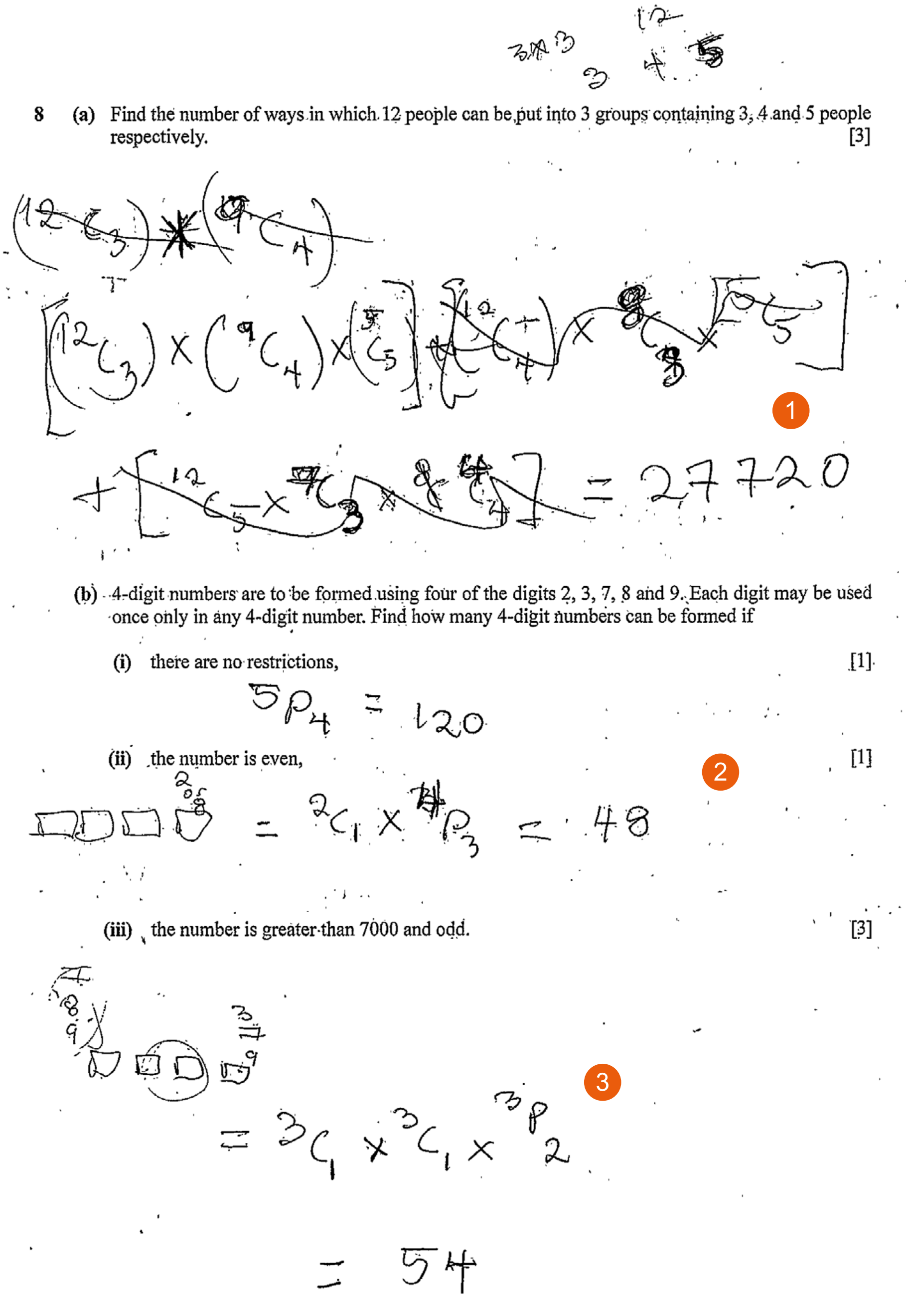
- Finding the equation of the tangent rather than the normal was a common error, highlighting the fact that candidates should check that they have answered the question set.
- Errors in evaluating or simplifying the derivative using the given value of x were common and lost accuracy marks. Rounding errors in the final answer when using decimals rather than surds were also common.

Question 8

Example Candidate Response – high	Examiner comments
 **8 (a)** Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively. [3] $^{12}C_3 \times {}^9C_4 \times {}^5C_5 = 27720$ ① OR $^{12}C_5 \times {}^7C_3 \times {}^4C_4 = 27720$ OR $^{12}C_4 \times {}^8C_5 \times {}^3C_3 = 27720$ ⇒ 83 160 ~~⇒ 484 060~~	① The candidate uses two correct combinations in a product of three combinations and is awarded two marks. However, they unnecessarily repeat the process and give an incorrect final answer. Mark for (a) = 2 out of 3
(b) 4-digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if **(i)** there are no restrictions, [1] ~~$^5P_4$~~ $^5P_4 = 120$ ②	② The candidate supplies a correct answer. Mark for (b)(i) = 1 out of 1
(ii) the number is even, [1] ~~$^3P_3 \times {}^2P_1 = 12$~~ _ $^4P_3$ _ 2 = 24 _ P_3 _ 8 = 24 = 48 ③	③ The candidate supplies a correct answer. Mark for (b)(ii) = 1 out of 1
(iii) the number is greater than 7000 and odd. [3] $^2P_1$ _ $^3P_2$ _ $^2P_1$ = 24 or 8 _ $^3P_2$ $^3P_1$ = 18 42 = 42 ④	④ The candidate supplies a correct answer and is awarded three marks. Mark for (b)(iii) = 3 out of 3
	Total mark awarded = 7 out of 8

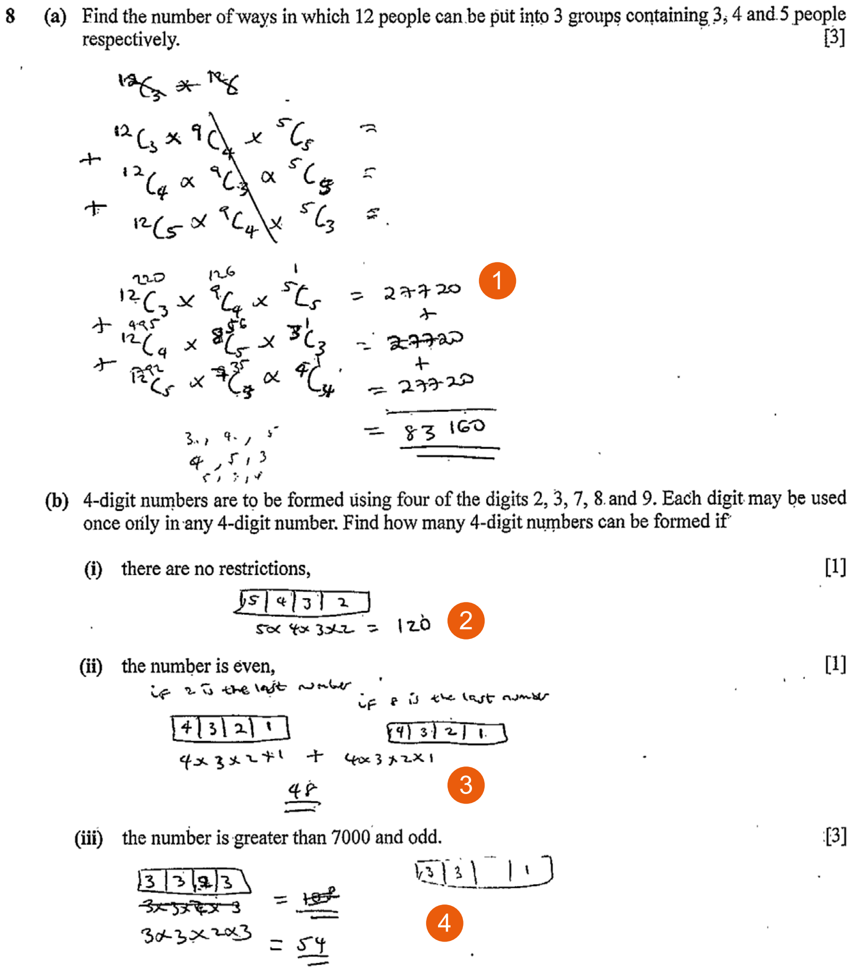
How the candidate could have improved their answer

(b)(iii) The candidate could have detailed what each of the two numbers obtained represented, for example, ‘starts with a 7 or a 9’ and ‘starts with an 8’.

Example Candidate Response – middle	Examiner comments
 12 3 4 5 **8 (a)** Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively. [3] ~~$(^{12}C_3)\times(^{9}C_4)$~~ $[(^{12}C_3)\times(^{9}C_4)\times(^{5}C_5)]$ ~~$+[^{12}C_4\times{}^{8}C_3\times{}^{5}C_5]$~~ ~~$+[^{12}C_5\times{}^{7}C_3\times{}^{4}C_4]$~~ $= 27720$ ①	① The candidate provides a correct solution for three marks. Mark for (a) = 3 out of 3
(b) 4-digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if **(i)** there are no restrictions, [1] $^5P_4 = 120$ **(ii)** the number is even, [1] ② $\square\square\square\overset{2,8}{\square} = {}^2C_1\times{}^4P_3 = 48$	② The candidate answers (b)(i) and (b)(ii) correctly and is awarded two marks. Mark for (b)(i) = 1 out of 1 Mark for (b)(ii) = 1 out of 1
(iii) the number is greater than 7000 and odd. [3] $\overset{7,8,9}{\square}\square\square\overset{3,7,9}{\square}$ ③ $= {}^3C_1\times{}^3C_1\times{}^3P_2$ $= 54$	③ The candidate does not consider the problem to be made up of more than one part and no further marks are awarded. Mark for (b)(iii) = 0 out of 3 **Total mark awarded = 5 out of 8**

How the candidate could have improved their answer

(b)(iii) The candidate needed to consider the problem as being made up of a number of parts, for example, 'starting with a 7 or a 9' and 'starting with an 8'.

Example Candidate Response – low	Examiner comments
 **8** **(a)** Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively. [3] $~~{}^{12}C_3 * {}^{12}C~~$ $^{12}C_3 \times {}^9C_4 \times {}^5C_5 =$ $+ \; {}^{12}C_4 \times {}^9C_3 \times {}^5C_5 =$ $+ \; {}^{12}C_5 \times {}^9C_4 \times {}^5C_3 =$ $\overset{220}{{}^{12}C_3} \times \overset{126}{{}^9C_4} \times \overset{1}{{}^5C_5} = 27720$ $+$ $+ \; \overset{495}{{}^{12}C_4} \times \overset{56}{{}^8C_5} \times \overset{1}{{}^3C_3} = ~~27720~~$ $+$ $+ \; \overset{792}{{}^{12}C_5} \times \overset{35}{{}^7C_3} \times \overset{1}{{}^4C_4} = 27720$ $= \underline{\underline{83\,160}}$ 3, 4, 5 4, 5, 3 5, 3, 4	① The candidate has two correct combinations in a product of three combinations and is awarded two marks. However, they unnecessarily repeat the process and give an incorrect final answer. Mark for (a) = 2 out of 3
(b) 4-digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if **(i)** there are no restrictions, [1] | 5 | 4 | 3 | 2 | $5 \times 4 \times 3 \times 2 = 120$	② The candidate produces the correct answer. Mark for (b)(i) = 1 out of 1
(ii) the number is even, [1] if 2 is the last number | 4 | 3 | 2 | 1 | if 8 is the last number | 4 | 3 | 2 | 1 | $4 \times 3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1$ $\underline{\underline{48}}$	③ The candidate produces another correct answer. Mark for (b)(ii) = 1 out of 1
(iii) the number is greater than 7000 and odd. [3] | 3 | 3 | 2 | 3 | $~~3 \times 3 \times 4 \times 3~~ = ~~108~~$ $3 \times 3 \times 2 \times 3 = \underline{\underline{54}}$ | 3 | 3 | | 1 |	④ The candidate does not consider that the problem is made up of more than one part and they are awarded no marks. Mark for (b)(iii) = 0 out of 3
	Total mark awarded = 4 out of 8

How the candidate could have improved their answer

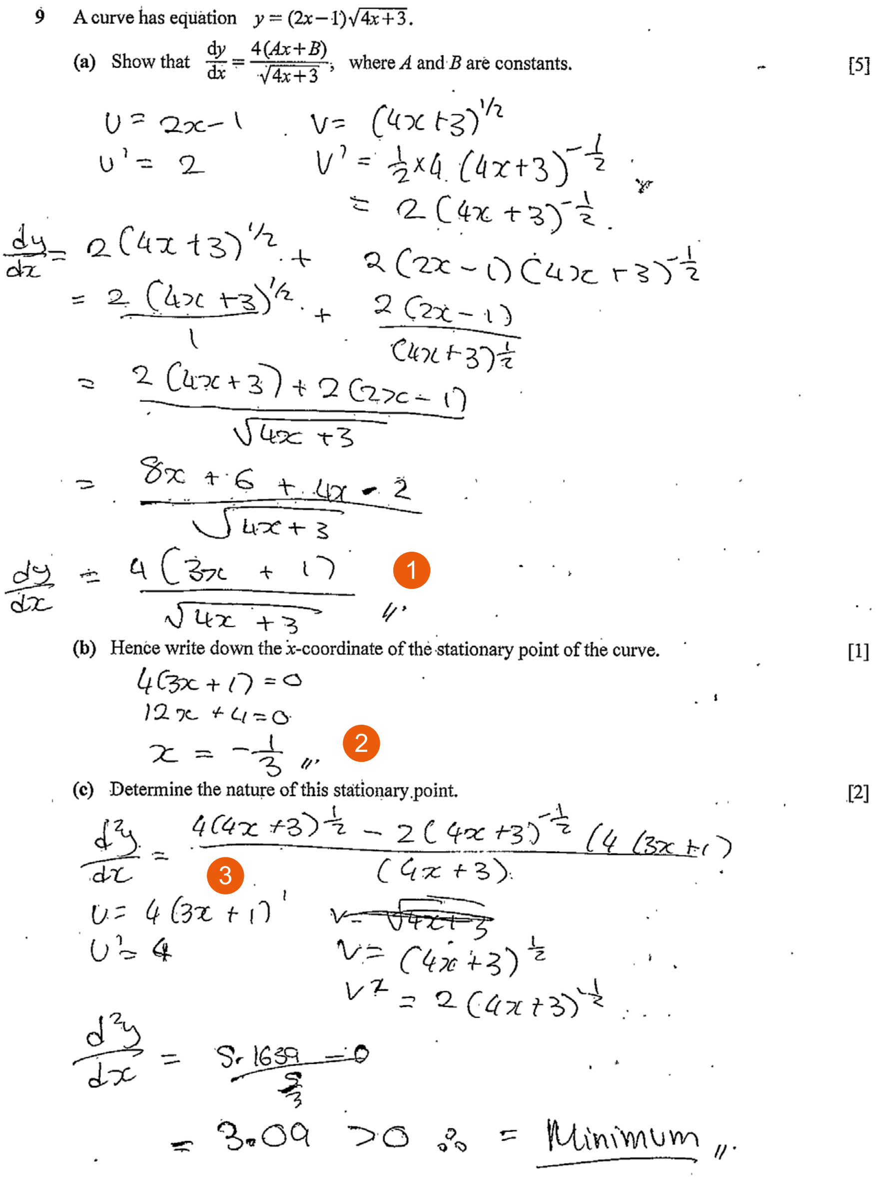
(b)(iii) The candidate needed to consider the problem as being made up of a number of parts, for example, ‘starting with a 7 or a 9’ and ‘starting with an 8’.

Common mistakes candidates made in this question

- **(a)** Many candidates obtained multiples of 27720 , not realising that all the combinations were covered by 27720 and that order was not important.
- **(b)(iii)** Many candidates did not consider that the problem was made up of a number of parts.

Question 9

Example Candidate Response – high



9 A curve has equation $y = (2x-1)\sqrt{4x+3}$.

(a) Show that $\frac{dy}{dx} = \frac{4(Ax+B)}{\sqrt{4x+3}}$, where A and B are constants. [5]

$U = 2x-1 \quad V = (4x+3)^{1/2}$

$U' = 2 \quad V' = \frac{1}{2} \times 4\,(4x+3)^{-\frac{1}{2}}$

$= 2(4x+3)^{-\frac{1}{2}}$

$\frac{dy}{dx} = 2(4x+3)^{1/2} + 2(2x-1)(4x+3)^{-\frac{1}{2}}$

$= \frac{2(4x+3)^{1/2}}{1} + \frac{2(2x-1)}{(4x+3)^{\frac{1}{2}}}$

$= \frac{2(4x+3)+2(2x-1)}{\sqrt{4x+3}}$

$= \frac{8x+6+4x-2}{\sqrt{4x+3}}$

$\frac{dy}{dx} = \frac{4(3x+1)}{\sqrt{4x+3}}$ // ①

(b) Hence write down the x-coordinate of the stationary point of the curve. [1]

$4(3x+1) = 0$

$12x + 4 = 0$

$x = -\frac{1}{3}$ // ②

(c) Determine the nature of this stationary point. [2]

$\frac{d^2y}{dx} = \frac{4(4x+3)^{\frac{1}{2}} - 2(4x+3)^{-\frac{1}{2}}(4(3x+1))}{(4x+3)}$ ③

$U = 4(3x+1)^{1} \quad$ ~~$v = \sqrt{4x+3}$~~

$U' = 4 \quad V = (4x+3)^{\frac{1}{2}}$

$V' = 2(4x+3)^{-\frac{1}{2}}$

$\frac{d^2y}{dx} = \frac{5.1639 - 0}{\frac{5}{3}}$

$= 3.09 > 0 \therefore =$ Minimum //

Examiner comments

① The candidate produces a correct solution and is awarded full marks.

Mark for (a) = 5 out of 5

② The candidate again produces a correct solution.

Mark for (b) = 1 out of 1

③ The candidate makes an error when calculating the second derivative. Although the stationary point is a minimum this answer is obtained from incorrect work. No marks are awarded.

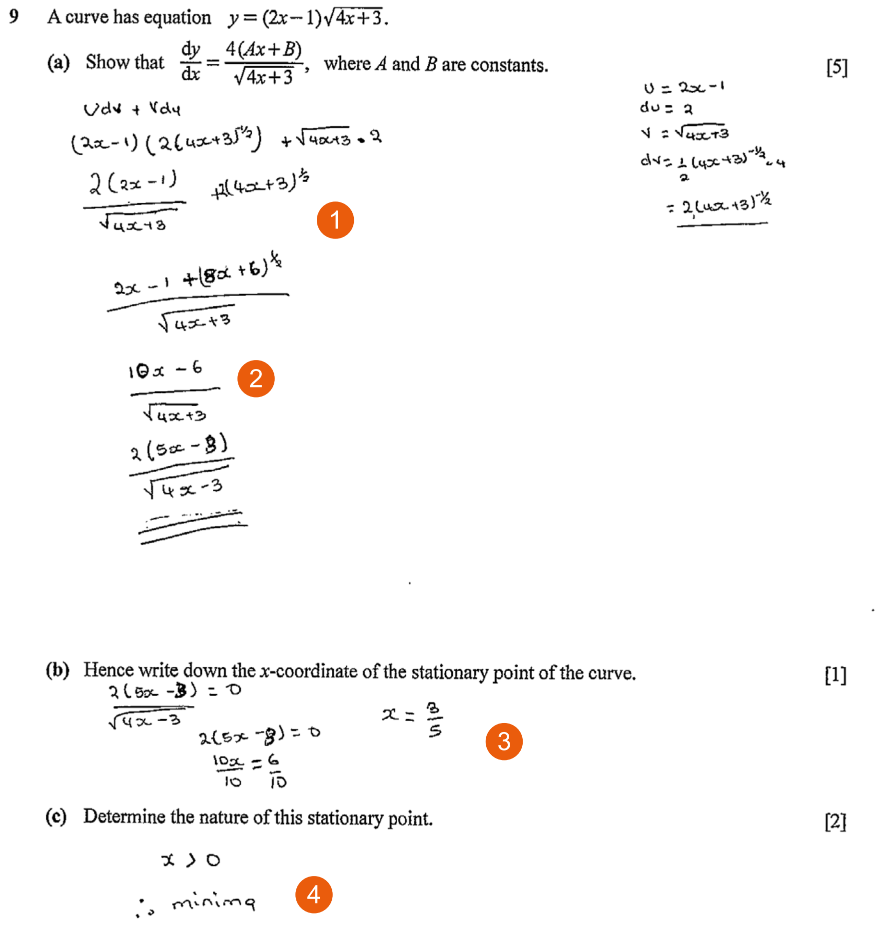
Mark for (c) = 0 out of 2

Total mark awarded = 6 out of 8

How the candidate could have improved their answer

(c) The candidate could have used a less lengthy method in attempting to find the nature of the stationary point. Considering either the gradient or the values of y at values either side of the stationary point would have been less involved, provided the results were displayed in a clear fashion.

Example Candidate Response – middle



9 A curve has equation $y = (2x-1)\sqrt{4x+3}$.

(a) Show that $\frac{dy}{dx} = \frac{4(Ax+B)}{\sqrt{4x+3}}$, where A and B are constants. [5]

$u = 2x - 1$

$du = 2$

$v = \sqrt{4x+3}$

$dv = \frac{1}{2}(4x+3)^{-1/2} \cdot 4$

$= 2(4x+3)^{-1/2}$

$udv + vdu$

$(2x-1)(2(4x+3)^{-1/2}) + \sqrt{4x+3} \cdot 2$

$\frac{2(2x-1)}{\sqrt{4x+3}} + 2(4x+3)^{1/2}$ ①

$\frac{2x-1+(8x+6)^{1/2}}{\sqrt{4x+3}}$

$\frac{10x-6}{\sqrt{4x+3}}$ ②

$\frac{2(5x-3)}{\sqrt{4x-3}}$

(b) Hence write down the x-coordinate of the stationary point of the curve. [1]

$\frac{2(5x-3)}{\sqrt{4x-3}} = 0$

$2(5x-3) = 0$

$\frac{10x}{10} = \frac{6}{10}$

$x = \frac{3}{5}$ ③

(c) Determine the nature of this stationary point. [2]

$x > 0$

$\therefore$ minima ④

Examiner comments

① The candidate differentiates the equation correctly, which is awarded three marks.

② The candidate does not use a valid method to simplify their expression in order to obtain the answer in the required form.

Mark for (a) = 3 out of 5

③ The candidate equates the numerator of their derivative to zero and solves it correctly. This is awarded a follow through mark.

Mark for (b) = 1 out of 1

④ The candidate supplies insufficient evidence of a valid method to determine the nature of the stationary point and no marks are awarded.

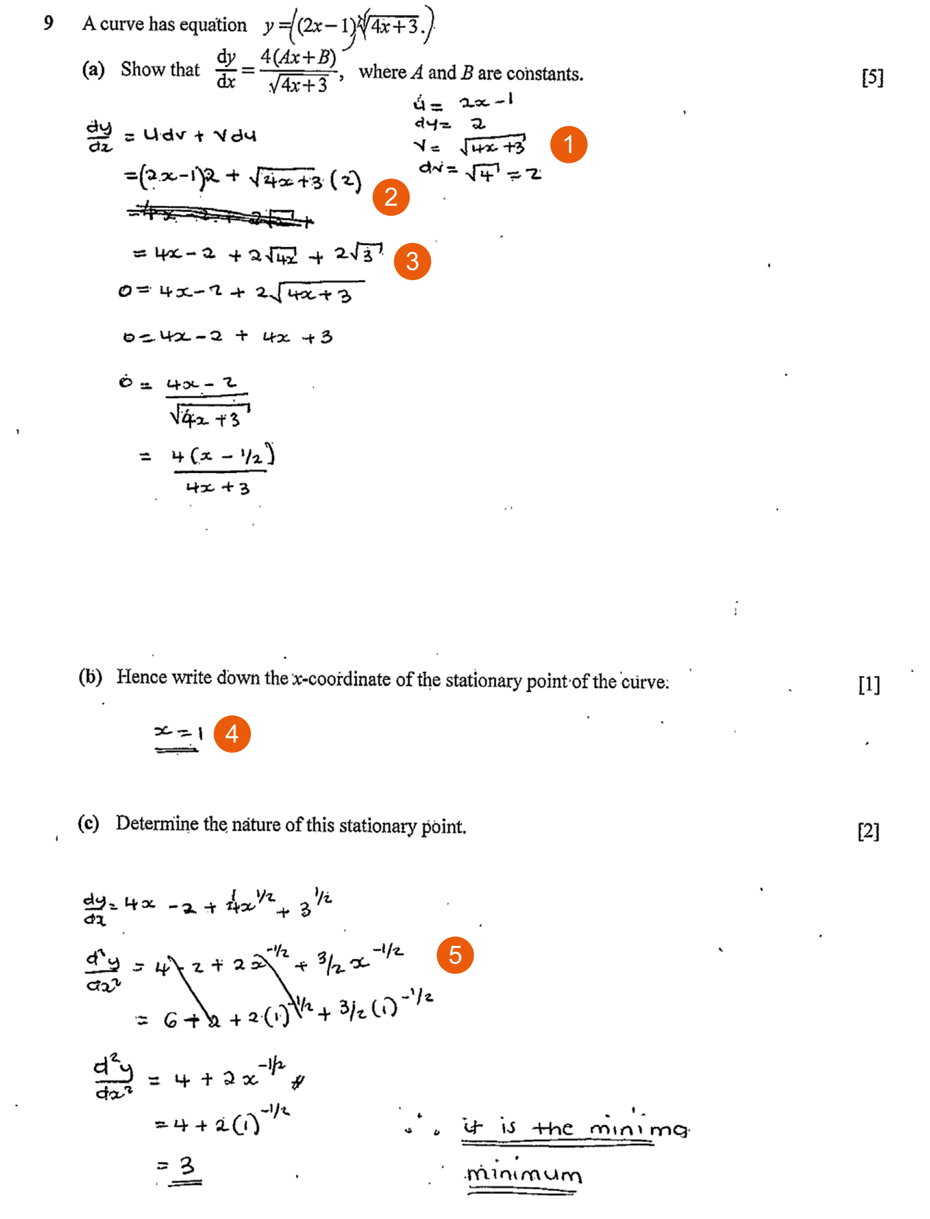
Mark for (c) = 0 out of 2

Total mark awarded = 4 out of 8

How the candidate could have improved their answer

(a) Correct simplification and manipulation of the algebraic expression was required. The candidate needed to be aware of the different ways of determining a stationary point and show one of them with sufficient working.

Example Candidate Response – low



9 A curve has equation $y = (2x-1)\sqrt{4x+3}$.

(a) Show that $\frac{dy}{dx} = \frac{4(Ax+B)}{\sqrt{4x+3}}$, where A and B are constants. [5]

$u = 2x - 1$

$du = 2$

$v = \sqrt{4x+3}$ ①

$dv = \sqrt{4} = 2$

$\frac{dy}{dx} = u\,dv + v\,du$

$= (2x-1)2 + \sqrt{4x+3}\,(2)$ ②

~~=4x−2+2√ +~~

$= 4x - 2 + 2\sqrt{4x} + 2\sqrt{3}$ ③

$0 = 4x - 2 + 2\sqrt{4x+3}$

$0 = 4x - 2 + 4x + 3$

$0 = \frac{4x-2}{\sqrt{4x+3}}$

$= \frac{4(x - 1/2)}{4x+3}$

(b) Hence write down the x-coordinate of the stationary point of the curve. [1]

$x = 1$ ④

(c) Determine the nature of this stationary point. [2]

$\frac{dy}{dx} = 4x - 2 + 4x^{1/2} + 3^{1/2}$

$\frac{d^2y}{dx^2} = 4 - 2 + 2x^{-1/2} + 3/2\,x^{-1/2}$ ⑤

$= 6 + 2 + 2(1)^{-1/2} + 3/2(1)^{-1/2}$

$\frac{d^2y}{dx^2} = 4 + 2x^{-1/2}$

$= 4 + 2(1)^{-1/2}$

$= 3$

∴ it is the minima

minimum

Examiner comments

① The candidate does not differentiate the square root of $(4x+3)$ correctly.

② The candidate attempts to differentiate the product. All the terms are correct apart from the differential of the square root term. Two marks are awarded.

③ The candidate cannot be awarded any further marks as their expression is not in a suitable form for simplification to the given form.

Mark for (a) = 2 out of 5

④ The candidate's result is not the solution that should be obtained when the numerator in (a) is equated to zero so a follow through mark cannot be awarded.

Mark for (b) = 0 out of 1

⑤ Since the candidate is using a first derivative which is not of the correct form, no method mark can be awarded.

Mark for (c) = 0 out of 2

Total mark awarded = 2 out of 8

How the candidate could have improved their answer

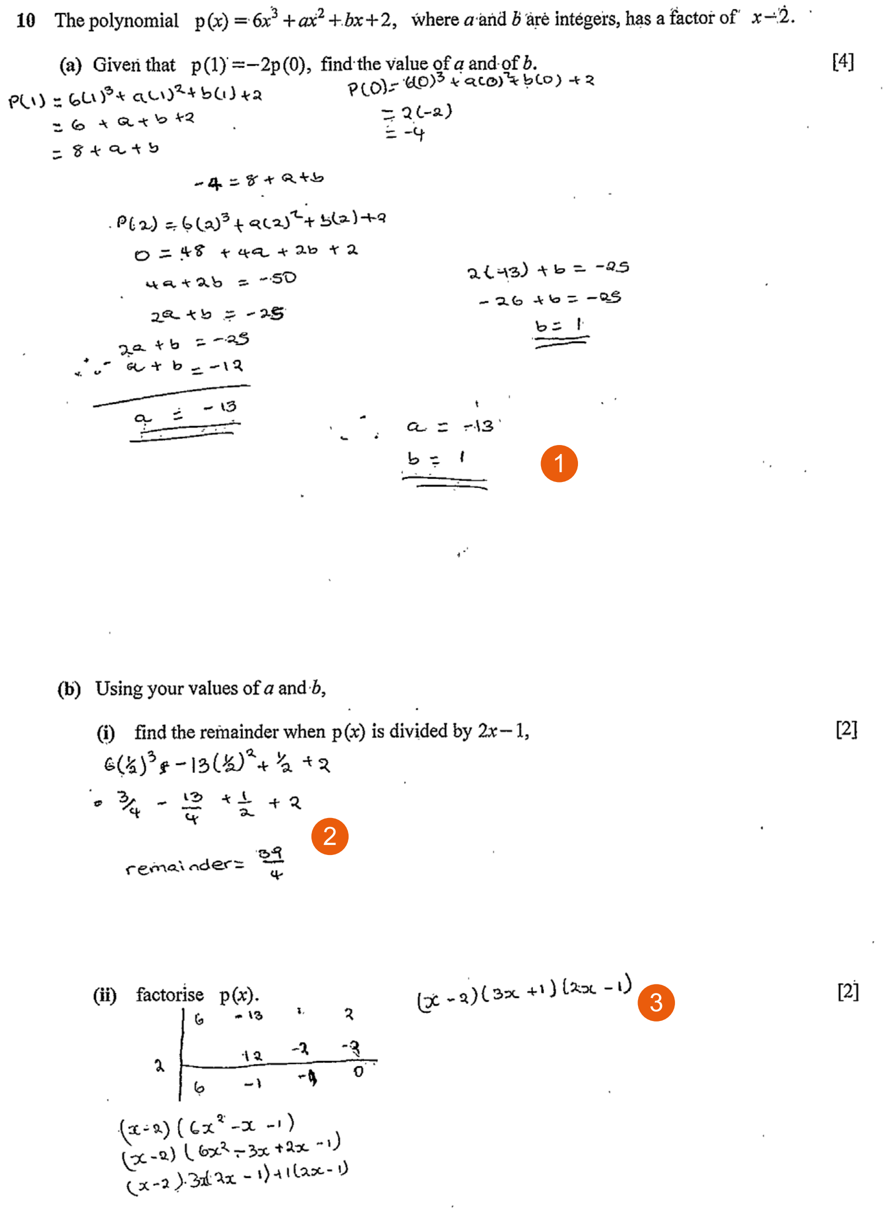
- The candidate could have differentiated the term $\sqrt{4x+3}$ by re-writing as $(4x+3)^{\frac{1}{2}}$ and using the chain rule.
- The candidate should be aware of the different ways of determining a stationary point and show one of them with sufficient working.

Common mistakes candidates made in this question

- **(a)** Most candidates realised that they needed to differentiate a product, however errors often occurred in the differentiation of $\sqrt{4x+3}$ and subsequent simplification.
- **(c)** It was not sufficient to state that the stationary point was a minimum without providing sufficient evidence. Very few candidates showed sufficient working if they were looking at the gradient or y value either side of the stationary point. Candidates also needed to ensure that they were not considering a value of $x < -0.75$. Errors in calculating the value of the second derivative for the appropriate value of x were common.

Question 10

Example Candidate Response – high



10 The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $x - 2$.

(a) Given that $p(1) = -2p(0)$, find the value of a and of b. [4]

$P(1) = 6(1)^3 + a(1)^2 + b(1) + 2$

$= 6 + a + b + 2$

$= 8 + a + b$

$P(0) = 6(0)^3 + a(0)^2 + b(0) + 2$

$= 2(-2)$

$= -4$

$-4 = 8 + a + b$

$P(2) = 6(2)^3 + a(2)^2 + b(2) + 2$

$0 = 48 + 4a + 2b + 2$

$4a + 2b = -50$

$2a + b = -25$

$2a + b = -25$

$a + b = -12$

$\underline{\underline{a = -13}}$

$2(-13) + b = -25$

$-26 + b = -25$

$\underline{\underline{b = 1}}$

$a = -13$

$\underline{\underline{b = 1}}$

①

(b) Using your values of a and b,

(i) find the remainder when $p(x)$ is divided by $2x - 1$, [2]

$6(\frac{1}{2})^3 - 13(\frac{1}{2})^2 + \frac{1}{2} + 2$

$\frac{3}{4} - \frac{13}{4} + \frac{1}{2} + 2$

remainder $= \frac{39}{4}$

②

(ii) factorise $p(x)$. [2]

	6	-13	1	2
2		12	-2	-2
	6	-1	-1	0

$(x-2)(6x^2 - x - 1)$

$(x-2)(6x^2 - 3x + 2x - 1)$

$(x-2)\,3x(2x-1) + 1(2x-1)$

$(x-2)(3x+1)(2x-1)$

③

Examiner comments

① The candidate gives a completely correct solution.

Mark for (a) = 4 out of 4

② The candidate uses a correct method to find the remainder for which a mark is awarded, but there is an error in the evaluation.

Mark for (b)(i) = 1 out of 2

③ The candidate correctly divides the polynomial by $x - 2$ to obtain the quadratic factor, which is then factorised. The product of the three correct factors is shown.

Mark for (b)(ii) = 2 out of 2

Total mark awarded = 7 out of 8

How the candidate could have improved their answer

(b)(i) A check of the evaluation should have resulted in the correct response of 0. The candidate should have been alerted to an error as $2x - 1$ appeared as a factor in **(b)(ii)**.

Example Candidate Response – middle

10 The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $x - 2$.

(a) Given that $p(1) = -2p(0)$, find the value of a and of b. [4]

$6(1)^3 + a(1)^2 + b(1) + 2 = 2(6(0)^3 + a(0)^2 + b(0) + 2)$

$6 + a + b + 2 = 2(2)$

$a + b = -4$ ①

$6(2)^3 + a(2)^2 + b(2) + 2 = 0$

$48 + 4a + 2b + 2 = 0$

$4a + 2b = \cancel{-46}\ -50$

$\cancel{4\left(\frac{-4}{b}\right) + 2b} = -50$

-16

$a + 17 = -4$ ③

$a = -21$

~~p(-2) = 6(~~

~~a = -4~~

~~b~~

$a + b = -4$

$4a + 2b = -50$ ②

$4a + 4b = -16$

$4a + 2b = -50$

$2b = 34$

$b = 17$

$a = -21$

$b = 17$

(b) Using your values of a and b,

(i) find the remainder when $p(x)$ is divided by $2x - 1$, [2]

~~$p\left(\frac{1}{2}\right)$~~ $6x^3 - 21x^2 + 17x + 2$

$p\left(\frac{1}{2}\right) = 6 \times \left(\frac{1}{2}\right)^3 - 21\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) + 2$

$= \frac{3}{4} - \frac{21}{4} + \frac{17}{2} + 2$ ④

$= \frac{43}{4} = 6$

(ii) factorise $p(x)$. [2]

$$\begin{array}{r|l} & 6x^2 - 9x - 1 \\ \hline x - 2 & 6x^3 - 21x^2 + 17x + 2 \\ & -\ 6x^2 - 12x^2 \\ \hline & -9x^2 + 17x \\ & -\ -9x^2 + 18x \\ \hline & -x + 2 \\ & -x + 2 \\ \hline & \end{array}$$

$p(x) = (x - 2)(6x^2 - 9x - 1)$

$= (x - 2)(6x^2 - 9x - 1)$

⑤

Examiner comments

① The candidate misses the minus sign on the right hand side leading to an incorrect equation for which no marks can be awarded.

② The candidate forms and simplifies the second equation correctly.

③ The candidate solves their simultaneous equations to obtain a value for a and for b.

Mark for (a) = 2 out of 4

④ The candidate is awarded a method mark for substituting $x = \frac{1}{2}$ into their polynomial to find the remainder but the answer is incorrect.

Mark for (b)(i) = 1 out of 2

⑤ The candidate is awarded a method mark for dividing their polynomial by $x - 2$ in order to obtain the associated quadratic factor. No further mark can be awarded even if the candidate attempts to factorise the quadratic factor because it is incorrect.

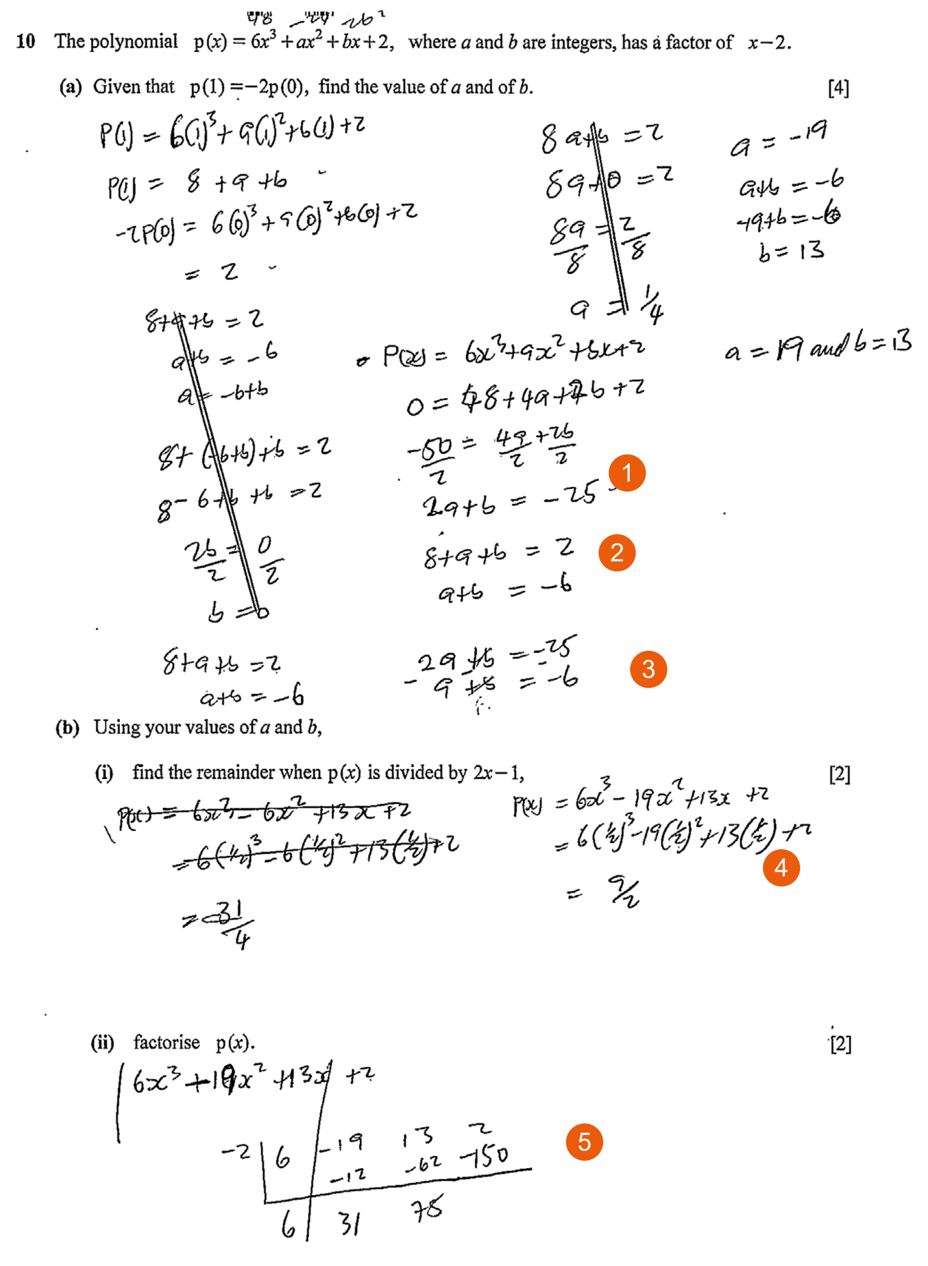
Mark for (b)(ii) = 1 out of 2

Total mark awarded = 4 out of 8

How the candidate could have improved their answer

(a) Checking the accuracy of the calculations would have ensured that the correct values of a and b were used in the rest of the question.

Example Candidate Response – low



48 −24 2b²

10 The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $x - 2$.

(a) Given that $p(1) = -2p(0)$, find the value of a and of b. [4]

$P(1) = 6(1)^3 + a(1)^2 + b(1) + 2$

$P(1) = 8 + a + b$

$-2P(0) = 6(0)^3 + a(0)^2 + b(0) + 2$

$= 2$

~~$8+a+b = 2$~~

~~$a+b = -6$~~

~~$a = -b+b$~~

~~$8+(-b+b)+b = 2$~~

~~$8-6+b+b = 2$~~

~~$\frac{2b}{2} = \frac{0}{2}$~~

~~$b = 0$~~

~~$8a+b = 2$~~

~~$8a+0 = 2$~~

~~$\frac{8a}{8} = \frac{2}{8}$~~

~~$a = \frac{1}{4}$~~

$P(x) = 6x^3 + ax^2 + bx + 2$

$0 = 48 + 4a + 2b + 2$

$\frac{-50}{2} = \frac{4a}{2} + \frac{2b}{2}$

$2a + b = -25$ ①

$8 + a + b = 2$ ②

$a + b = -6$

$2a + b = -25$

$-\ a + b = -6$ ③

$8 + a + b = 2$

$a + b = -6$

$a = -19$

$a + b = -6$

$-19 + b = -6$

$b = 13$

$a = 19$ and $b = 13$

(b) Using your values of a and b,

(i) find the remainder when $p(x)$ is divided by $2x - 1$, [2]

~~$p(x) = 6x^3 - 6x^2 + 13x + 2$~~

~~$= 6(\frac{1}{2})^3 - 6(\frac{1}{2})^2 + 13(\frac{1}{2}) + 2$~~

$= \frac{31}{4}$

$P(x) = 6x^3 - 19x^2 + 13x + 2$

$= 6(\frac{1}{2})^3 - 19(\frac{1}{2})^2 + 13(\frac{1}{2}) + 2$ ④

$= \frac{9}{2}$

(ii) factorise $p(x)$. [2]

$6x^3 + 19x^2 + 13x + 2$

$$\begin{array}{r|r|rrr} -2 & 6 & -19 & 13 & 2 \\ & & -12 & -62 & -150 \\ \hline & 6 & 31 & 78 & \end{array}$$ ⑤

Examiner comments

① The candidate forms and simplifies a correct equation. One mark is awarded.

② Although the left hand side of the equation is correct , the candidate does not multiply the right-hand side by -2 leading to an incorrect equation.

③ The candidate solves their simultaneous equations to find a value for a and for b. A method mark is awarded, but the answer is incorrect.

Mark for (a) = 2 out of 4

④ The candidate substitutes $x = \frac{1}{2}$ into their polynomial in order to find the remainder and is awarded the method mark, but the answer is incorrect.

Mark for (b)(i) = 1 out of 2

⑤ The candidate makes an attempt at finding the quadratic factor by using synthetic division, but this is incomplete and is not awarded the method mark.

Mark for (b)(ii) = 0 out of 2

Total mark awarded = 3 out of 8

How the candidate could have improved their answer

- **(a)** Checking the accuracy of the calculations would have ensured that the correct values of a and b were used in the rest of the question.
- **(b)(ii)** Use of algebraic long division may have been awarded a method mark.

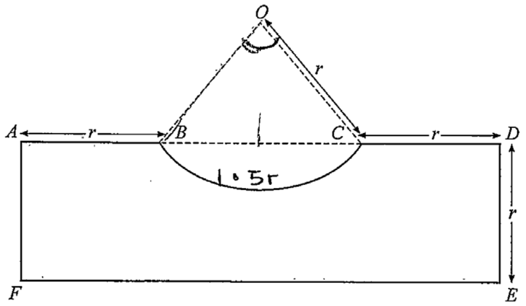
Common mistakes candidates made in this question

- **(a)** Most candidates were able to obtain the equation using the factor theorem and $x - 2$. More errors occurred when obtaining the second equation, with candidates often omitting either the -2 or $-2\mathrm{p}$ from the right hand side of the given relationship.
- **(b)(i)**, **b(ii)** Use of synthetic division often led candidates to make errors, especially when dividing by $2x - 1$.

Question 11

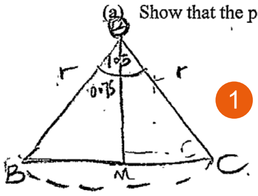
Example Candidate Response – high

11 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows the rectangle *ADEF*, where $AF = DE = r$. The points *B* and *C* lie on *AD* such that $AB = CD = r$. The curve *BC* is an arc of the circle, centre *O*, radius *r* and has a length of 1.5*r*.

(a) Show that the perimeter of the shaded region is $(7.5 + 2\sin 0.75)r$. [5]



①

$\sin(0.75) = \frac{MB}{OB} = \frac{MB}{r}$

$MB = r\sin(0.75)$ ②

$MB + MC = 2r\sin(0.75)$

Perimeter of Shaded region $= AB + \text{curve} + CD + DE + EF + AF$

$= r + 1.5r + r + r + 2r + 2r\sin 0.75 + r$

$= 6r + 1.5r + 2r\sin(0.75)$

$EF = 2r + 2r\sin 0.75$

$= 7.5r + 2r\sin(0.75)$ ③

$= (7.5 + 2\sin(0.75))r$

Length of arc $= r\theta$

$r\theta = 1.5r$

$\theta = 1.5$ ④

Examiner comments

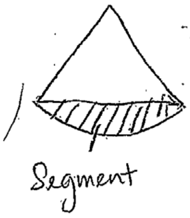
① The candidate draws an explanatory diagram which makes it clear what is being referred to.

② The candidate correctly calculates the length of BC.

③ The candidate calculates the perimeter is correctly with sufficient detail shown to be awarded all the marks. This is needed, as the answer is given in the question.

④ The candidate identifies the angle of 1.5 radians and is awarded one mark.

Mark for (a) = 5 out of 5

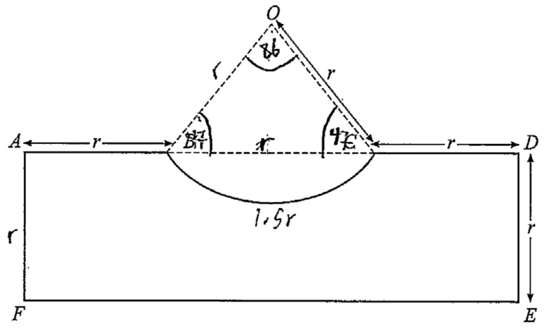
Example Candidate Response – high, continued	**Examiner comments**
(b) Find the area of the shaded region, giving your answer in the form kr^2, where k is a constant correct to 2 decimal places. [4]  ∴ Area of Segment = Area of Sector – Area of triangle $\theta = 1.5^c$ $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ $= \frac{1}{2}\times r^2\times 1.5 - \frac{1}{2}\times r^2\times \sin(1.5)$ ⑤ $= 0.75r^2 - 0.5r^2 = 0.25r^2$	⑤ The candidate calculates the area of the segment correctly.
Area of shaded region = Area of rectangle – area of segment $= DE \times EF$ – area of segment ⑥ $= r(2r + 2r\sin 0.75) - (0.25r^2)$ $= r(2r + 1.4r) - 0.25r^2$ $= r(3.4r) - 0.25r^2$ $= 3.4r^2 - 0.25r^2$	⑥ The candidate is awarded a method mark for the correct plan and another for the correct calculation of the area of the rectangle.
$= 3.15r^2$ $k\cancel{r^2} = 3.15\cancel{r^2}$ ⑦ $k = 3.15$	⑦ The candidate works with rounded values and this leads to an inaccurate answer. No further marks are awarded. Mark for (b) = 3 out of 4
	Total mark awarded = 8 out of 9

How the candidate could have improved their answer

(b) The candidate needed to work with more accurate figures. It is preferable to work with calculations correct to 4 significant figures and then round the final answer to 3 significant figures unless a different level of accuracy is specified.

Example Candidate Response – middle

11 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows the rectangle *ADEF*, where $AF = DE = r$. The points *B* and *C* lie on *AD* such that $AB = CD = r$. The curve *BC* is an arc of the circle, centre *O*, radius *r* and has a length of 1.5*r*.

(a) Show that the perimeter of the shaded region is $(7.5 + 2\sin 0.75)r$. [5]

$s = r\theta$

$1.5r = r\theta$

$\theta = 1.5$ ①

~~$\cos 47 = \frac{r}{r}$~~

$\sin 43^\circ = \frac{0}{r}$ ②

$r \cdot \sin 43^\circ = 0$

$2r\sin 43 = BC$

$42.97 \times \frac{\pi}{180} = 0.75$

~~$1.5r + 6r + 4r\sin 43$~~

~~$\sin 0.75$~~

$1.5r + 6r + 2\sin 0.75$

$7.5r + 2\sin 0.75$

$\underline{(7.5 + 2\sin 0.75)r}$

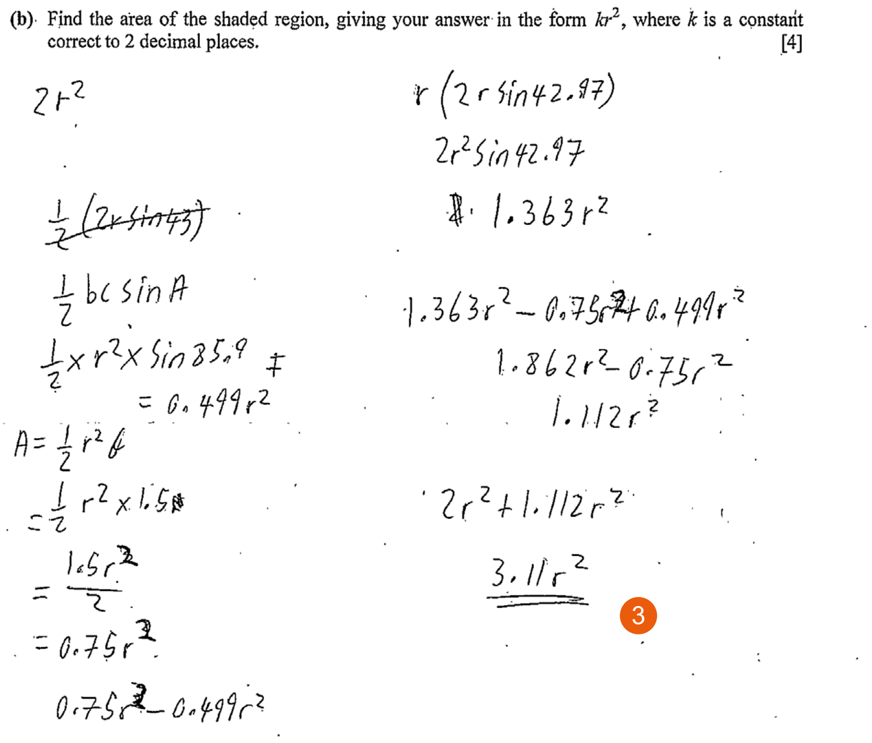
Examiner comments

① The angle, in radians, is identified correctly by the candidate and one mark is awarded.

② The question asks the candidate to show the given result. This means that they are expected to provide sufficient detail to do this. *O* is not identified as being the midpoint of *BC*. The length of *BC* can easily be deduced from the given result, hence the need to show all the relevant working. No further marks are available.

Mark for (a) = 1 out of 5

Example Candidate Response – middle, continued



(b) Find the area of the shaded region, giving your answer in the form kr^2, where k is a constant correct to 2 decimal places. [4]

$2r^2$

~~$\frac{1}{2}(2r\sin 43)$~~

$\frac{1}{2}bc\sin A$

$\frac{1}{2}\times r^2\times\sin 35.9$ ~~=~~

$= 0.499r^2$

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\times 1.5$

$= \frac{1.5r^2}{2}$

$= 0.75r^2$

$0.75r^2 - 0.499r^2$

$r(2r\sin 42.97)$

$2r^2\sin 42.97$

$1.363r^2$

$1.363r^2 - 0.75r^2 + 0.499r^2$

$1.862r^2 - 0.75r^2$

$1.112r^2$

$2r^2 + 1.112r^2$

$\underline{\underline{3.11r^2}}$

③

Examiner comments

③ The candidate shows a completely correct solution. The candidate works to sufficient accuracy so that the final answer is correct to 3 significant figures. Four marks are awarded.

Mark for (b) = 4 out of 4

Total mark awarded = 5 out of 9

How the candidate could have improved their answer

(a) The candidate needed to ensure that any unknown quantities they introduced into their solution were defined.

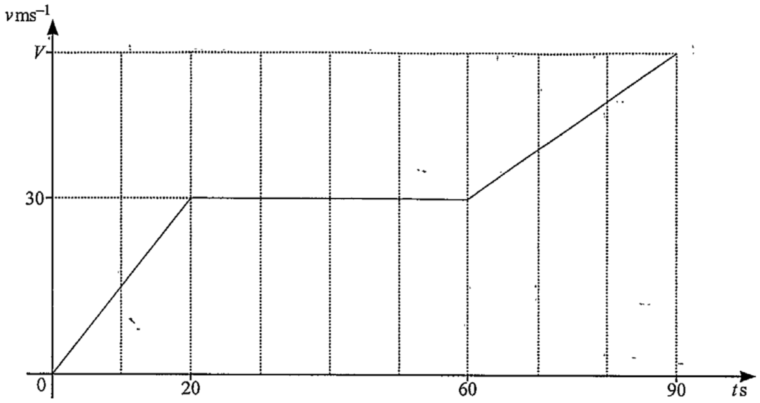
Common mistakes candidates made in this question

- **(a)** The most common candidate error was not giving enough detail when showing the given result. Unknown quantities introduced by candidates were often not defined.
- **(b)** Candidates commonly lacked a clear plan. Many also did not work to the required level of accuracy.

Question 12

Example Candidate Response – high

12 (a)



The diagram shows the velocity–time graph of a particle P that travels 2775 m in 90 s, reaching a final velocity of V ms^{-1}.

(i) Find the value of V. [3]

$\frac{1}{2}\times 20 \times 30 + 40 \times 30 \times \frac{1}{2}(30+v) \times 30$

$300 + 1200 + 15(30+v)$

$1500 + 15(30+v) = 2775$

$15(30+v) = 2775 - 1500$

$15(30+v) = 1275$

$30 + v = 85$

$v = 85 - 30$

$\underline{v = 55}$

①

(ii) Write down the acceleration of P when $t = 40$. [1]

~~45~~ ~~ms^{-2}~~ $\frac{5}{6}\ \text{ms}^{-2}$ ②

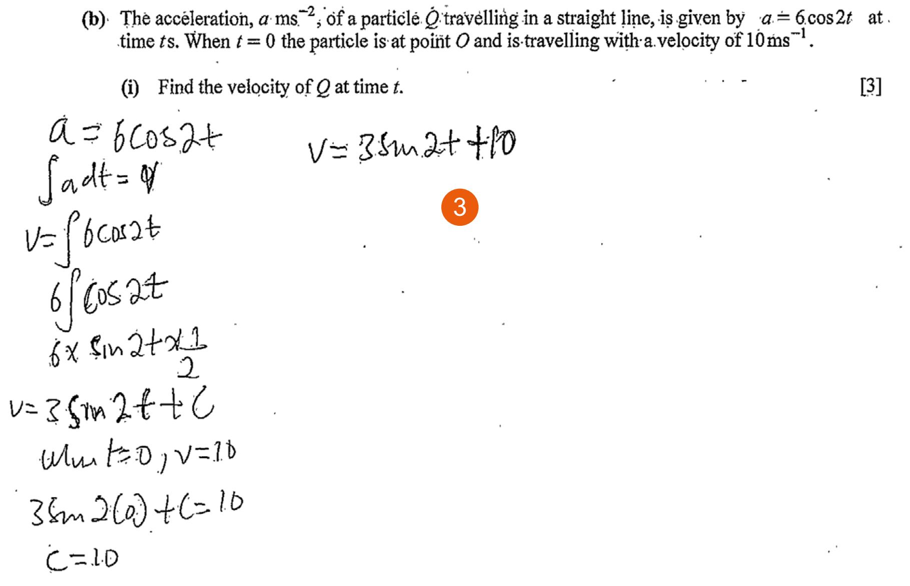
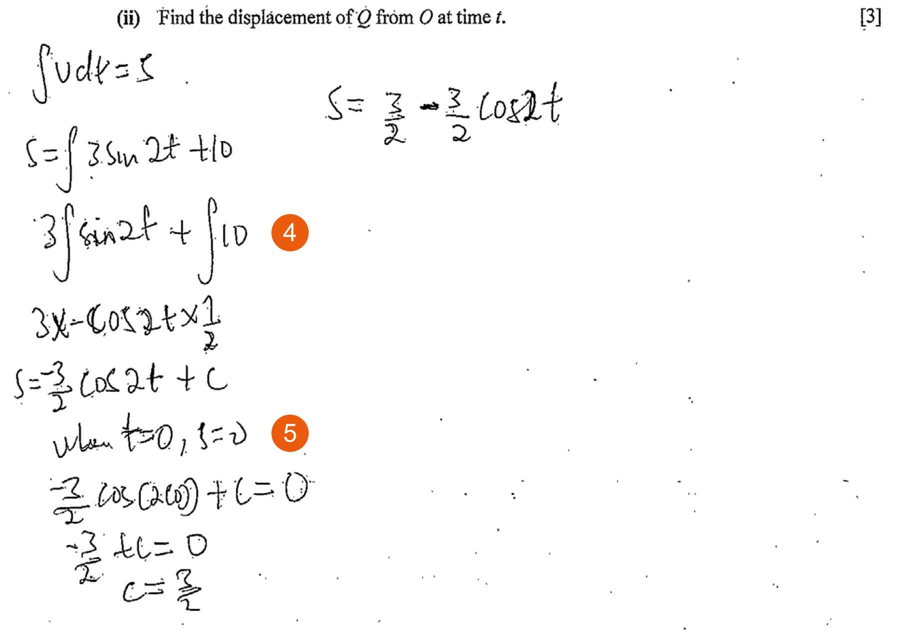
Examiner comments

① The candidate presents a completely correct solution, clearly showing the areas of the triangle, the rectangle and the trapezium. They are awarded full marks.

Mark for (a)(i) = 3 out of 3

② The candidate does not calculate the gradient of the line when $t = 40$.

Mark for (a)(ii) = 0 out of 1

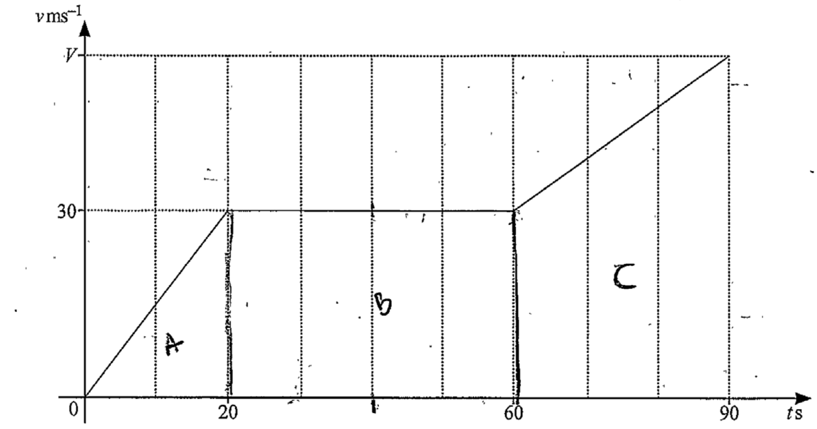
Example Candidate Response – high, continued	Examiner comments
 **(b)** The acceleration, a ms^{-2}, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When $t = 0$ the particle is at point O and is travelling with a velocity of 10 ms^{-1}. **(i)** Find the velocity of Q at time t. [3] $a = 6\cos 2t$ $\int a\,dt = v$ $v = \int 6\cos 2t$ $6\int \cos 2t$ $6 \times \sin 2t \times \frac{1}{2}$ $v = 3\sin 2t + C$ when $t = 0$, $v = 10$ $3\sin 2(0) + C = 10$ $C = 10$ $v = 3\sin 2t + 10$ ③	③ The candidate produces a correct solution with all working clearly shown. Mark for (b)(i) = 3 out of 3
 **(ii)** Find the displacement of Q from O at time t. [3] $\int v\,dt = s$ $s = \int 3\sin 2t + 10$ $3\int \sin 2t + \int 10$ ④ $3 \times -\cos 2t \times \frac{1}{2}$ $s = -\frac{3}{2}\cos 2t + C$ when $t = 0$, $s = 0$ ⑤ $-\frac{3}{2}\cos(2(0)) + C = 0$ $-\frac{3}{2} + C = 0$ $C = \frac{3}{2}$ $s = \frac{3}{2} - \frac{3}{2}\cos 2t$	④ The candidate attempts to integrate their expression for the velocity and this is awarded a method mark. ⑤ The candidate uses a correct method to find the value of the arbitrary constant. This is awarded one mark, but the final answer is incorrect. Mark for (b)(ii) = 2 out of 3 **Total mark awarded = 8 out of 10**

How the candidate could have improved their answer

- **(a)(i)** The candidate needed to realise that the gradient of the line was needed.
- **(b)(ii)** The candidate should not have omitted integration of the constant term.

Example Candidate Response – middle

12 (a)



The diagram shows the velocity–time graph of a particle P that travels 2775 m in 90 s, reaching a final velocity of V ms^{-1}.

(i) Find the value of V. [3]

Area A + Area B + Area C = 2775

Area A = 20 × 30 = 600

Area B = 40 × 30 = 1200

Area C = $\frac{1}{\cancel{2}}(30+V)\cancel{30}$

= $15(30+V)$

$600+1200+15(30+V) = 2775$ ①

$15(30+V) = 975$

$30+V = 65$

$V = 35$

(ii) Write down the acceleration of P when $t = 40$. [1]

0 ②

Examiner comments

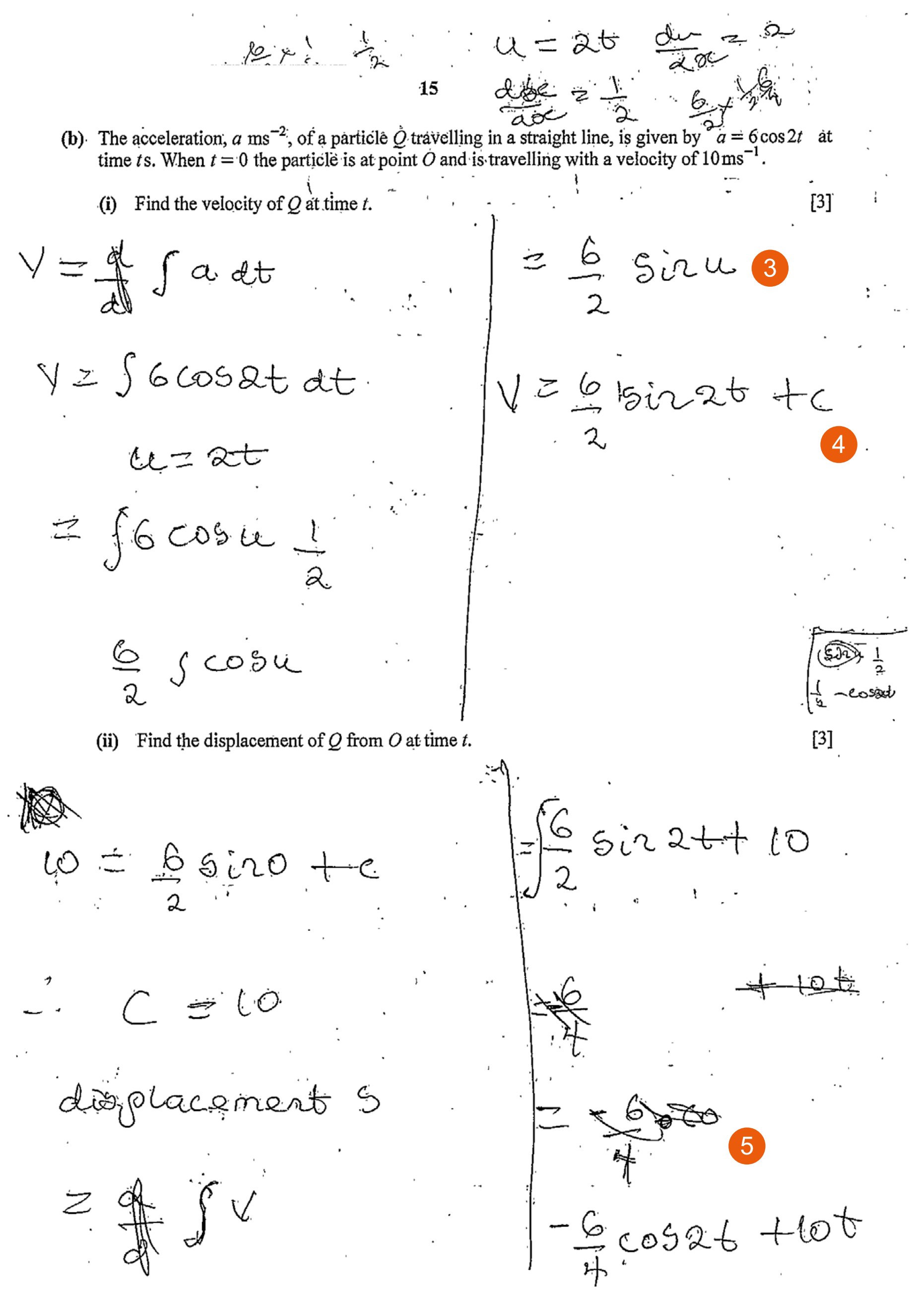
① The candidate attempts to calculate the area under the graph and equate it to 2775. There is an error in the calculation of the area the candidate marks as A, but both method marks are still awarded.

Mark for (a)(i) = 2 out of 3

② The candidate gives the correct acceleration.

Mark for (a)(ii) = 1 out of 1

Example Candidate Response – middle, continued



$\frac{1}{2}$ $u = 2t$ $\frac{du}{dx} = 2$

15 $\frac{dt}{dx} = \frac{1}{2}$ $\frac{6}{2}$

(b) The acceleration, a ms^{-2}, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When $t = 0$ the particle is at point O and is travelling with a velocity of 10 ms^{-1}.

(i) Find the velocity of Q at time t. [3]

$V = \cancel{\frac{d}{dt}} \int a\,dt$

$V = \int 6\cos 2t\,dt$

$u = 2t$

$= \int 6\cos u \frac{1}{2}$

$\frac{6}{2} \int \cos u$

$= \frac{6}{2}\sin u$ ③

$V = \frac{6}{2}\sin 2t + c$ ④

$\sin$ $\frac{1}{2}$

$\frac{1}{2}$ $-\cos 2t$

(ii) Find the displacement of Q from O at time t. [3]

$10 = \frac{6}{2}\sin 0 + c$

$\therefore C = 10$

displacement S

$= \cancel{\frac{d}{dt}} \int V$

$= \int \frac{6}{2}\sin 2t + 10$

$\cancel{\frac{6}{4}}$ ~~+10t~~

$= \cancel{\frac{6\cos}{4}}$ ⑤

$-\frac{6}{4}\cos 2t + 10t$

Examiner comments

③ The candidate integrates the trigonometric term correctly. Even though the coefficient is not simplified at this point, the method mark is still awarded.

④ No attempt is made by the candidate to find the value of the arbitrary constant in this part of the question. Although the value of the arbitrary constant is found in (ii), the mark is not awarded retrospectively. The expression for the displacement is never written out.

Mark for (b)(i) = 1 out of 3

⑤ The candidate is awarded a method mark for integrating the expression, but does not consider the arbitrary constant so no further marks are awarded.

Mark for (b)(ii) = 1 out of 3

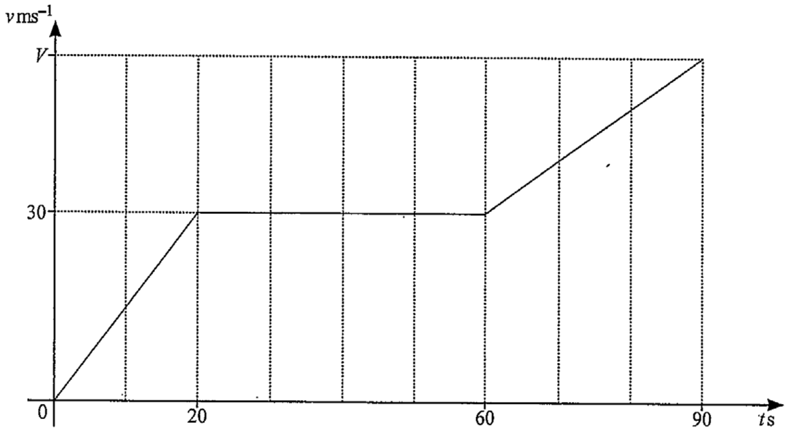
Total mark awarded = 5 out of 10

How the candidate could have improved their answer

- **(a)(i)** Checking the working may have alerted to the candidate that there was an error.
- **(b)(i)** The candidate needed to calculate the arbitrary constant in the correct part of the question and write out the expression for the velocity, which would have possibly added two further marks.
- **(b)(ii)** The candidate should have considered another arbitrary constant.

Example Candidate Response – low

12 (a)



The diagram shows the velocity–time graph of a particle P that travels 2775 m in 90 s, reaching a final velocity of V ms^{-1}.

(i) Find the value of V. [3]

distance travelled = area under graph

$2775 = \frac{1}{2}(30 + V)30$

$2775 = 15(30 + V)$

$2775 = 450 + 15V$ ①

$\frac{15V}{15} = \frac{2325}{15}$

$V = 155 \text{ ms}^{-1}$

(ii) Write down the acceleration of P when $t = 40$. [1]

$a = \frac{v - u}{40}$

$a = \frac{30 - 30}{40}$ ②

$a = 0 \text{ ms}^{-2}$

Examiner comments

① The candidate does not consider the total area under the graph so no marks can be awarded.

Mark for (a)(i) = 0 out of 3

② The candidate gives a correct solution. One mark is awarded.

Mark for (a)(ii) = 1 out of 1

Example Candidate Response – low, continued	**Examiner comments**
(b) The acceleration, a ms^{-2}, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When $t = 0$ the particle is at point O and is travelling with a velocity of 10 ms^{-1}. S V a **(i)** Find the velocity of Q at time t. [3] $v = \int a$ $= \int 6 \cos 2t \ dx$ $\underline{\underline{v = 3 \sin 2t}}$ ③	③ A method mark is awarded as the candidate integrates the trigonometric term correctly, but there is no consideration of the arbitrary constant so no further marks are awarded. Mark for (b)(i) = 1 out of 3
(ii) Find the displacement of Q from O at time t. [3] $S = \int v$ $= \int 3 \sin 2t$ $\underline{\underline{S = \int - \frac{3}{2} \cos 2t}}$ ④	④ A method mark is awarded as the candidate integrates the trigonometric term correctly, but there is no consideration of the arbitrary constant so no further marks are awarded. Mark for (b)(ii) = 1 out of 3
	Total mark awarded = 3 out of 10

How the candidate could have improved their answer

- **(a)(i)** The candidate needed to consider the total area under the graph. They stated 'distance = area under graph', but they only calculated the area of the trapezium on the right-hand side.
- **(b)** In both parts the candidate needed to consider the arbitrary constant and attempt to find it in each.

Common mistakes candidates made in this question

- **(a)(i)** Calculation errors were common, as were errors when candidates attempted to use the equations of linear motion even though they are not on the syllabus.
- Many candidates did not realise that they needed to find the gradient of the straight line between $t = 20$ and $t = 60$. Of those that did, many still did not obtain zero as they considered there to be a difference in the v-coordinates.
- **(b)** Many candidates were not able to integrate the trigonometric term correctly, with errors in the coefficient of the term being common. Some candidates mistakenly differentiated. The consideration and calculation of the arbitrary constant was missing in many solutions.

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: info@cambridgeinternational.org   www.cambridgeinternational.org